Binary Representation

Major: All Engineering Majors

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http://numericalmethods.eng.usf.edu

Transforming Numerical Methods Education for STEM Undergraduates

Binary Representation

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How a Decimal Number is Represented

$$257.76 = 2\times10^{2} + 5\times10^{1} + 7\times10^{0} + 7\times10^{-1} + 6\times10^{-2}$$

Base 2

$$(1011.0011)_2 = \left(\begin{array}{l} (1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0) \\ +(0\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3} + 1\times 2^{-4}) \end{array} \right)_{10}$$

$$= 11.1875$$

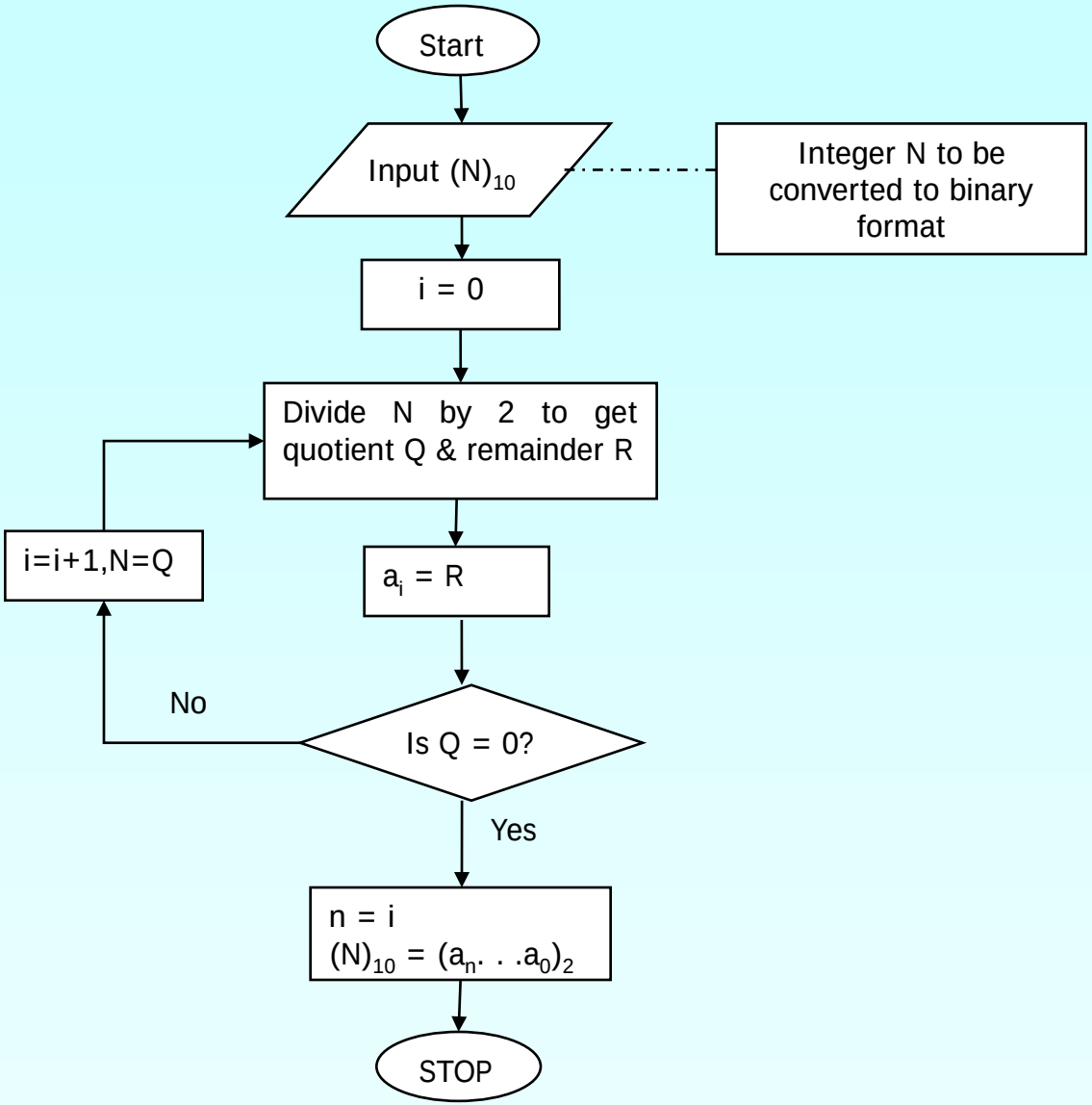
Convert Base 10 Integer to binary representation

Table 1 Converting a base-10 integer to binary representation.

	Quotient	**Remainder**
11/2	5	$1 = a_0$
5/2	2	$1 = a_1$
2/2	1	$0 = a_2$
1/2	0	$1 = a_3$

Hence

$$(11)_{10} = (a_3 a_2 a_1 a_0)_2$$
$$= (1011)_2$$


Start
Input $(N)_{10}$
Integer N to be converted to binary format
i = 0
Divide N by 2 to get quotient Q & remainder R
a_i = R
Is Q = 0?
No
i=i+1,N=Q
Yes
n = i
$(N)_{10} = (a_n . . . a_0)_2$
STOP

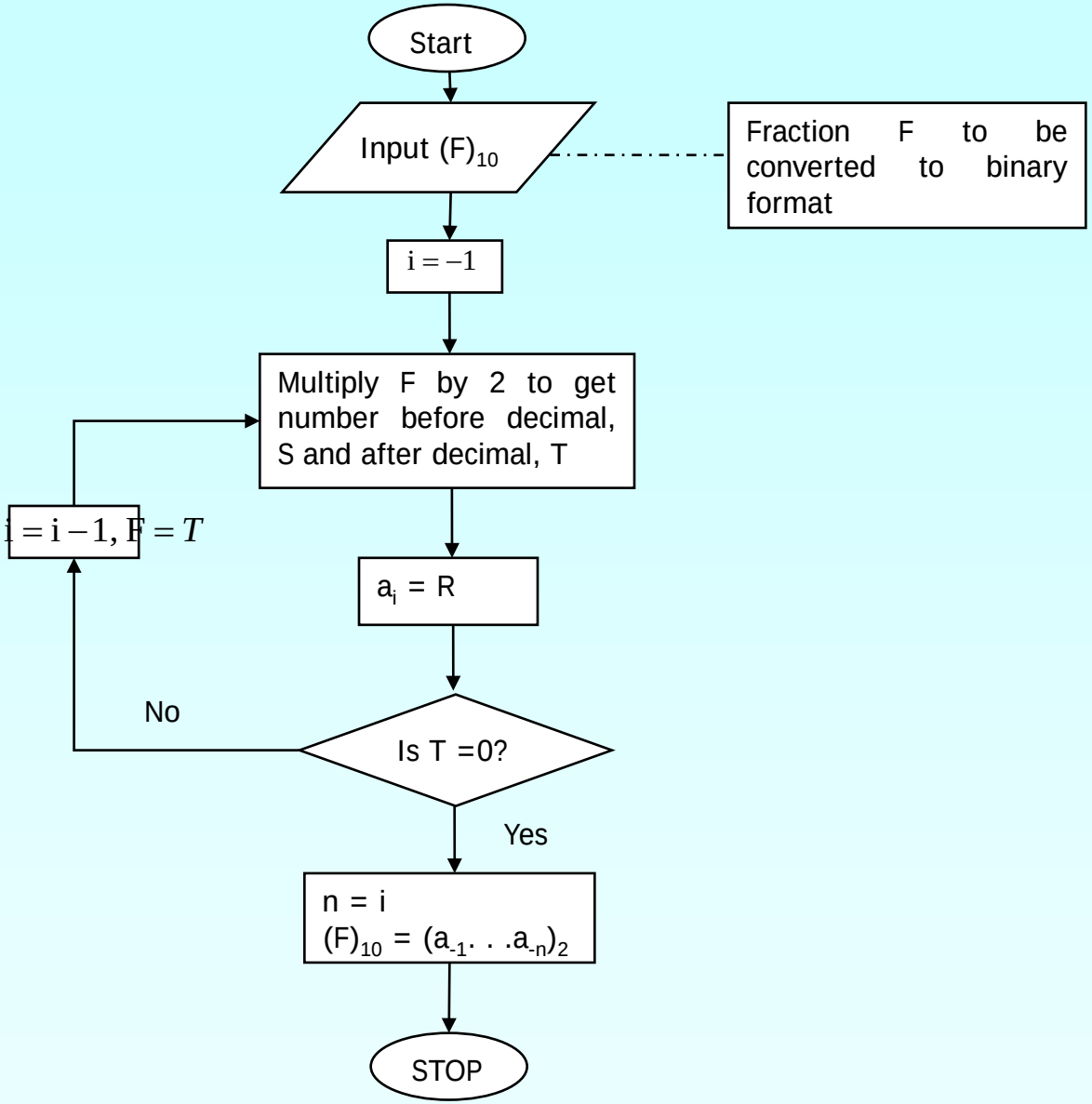
Fractional Decimal Number to Binary

Table 2. Converting a base-10 fraction to binary representation.

	Number	Number after decimal	Number before decimal
0.1875×2	0.375	0.375	$0 = a_{-1}$
0.375×2	0.75	0.75	$0 = a_{-2}$
0.75×2	1.5	0.5	$1 = a_{-3}$
0.5×2	1.0	0.0	$1 = a_{-4}$

Hence

$$(0.1875)_{10} = (a_{-1}a_{-2}a_{-3}a_{-4})_2$$
$$= (0.0011)_2$$


Start
Input $(F)_{10}$
Fraction F to be converted to binary format
$i = -1$
Multiply F by 2 to get number before decimal, S and after decimal, T
$i = i - 1, F = T$
a_i = R
No
Is T =0?
Yes
n = i
$(F)_{10} = (a_{-1} \ldots a_{-n})_2$
STOP

Decimal Number to Binary

$$(11.1875)_{10} = (\ \ ?.?\ \ )_2$$

Since

$$(11)_{10} = (1011)_2$$

and

$$(0.1875)_{10} = (0.0011)_2$$

we have

$$(11.1875)_{10} = (1011.0011)_2$$

All Fractional Decimal Numbers Cannot be Represented Exactly

Table 3. Converting a base-10 fraction to approximate binary representation.

	Number	Number after decimal	Number before Decimal
0.3×2	0.6	0.6	$0 = a_{-1}$
0.6×2	1.2	0.2	$1 = a_{-2}$
0.2×2	0.4	0.4	$0 = a_{-3}$
0.4×2	0.8	0.8	$0 = a_{-4}$
0.8×2	1.6	0.6	$1 = a_{-5}$

$$(0.3)_{10} \approx (a_{-1}a_{-2}a_{-3}a_{-4}a_{-5})_2 = (0.01001)_2 = 0.28125$$

Another Way to Look at Conversion

Convert $(11.1875)_{10}$ to base 2

$$
\begin{aligned}
(11)_{10} &= 2^3 + 3 \\
&= 2^3 + 2^1 + 1 \\
&= 2^3 + 2^1 + 2^0 \\
&= 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0 \\
&= (1011)_2
\end{aligned}
$$

$$
\begin{aligned}
(0.1875)_{10} &= 2^{-3} + 0.0625 \\
&= 2^{-3} + 2^{-4} \\
&= 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} \\
&= (.0011)_2
\end{aligned}
$$

$$
(11.1875)_{10} = (1011.0011)_2
$$

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/binary_representation.html

THE END

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