

Introduction to Fourier Series

Part: Introduction to Fourier Series

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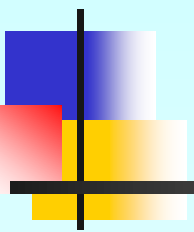
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Lecture # 1

Chapter 11.01: Introduction to Fourier Series

Major: All Engineering Majors

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Numerical Methods for STEM undergraduates



Background

The following relationships can be readily established

$$\int_0^T \sin(k\omega_0 t) dt = \int_0^T \cos(k\omega_0 t) dt = 0 \quad (1)$$

$$\int_0^T \sin^2(k\omega_0 t) dt = \int_0^T \cos^2(k\omega_0 t) dt = \frac{T}{2} \quad (2)$$



Background cont.

$$\int_0^T \cos(kw_0 t) \sin(gw_0 t) dt = 0 \quad (3)$$

$$\int_0^T \sin(kw_0 t) \sin(gw_0 t) dt = 0 \quad (4)$$

$$\int_0^T \cos(kw_0 t) \cos(gw_0 t) dt = 0 \quad (5)$$



Background cont.

$$\omega_0 = 2\pi f \quad (6)$$

$$f = \frac{1}{T} \quad (7)$$

Where f and T represents the frequency in (cycles/time) and period (in seconds) respectively.

A periodic function with a period T should satisfy the following equation:

$$f(t + T) = f(t) \quad (8)$$



Background cont.

Example 1

Let

$$A = \int_0^T \sin(kw_0 t) dt \quad (9)$$

$$= -\left(\frac{1}{kw_0}\right) [\cos(kw_0 t)]_0^T$$



Background cont.

$$\begin{aligned} A &= \left(\frac{-1}{k\omega_0} \right) [\cos(k\omega_0 T) - \cos(0)] & (10) \\ &= \left(\frac{-1}{k\omega_0} \right) [\cos(k2\pi) - 1] \\ &= 0 \end{aligned}$$



Background cont.

Example 2

$$\text{Let } B = \int_0^T \sin^2(k\omega_0 t) dt \quad (11)$$

Recall

$$\sin^2(\alpha) = \frac{1 - \cos(2\alpha)}{2} \quad (12)$$

$$B = \int_0^T \left[\frac{1}{2} - \frac{1}{2} \cos(2k\omega_0 t) \right] dt \quad (13)$$



Background cont.

$$= \left[\left(\frac{1}{2} \right) t - \left(\frac{1}{2} \right) \left(\frac{1}{2kw_0} \right) \sin(2kw_0 t) \right]_0^T$$

$$B = \left[\frac{T}{2} - \frac{1}{4kw_0} \sin(2kw_0 T) \right] - [0] \quad (14)$$



Background cont.

$$= \frac{T}{2} - \left(\frac{1}{4k\omega_0} \right) \sin(2k * 2\pi)$$

$$= \frac{T}{2}$$

Example 3

Let

$$C = \int_0^T \sin(g\omega_0 t) \cos(k\omega_0 t) dt \quad (15)$$



Background cont.

Recall that

$$\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \sin(\beta) \cos(\alpha) \quad (16)$$

$$C = \int_0^T [\sin[(g + k)w_0 t] - \sin(kw_0 t) \cos(gw_0 t)] dt \quad (17)$$



Background cont.

$$= \int_0^T \sin[(g + k)w_0 t] dt - \int_0^T \sin(kw_0 t) \cos(gw_0 t) dt \quad (18)$$

$$C = 0 - \int_0^T \sin(kw_0 t) \cos(gw_0 t) dt \quad (19)$$

Adding Equations (15), (19),

$$\begin{aligned} 2C &= \int_0^T \sin(gw_0 t) \cos(kw_0 t) dt - \int_0^T \sin(kw_0 t) \cos(gw_0 t) dt \\ &= \int_0^T \sin[(gw_0 t) - (kw_0 t)] dt = \int_0^T \sin[(g - k)w_0 t] dt \quad (20) \end{aligned}$$



Background cont.

$$2C = 0,$$

since the right side of the above equation is zero
Thus,

$$C = \int_0^T \sin(gw_0 t) \cos(kw_0 t) dt = 0 \quad (21)$$

THE END

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Acknowledgement

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