Introduction to Fourier Series

Part: Introduction to Fourier Series

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Lecture # 1

Chapter 11.01: Introduction to Fourier Series

Major: All Engineering Majors

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Numerical Methods for STEM undergraduates

Background

The following relationships can be readily established

$$\int_0^T \sin(kw_0 t)\,dt = \int_0^T \cos(kw_0 t)\,dt = 0 \qquad (1)$$

$$\int_0^T \sin^2(kw_0 t)\,dt = \int_0^T \cos^2(kw_0 t)\,dt = \frac{T}{2} \qquad (2)$$

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Background cont.

$$\int_0^T \cos(kw_0t)\sin(gw_0t)dt = 0 \qquad (3)$$

$$\int_0^T \sin(kw_0t)\sin(gw_0t)dt = 0 \qquad (4)$$

$$\int_0^T \cos(kw_0t)\cos(gw_0t)dt = 0 \qquad (5)$$

Background cont.

$$w_0 = 2\pi f \quad (6)$$

$$f = \frac{1}{T} \quad (7)$$

Where f and T represents the frequency in (cycles/time) and period (in seconds) respectively.

A periodic function with a period T should satisfy the following equation:

$$f(t+T) = f(t) \quad (8)$$

Background cont.

Example 1

Let

$$A = \int_0^T \sin(kw_0 t)\,dt \qquad (9)$$

$$= -\left(\frac{1}{kw_0}\right)\left[\cos(kw_0 t)\right]_0^T$$

Background cont.

$$A = \left(\frac{-1}{kw_0}\right)\left[\cos(kw_0T) - \cos(0)\right] \qquad (10)$$

$$= \left(\frac{-1}{kw_0}\right)\left[\cos(k2\pi) - 1\right]$$

$$= 0$$

Background cont.

Example 2

Let $$B = \int_0^T \sin^2(kw_0 t)\,dt \qquad (11)$$

Recall

$$\sin^2(\alpha) = \frac{1-\cos(2\alpha)}{2} \qquad (12)$$

$$B = \int_o^T \left[\frac{1}{2} - \frac{1}{2}\cos(2kw_0 t)\right] dt \qquad (13)$$

Background cont.

$$=\left[\left(\frac{1}{2}\right)t-\left(\frac{1}{2}\right)\left(\frac{1}{2kw_0}\right)\sin(2kw_0t)\right]_0^T$$

$$B=\left[\frac{T}{2}-\frac{1}{4kw_0}\sin(2kw_0T)\right]-[0] \qquad (14)$$

Background cont.

$$= \frac{T}{2} - \left(\frac{1}{4kw_0}\right)\sin(2k * 2\pi)$$

$$= \frac{T}{2}$$

Example 3

Let

$$C = \int_0^T \sin(gw_0 t)\cos(kw_0 t)dt \qquad (15)$$

Background cont.

Recall that

$$\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\sin(\beta)\cos(\alpha) \qquad (16)$$

$$C=\int_0^T\left[\sin\left[\left(g+k\right)w_0t\right]-\sin(kw_0t)\cos(gw_0t)\right]dt \qquad (17)$$

Background cont.

$$= \int_0^T \sin\left[(g+k)w_0 t\right]dt - \int_0^T \sin(kw_0 t)\cos(gw_0 t)dt \quad (18)$$

$$C = 0 - \int_0^T \sin(kw_0 t)\cos(gw_0 t)dt \quad (19)$$

Adding Equations (15), (19),

$$2C = \int_0^T \sin(gw_0 t)\cos(kw_0 t)dt - \int_0^T \sin(kw_0 t)\cos(gw_0 t)dt$$

$$= \int_0^T \sin\left[\left(gw_0 t\right) - (kw_0 t)\right]dt = \int_0^T \sin\left[(g-k)w_0 t\right]dt \quad (20)$$

Background cont.

$$2C = 0,$$

since the right side of the above equation is zero
Thus,

$$C = \int_{o}^{T} \sin(gw_0 t)\cos(kw_0 t)dt = 0 \qquad (21)$$

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THE END

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This material is based upon work supported by the National Science Foundation under Grant # 0717624. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.