Chapter 03.00F

Physical Problem for Nonlinear Equations
Industrial Engineering

Problem Statement

You have been recently employed by a start-up computer assembly company called the "MOM AND POP COMPUTER SHOP". As a recent graduate with a bachelor's degree in industrial engineering, you have been asked by the president, to determine the minimum number of computers that the shop will have to sell to make a profit during the first year in business.

Solution

First, it is important to determine the first costs or capital costs, CC, associated with starting the business. The capital costs include such items as:
  - computer assembly and diagnostic equipment,
  - office furniture,
  - workbenches, and
  - initial inventory purchase, to name a few.

It is assumed that the capital costs, CC, associated with the business are

$$CC = \$20,000$$

Second, it is important to determine the repeated costs or operating costs, OC, associated with operating the business for the first year. Operating costs differ from capital costs mainly in that operating costs must be paid repeatedly (every year in this instance), in contrast to capital costs, which involve a one-time payment. Operating costs can be broken down into three major classifications:
  1. indirect costs or fixed costs, FC
  2. direct or variable costs, VC, and
  3. semivariable or regulated costs, RC,

where

$$OC = FC + VC + RC$$

Fixed costs include items that are not generally a function of the production level, such as:
  - building lease,
  - real estate taxes,
  - utilities (DSL or cable modem, basic electric and local phone),

- insurance, and
- marketing, to name a few.

These fixed costs could, however, increase say if the production level reaches a point when a larger building is needed or when more phone lines needs to be installed. It is assumed that the fixed costs, FC, associated with the business are

$$FC = \$15,000$$

Variable costs include items that are a direct function, often linear, of the production level, such as:

- materials (computer components),
- utilities (production electric and long distance phone),
- labor (also includes supervision and payroll charges),
- maintenance, and
- distribution (packaging and shipping), to name a few.

It is assumed that the variable costs, VC, associated with the business are

$$VC = \$625n$$

where n is the number of computers produced. Regulated costs include items that are also a direct function, though often non-linear, of the production level, such as:

- labor (also includes supervision and payroll charges),

For instance, if production levels reach a certain level, another employee may need to be hired. It is assumed that the regulated costs, RC, associated with the business are

$$RC = \$30n^{1.5}$$

Combining the fixed, variable and regulated costs gives the operating cost, that is,

$$CC = FC + VC + RC$$
$$CC = \$15,000 + \$500n + 30n^{1.5}$$

And the total cost, TC, to operate the business for the first year is the capital costs plus the operating costs, that is

$$TC = CC + OC$$
$$TC = \$20,000 + \$15,000 + \$625n + \$30n^{1.5}$$
$$TC - \$35,000 + \$625n + \$30n^{1.5}$$

All of the costs (capital, fixed, variable, regulated, operating, and total) are shown below in Figure 1.

Next, it is important to determine the total sales, TS, associated with operating the business for the first year. Total sales can be broken down into two major classifications:

1. product sales, PS, and
2. discounted sales, DS,

where

$$TS = PS - DS$$

If they sell n computers, their product sales, PS, are

$$PS = \$15000n$$

This represents the $1500 selling price for each computer. It is assumed that the discounted sales, DS, are

$$DS = \$10n^{1.5}$$

which represents a discount as the number of computers sold increases. The total sales is then given by

$$TS = PS - DS$$

$$TS = \$1500n - \$10n^{1.5}$$

All of the sales (product, discounted, and total) revenue are shown below in Figure 2.
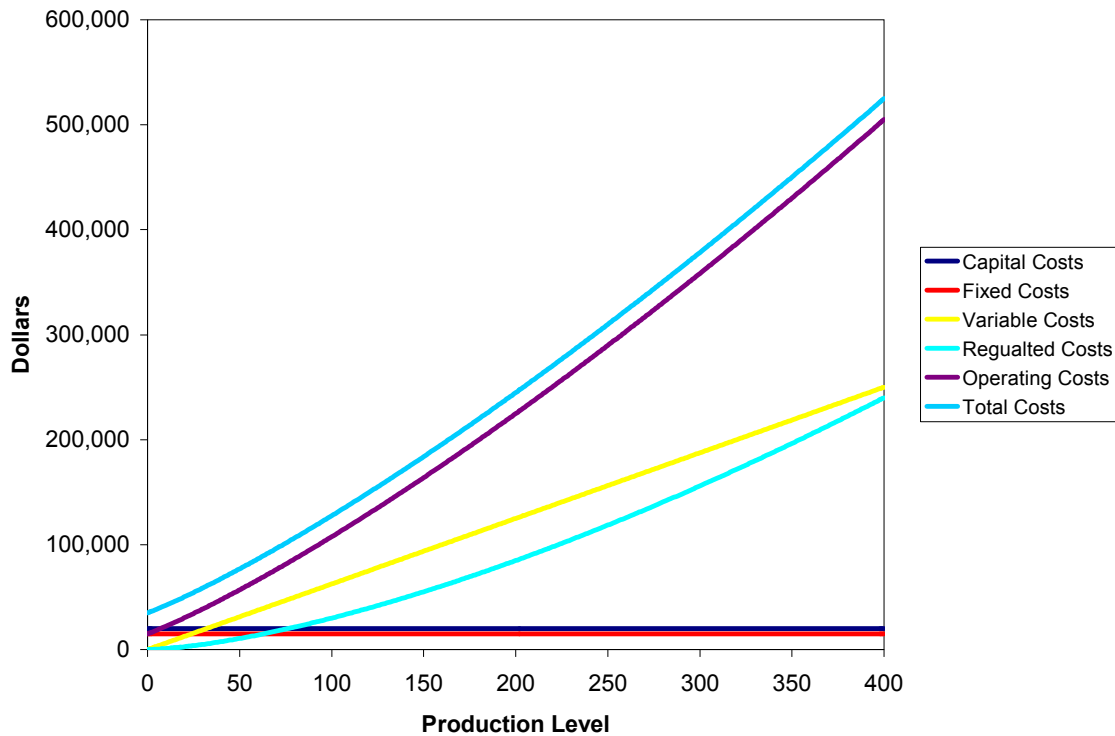

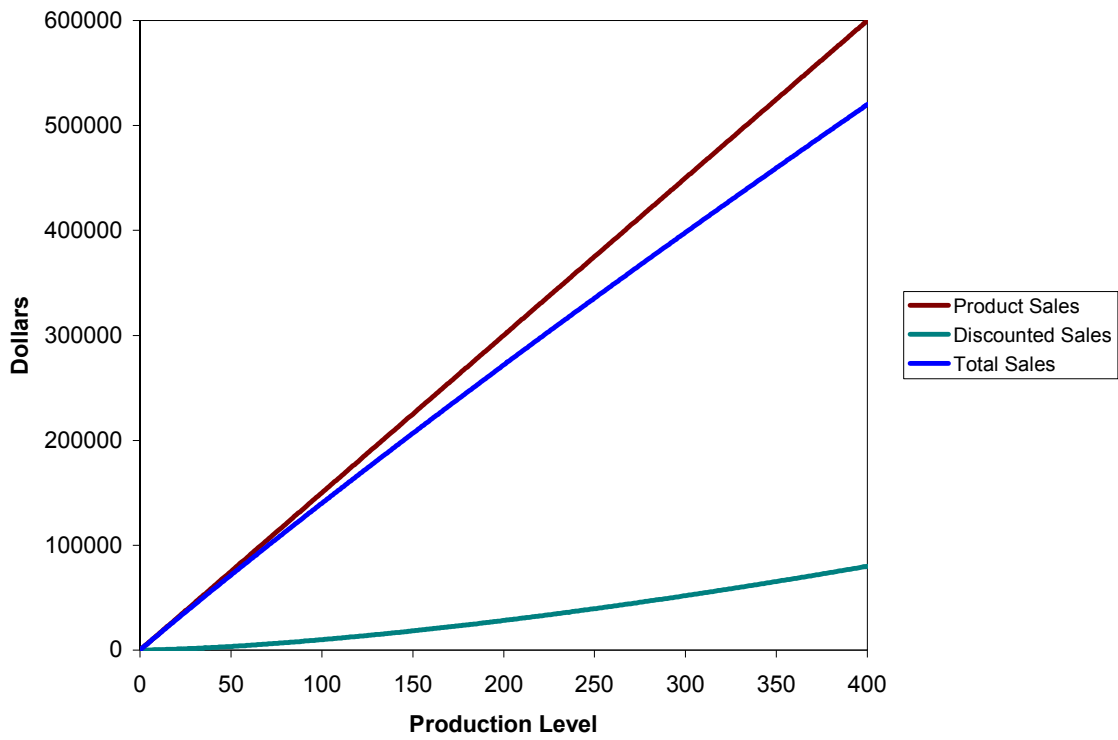**Figure 1** Capital, Fixed, Variable, Regulated, Operating, And Total Costs

Figure 2   Product, Discounted, And Total Sales

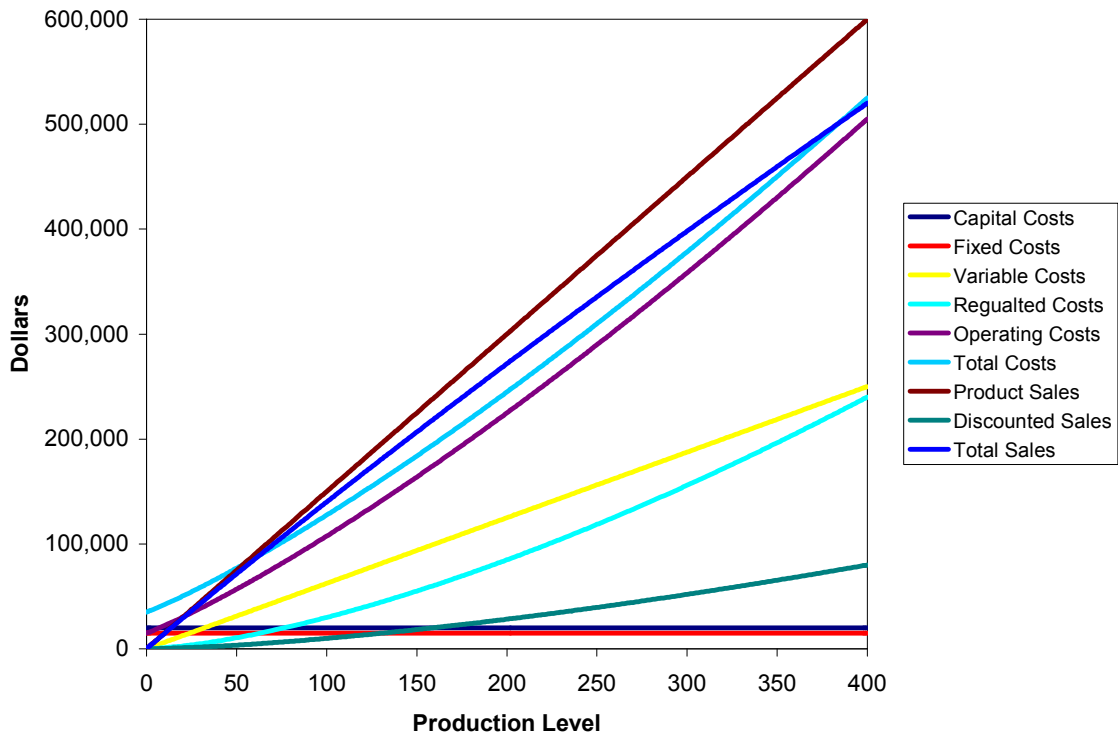All of the costs and sales revenue are shown next in Figure 3.

Figure 3   All of the Costs and Sales Revenue

At the break-even point there is no profit, so the capital cost plus the product cost equals the product sales (see Figure 4).

$$TC = TS$$

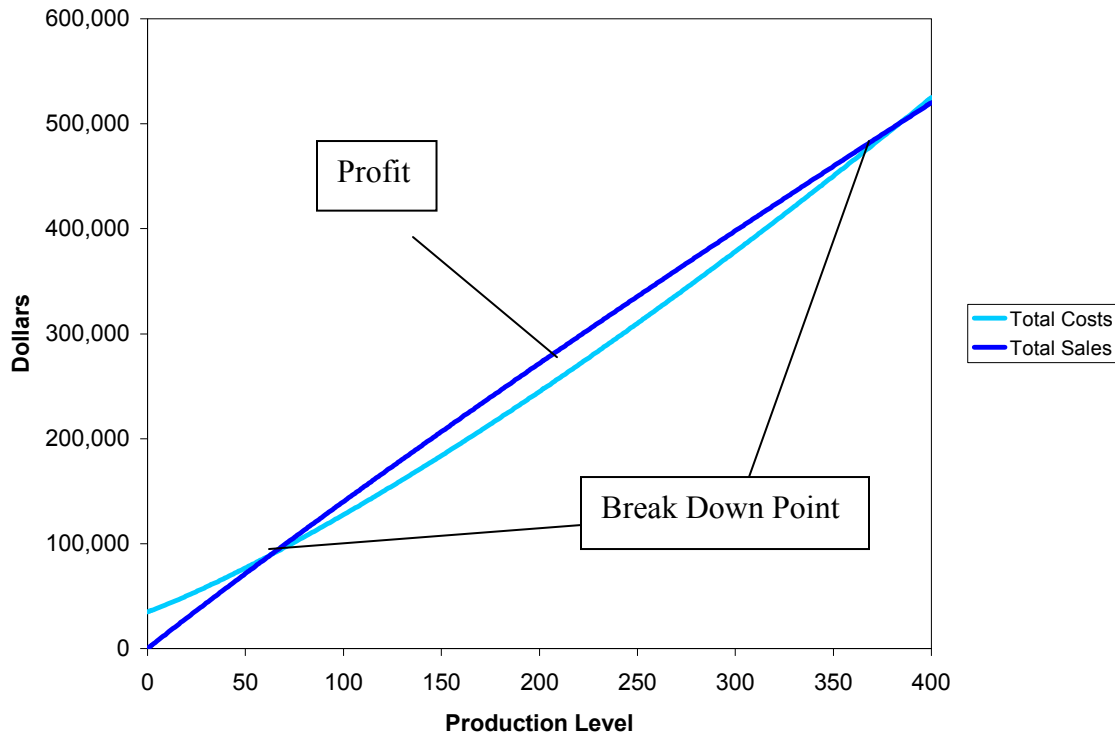$$\$35,000 + \$625n + \$30n^{1.5} = \$1500n - \$10n^{1.5}$$

Figure 4 Break-Even Point and Profit Margin

Simplifying the previous equation yields the following nonlinear equation that needs to solved.

$$\$35,000 - \$875n + \$40n^{1.5} = 0$$

The value of n is the minimum number of computers that the shop will have to sell to make a profit. This is called the *break-even point*.

Questions
1. Will some of the roots be complex for the above nonlinear equation?
2. Using numerical techniques, we can solve this equation and any other equation of the form $f(x) = 0$. Solve the above equation by all the methods you have learned assuming you want at least 3 significant digits to be correct in your answer.
3. How can you use the knowledge of the problem to develop initial guess(es) for the numerical methods?

4. Determine the production level that produces the most profit.
5. Determine the second break-even point, after which a loss is realized.
6. Reformulate the problem to determine the break-even point for the second year in business assuming that no capital costs are incurred.

Topic	NONLINEAR EQUATIONS
Sub Topic	Physical Problem
Summary	A physical problem of determining how many computers a business would have to sell to turn a profit. To find this number of computers to be sold, the physical model is a nonlinear equation.
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