# Chapter 09.03
# Multidimensional Direct Search Method

*After reading this chapter, you should be able to:*
1. *Understand the fundamentals of the multidimensional direct search methods*
2. *Understand how the coordinate cycling search method works*
3. *Solve multi-dimensional optimization problems using the coordinate cycling search method*

**Optimization Techniques**

Methods for finding optimal solutions in multidimensional spaces are not too different than their cousins used in finding optimal solutions in a single dimension. The trade-off between general applicability versus computational complexity also exists in multidimensional optimization. The multidimensional direct search methods we will cover in this chapter, like the one-dimensional Golden Section Search method (http://numericalmethods.eng.usf.edu/topics/opt_goldensearch.html), does not require a differentiable function. These methods are sometimes referred to as Zeroth Order Algorithms because it is not required to differentiate the optimization function.

Probably the most obvious solution to an optimization problem in multidimensional space is to systematically evaluate every possible solution and select the maximum or the minimum depending on our objective. This is a very generally applicable approach and may even be useful if the solution space is relatively small. However, as the dimensions of the problem space, (number of independent variables), increase, the computational complexity of this solution approach quickly becomes unmanageable. Therefore, we are interested in methods that intelligently search through the solution space to find an optimal solution without enumerating all possible solutions.

It is important to note that some of the popular optimization techniques you may have heard of such as simulated annealing, tabu search, neural networks and genetic algorithms all fall under this family of optimization techniques.

**What is the Coordinate Cycling Search Method and How Does it Work?**

The coordinate cycling search method, starts from an initial point and looks for an optimal solution along each coordinate direction iteratively. For example, using a function $f(x, y)$ with two independent variables $x$ and $y$, and starting at point $(x_0, y_0)$; the first iteration will move along direction (1, 0), until an optimal solution is found for the function $f(x, y_0)$. The next search involves searching along the direction (0,1) to determine the optimal value for the function $f(x_1, y)$ where $x_1$ is the solution found in the previous search. Once searches in all directions are completed, the process is repeated in the next cycle. The search will

continue until convergence occurs or a predetermined error limit is met. The search along each coordinate direction can be conducted by using anyone of the one-dimensional search techniques previously covered. A visual representation of how the search converges is shown below in Figure1.
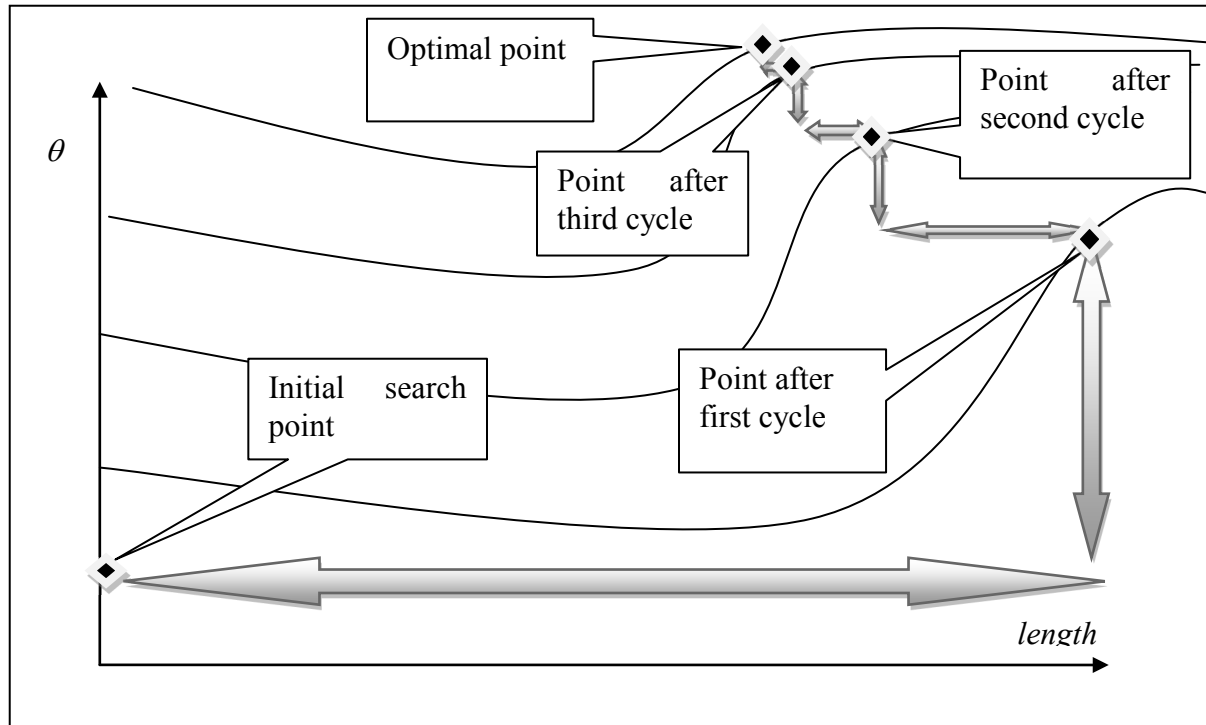


**Figure 1  Visual Representation of a Multidimensional Search**

**Example 1**

Consider Figure 2 below. The cross-sectional area $A$ of a gutter with a base length $b$ and an edge length of $l$ is given by

$$A = \frac{1}{2}(b + b + 2l\cos\theta)l\sin\theta$$

Assuming that the width of the material to be bent into the gutter shape is 6 inches, find the angle $\theta$ and edge length $l$ which maximizes the cross-sectional area of the gutter.
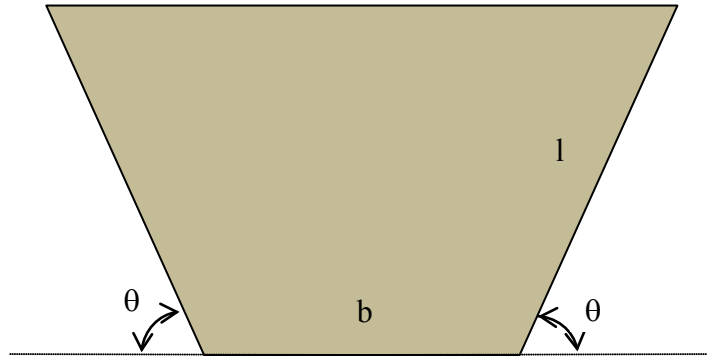
**Figure 2** Cross section of the gutter

**Solution**

Recognizing that the base length b can be expressed as $b = 6 - 2l$, we can re-write the area function to be optimized in terms of two independent variables giving

$$f(l,\theta) = (6 - 2l + l\cos\theta)l\sin\theta .$$

Let us consider an initial point $(0,\frac{\pi}{6})$. We will use the Golden Section Search method to determine the optimal solution along direction $(1,0)$ namely the independent variable corresponding to the length of each side. To use the Golden Section Search method, we will use 0 and 3 as the lower and upper bounds, respectively for the search region (Can you determine why we are using 3 as the upper bound?) and look for the optimal solution of the function $f(l,0.52360)$ with a convergence limit of $\varepsilon < 0.05$. Table 1 below shows the iterations of the Golden Section Search method in the $(1,0)$ direction. The maximum area of $3.6964\ in^2$ is obtained at point $(2.6459,0.52360)$ .

**Table 1** Summary of the Golden Section Search iterations along direction $(1,0)$ for Example 1. Here $\theta = 0.52360$ and $f(x_i) = (6 - 2l + l\cos(0.52360))l\sin(0.52360))$

| Iteration | $x_l$ | $x_u$ | $x_1$ | $x_2$ | $f(x_1)$ | $f(x_2)$ | $\varepsilon$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.0000 | 3.0000 | 1.8541 | 1.1459 | 3.6143 | 2.6941 | 3.0000 |
| 2 | 1.1459 | 3.0000 | 2.2918 | 1.8541 | 3.8985 | 3.6143 | 1.8541 |
| 3 | 1.8541 | 3.0000 | 2.5623 | 2.2918 | 3.9655 | 3.8985 | 1.1459 |
| 4 | 2.2918 | 3.0000 | 2.7295 | 2.5623 | 3.9654 | 3.9655 | 0.7082 |
| 5 | 2.2918 | 2.7295 | 2.5623 | 2.4590 | 3.9655 | 3.9497 | 0.4377 |
| 6 | 2.4590 | 2.7295 | 2.6262 | 2.5623 | 3.9692 | 3.9655 | 0.2705 |
| 7 | 2.5623 | 2.7295 | 2.6656 | 2.6262 | 3.9692 | 3.9692 | 0.1672 |
| 8 | 2.5623 | 2.6656 | 2.6262 | 2.6018 | 3.9692 | 3.9683 | 0.1033 |
| 9 | 2.6018 | 2.6656 | 2.6412 | 2.6262 | 3.9694 | 3.9692 | 0.0639 |
| 10 | 2.6262 | 2.6656 | 2.6506 | 2.6412 | 3.9694 | 3.9694 | 0.0395 |

To search along the (0,1) direction corresponding to the angle $\theta$, we again use the Golden Section Search method, but in this case using the function $f(2.6459,\theta)$. Table 2 below shows the iterations of the Golden Section Search method in the (0,1) direction. Note that at the new optimal point $(2.6459,0.8668)$, the approximation of the maximum area is improved to $4.8823\,in^2$.

**Table 2** Summary of the Golden Section Search iterations along direction (0,1). Here $l = 2.6459$ and $f(x_i) = (6 - 2\times 2.6549 + 2.6549 \times \cos\theta) \times 2.6549 \times \sin\theta$

| Iteration | $x_l$ | $x_u$ | $x_1$ | $x_2$ | $f(x_1)$ | $f(x_2)$ | $\varepsilon$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.0000 | 1.5714 | 0.9712 | 0.6002 | 4.8084 | 4.3215 | 1.5714 |
| 2 | 0.6002 | 1.5714 | 1.2005 | 0.9712 | 4.1088 | 4.8084 | 0.9712 |
| 3 | 0.6002 | 1.2005 | 0.9712 | 0.8295 | 4.8084 | 4.8689 | 0.6002 |
| 4 | 0.6002 | 0.9712 | 0.8295 | 0.7419 | 4.8689 | 4.7533 | 0.3710 |
| 5 | 0.7419 | 0.9712 | 0.8836 | 0.8295 | 4.8816 | 4.8689 | 0.2293 |
| 6 | 0.8295 | 0.9712 | 0.9171 | 0.8836 | 4.8672 | 4.8816 | 0.1417 |
| 7 | 0.8295 | 0.9171 | 0.8836 | 0.8630 | 4.8816 | 4.8820 | 0.0876 |
| 8 | 0.8295 | 0.8836 | 0.8630 | 0.8502 | 4.8820 | 4.8790 | 0.0541 |
| 9 | 0.8502 | 0.8836 | 0.8708 | 0.8630 | 4.8826 | 4.8820 | 0.0334 |

After completing these two iterations, we use the optimal point to start a new cycle. Table 3 shows the first set of iterations for the second cycle.

**Table 3** Summary of the Golden Section Search iterations along direction (1,0)

| Iteration | $x_l$ | $x_u$ | $x_1$ | $x_2$ | $f(x_1)$ | $f(x_2)$ | $\varepsilon$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.0000 | 3.0000 | 1.8541 | 1.1459 | 4.9354 | 3.8871 | 3.0000 |
| 2 | 1.1459 | 3.0000 | 2.2918 | 1.8541 | 5.0660 | 4.9354 | 1.8541 |
| 3 | 1.8541 | 3.0000 | 2.5623 | 2.2918 | 4.9491 | 5.0660 | 1.1459 |
| 4 | 1.8541 | 2.5623 | 2.2918 | 2.1246 | 5.0660 | 5.0627 | 0.7082 |
| 5 | 2.1246 | 2.5623 | 2.3951 | 2.2918 | 5.0391 | 5.0660 | 0.4377 |
| 6 | 2.1246 | 2.3951 | 2.2918 | 2.2279 | 5.0660 | 5.0715 | 0.2705 |
| 7 | 2.1246 | 2.2918 | 2.2279 | 2.1885 | 5.0715 | 5.0708 | 0.1672 |
| 8 | 2.1885 | 2.2918 | 2.2523 | 2.2279 | 5.0704 | 5.0715 | 0.1033 |
| 9 | 2.1885 | 2.2523 | 2.2279 | 2.2129 | 5.0715 | 5.0716 | 0.0639 |
| 10 | 2.1885 | 2.2279 | 2.2129 | 2.2035 | 5.0716 | 5.0714 | 0.0395 |

Here $\theta = 0.8668$ and $f(x_i) = (6 - 2l + l\cos(0.8668)l\sin(0.8668))$. Note that we still use the initial intervals chosen for $x_l$ and $x_u$ values throughout the cycles.

Since this is a two-dimensional search problem, the two searches along the two dimensions completes the first cycle. In the next cycle, we return to the first dimension for which we conducted a search, namely $l$, and start the second cycle with a search along this dimension.

Namely, look for the optimal solution of the function $f(l,0.8668)$. Each cycle consists of enough iterations to satisfy the predetermined convergence limit.

After the fifth cycle, the optimal solution of $(2.0016,1.0420)$ with an area of $5.1960\,in^2$ is obtained. The optimal solution to the problem happens at exactly $60^0$ which is $1.0472$ radians, having an edge and base length of $2\,in$. The area of the gutter at this point is $5.1962\,in^2$. Therefore folding the sheet metal in such a way that the base is $2\,in$ and the sides are $2\,in$ at an angle of $60^0$ maximizes the area of the gutter.

| OPTIMIZATION | |
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