Numerical Methods

Newton's Method for One -Dimensional Optimization -Theory

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Old Dominion University

USF University of South Florida

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Newton's Method-Overview

- Open search method
- A good initial estimate of the solution is required
- The objective function must be twice differentiable
- Unlike Golden Section Search method
  - Lower and upper search boundaries are not required (open vs. bracketing)
  - May not converge to the optimal solution

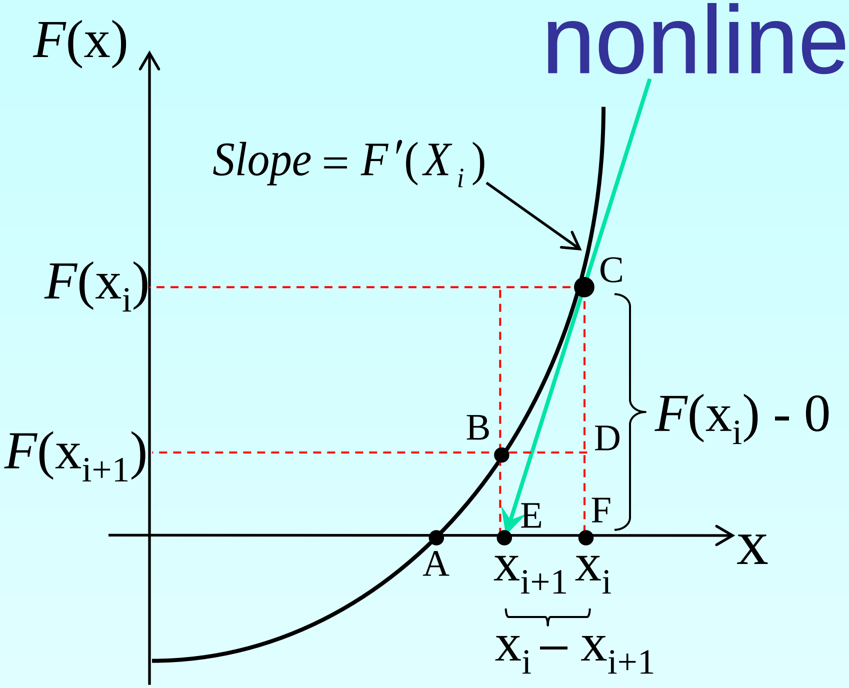
Newton's Method-How it works

- The derivative of the function $f_{Opt.}(x)$,Nonlinear root finding equation $f^{'}(x) = 0 = F(x)$ at the function's maximum and minimum.
- The minima and the maxima can be found by applying the Newton-Raphson method to the derivative, essentially obtaining

$$x_{i+1} = x_i - \frac{f^{'}(x_i)}{f^{"}(x_i)}$$

- Next slide will explain how to get/derive the above formula

Newton's Method-To find root of a nonlinear equation



Slope @ pt. C $\approx \dfrac{F(X_i) - F(X_{i+1})}{X_i - X_{i+1}}$

We "wish" that in the next iteration x_{i+1} will be the root, or $F(X_{i+1}) = 0$.

<u>Thus</u>:

Slope @ pt. C = $\dfrac{F(X_i) - 0}{X_i - X_{i+1}}$

Or $F'(X_i) = \dfrac{F(X_i)}{X_i - X_{i+1}}$

Hence: $$X_{i+1} = X_i - \frac{F(X_i)}{F'(X_I)}$$ N-R Equation

Newton's Method-To find root of a nonlineat equation

- If $F(x) \equiv f'(x)$,then $X_{i+1} = X_i - \frac{f'(X_i)}{f''(X_i)}$.
- For Multi-variable case ,then N-R method becomes

$$\vec{X}_{i+1} = \vec{X}_i - [f''(X_i)]^{-1} \times \nabla\vec{f}(X_i)$$

Newton's Method-Algorithm

Initialization: Determine a reasonably good estimate for the maxima or the minima of the function $f(x)$.

Step **1.** Determine $f'(x)$ and $f''(x)$.

Step **2.** Substitute x_i (initial estimate x_0 for the first iteration) and $f'(x)$ into $f''(x)$

$$x_{i+1} = x_i - \frac{f'(x_i)}{f''(x_i)}$$

to determine x_{i+1} and the function value in iteration *i*.

Step **3.** If the value of the first derivative of the function is zero then you have reached the optimum (maxima or minima). Otherwise, repeat Step 2 with the new value of x_i

THE END

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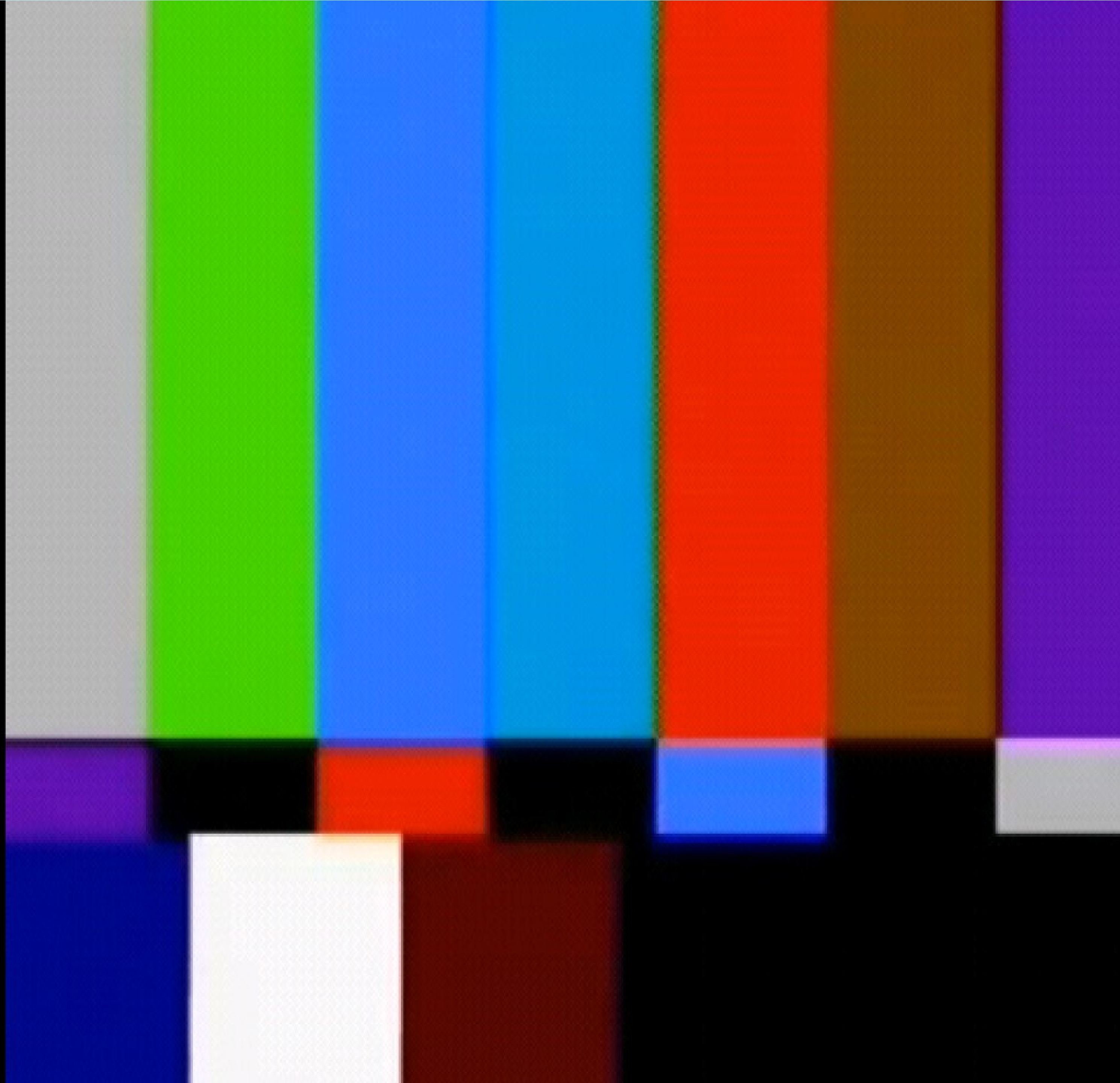

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Newton's Method for One - Dimensional Optimization - Example

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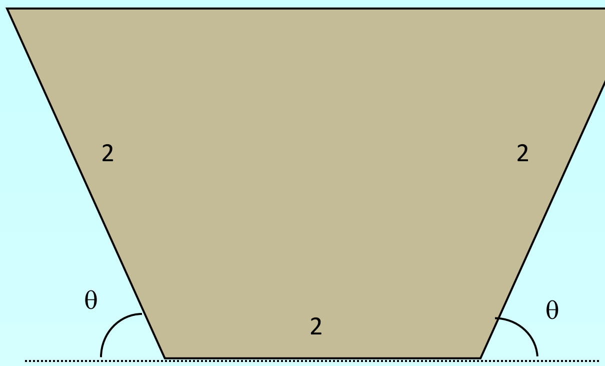
Example



The cross-sectional area A of a gutter with equal base and edge length of 2 is given by

$$A = 4\sin\theta(1+\cos\theta)$$

Find the angle θ which maximizes the cross-sectional area of the gutter.

Solution

The function to be maximized is $f(\theta) = 4\sin\theta(1+\cos\theta)$

$$f'(\theta) = 4(\cos\theta + \cos^2\theta - \sin^2\theta)$$

$$f''(\theta) = -4\sin\theta(1+4\cos\theta)$$

Iteration 1: Use $\theta_0 = \frac{\pi}{4} = 0.7854 rad$ as the initial estimate of the solution

$$\theta_1 = \frac{\pi}{4} - \frac{4(\cos\frac{\pi}{4} + \cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{4})}{-4\sin\frac{\pi}{4}(1+4\cos\frac{\pi}{4})} = 1.0466$$

$$f(1.0466) = 5.196151$$

Solution Cont.

Iteration 2:

$$\theta_2 = 1.0466 - \frac{4(\cos 1.0466 + \cos^2 1.0466 - \sin^2 1.0466)}{-4\sin 1.0466(1 + 4\cos 1.0466)} = 1.0472$$

Summary of iterations

Iteration	θ	$f'(\theta)$	$f''(\theta)$	θ *estimate*	$f(\theta)$
1	0.7854	2.8284	-10.8284	1.0466	5.1962
2	1.0466	0.0062	-10.3959	1.0472	5.1962
3	1.0472	1.06E-06	-10.3923	1.0472	5.1962
4	1.0472	3.06E-14	-10.3923	1.0472	5.1962
5	1.0472	1.3322E-15	-10.3923	1.0472	5.1962

Remember that the actual solution to the problem is at 60 degrees or 1.0472 radians.

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