Chapter 05.03
Newton's Divided Difference Interpolation

After reading this chapter, you should be able to:
1. *derive Newton's divided difference method of interpolation,*
2. *apply Newton's divided difference method of interpolation, and*
3. *apply Newton's divided difference method interpolants to find derivatives and integrals.*

What is interpolation?

Many times, data is given only at discrete points such as (x_0, y_0), (x_1, y_1), ..., (x_{n-1}, y_{n-1}), (x_n, y_n). So, how then does one find the value of y at any other value of x? Well, a continuous function $f(x)$ may be used to represent the $n+1$ data values with $f(x)$ passing through the $n+1$ points (Figure 1). Then one can find the value of y at any other value of x. This is called *interpolation*.

Of course, if x falls outside the range of x for which the data is given, it is no longer interpolation but instead is called *extrapolation*.

So what kind of function $f(x)$ should one choose? A polynomial is a common choice for an interpolating function because polynomials are easy to

(A) evaluate,
(B) differentiate, and
(C) integrate,

relative to other choices such as a trigonometric and exponential series.

Polynomial interpolation involves finding a polynomial of order n that passes through the $n+1$ points. One of the methods of interpolation is called Newton's divided difference polynomial method. Other methods include the direct method and the Lagrangian interpolation method. We will discuss Newton's divided difference polynomial method in this chapter.

Newton's Divided Difference Polynomial Method

To illustrate this method, linear and quadratic interpolation is presented first. Then, the general form of Newton's divided difference polynomial method is presented. To illustrate the general form, cubic interpolation is shown in Figure 1.

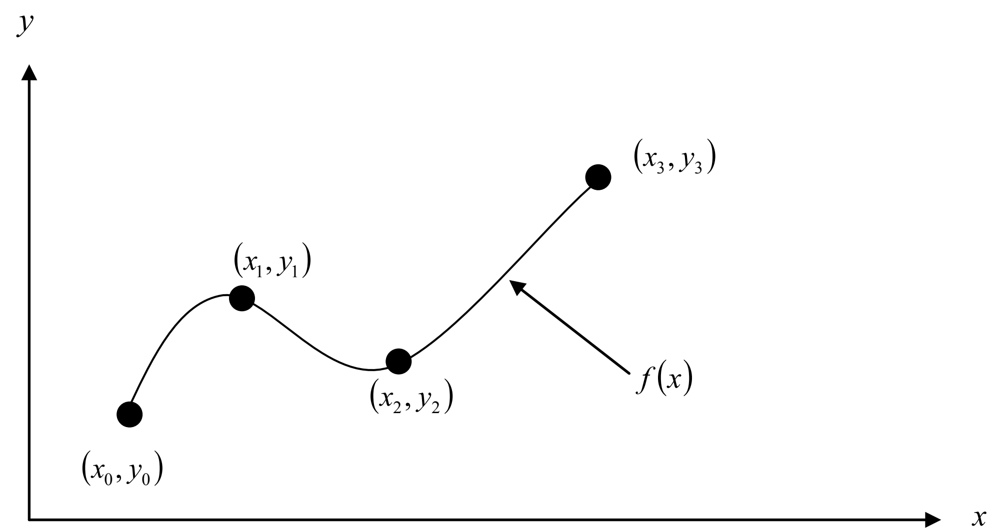

Figure 1 Interpolation of discrete data.

<u>Linear Interpolation</u>

Given (x_0, y_0) and (x_1, y_1), fit a linear interpolant through the data. Noting $y = f(x)$ and $y_1 = f(x_1)$, assume the linear interpolant $f_1(x)$ is given by (Figure 2)

$$f_1(x) = b_0 + b_1(x - x_0)$$

Since at $x = x_0$,

$$f_1(x_0) = f(x_0) = b_0 + b_1(x_0 - x_0) = b_0$$

and at $x = x_1$,

$$f_1(x_1) = f(x_1) = b_0 + b_1(x_1 - x_0)$$
$$= f(x_0) + b_1(x_1 - x_0)$$

giving

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

So

$$b_0 = f(x_0)$$
$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

giving the linear interpolant as

$$f_1(x) = b_0 + b_1(x - x_0)$$
$$f_1(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0)$$

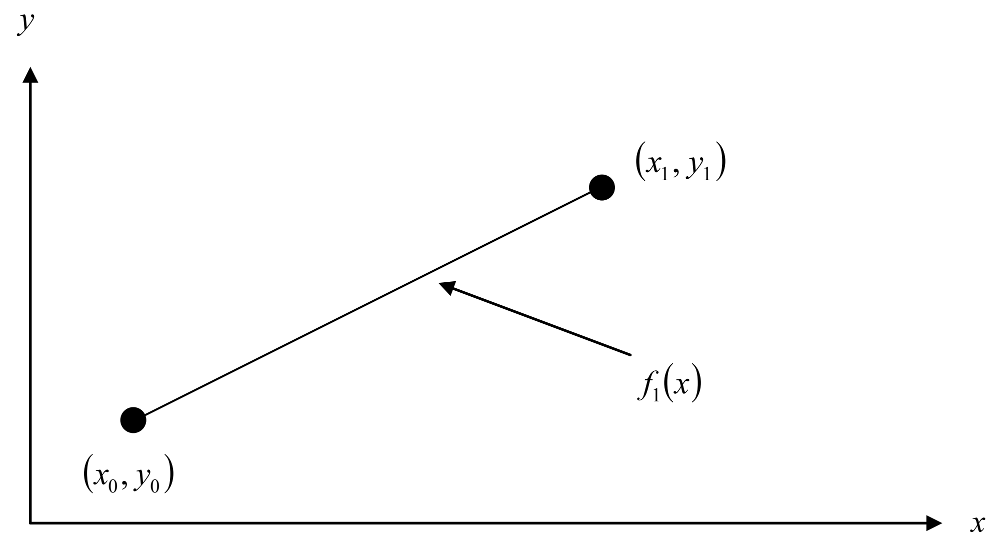

Figure 2 Linear interpolation.

Example 1

The geometry of a cam is given in Figure 3. A curve needs to be fit through the seven points given in Table 1 to fabricate the cam.

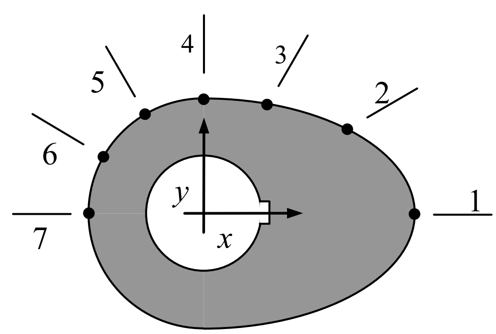

Figure 3 Schematic of cam profile.

Table 1 Geometry of the cam.

Point	x (in.)	y (in.)
1	2.20	0.00
2	1.28	0.88
3	0.66	1.14
4	0.00	1.20
5	–0.60	1.04
6	–1.04	0.60
7	–1.20	0.00

If the cam follows a straight line profile from $x=1.28$ to $x=0.66$, what is the value of y at $x=1.10$ using Newton's divided difference method of interpolation and a first order polynomial.

Solution

For linear interpolation, the value of y is given by

$$y(x)=b_0+b_1(x-x_0)$$

Since we want to find the value of y at $x=1.10$, using the two points $x=1.28$ and $x=0.66$, then

$$x_0=1.28,\ \ y(x_0)=0.88$$
$$x_1=0.66,\ \ y(x_1)=1.14$$

gives

$$\begin{aligned} b_0 &= y(x_0) \\ &= 0.88 \\ b_1 &= \frac{y(x_1)-y(x_0)}{x_1-x_0} \\ &= \frac{1.14-0.88}{0.66-1.28} \\ &= -0.41935 \end{aligned}$$

Hence

$$\begin{aligned} y(x) &= b_0+b_1(x-x_0) \\ &= 0.88-0.41935(x-1.28), \qquad 0.66 \le x \le 1.28 \end{aligned}$$

At $x=1.10$

$$\begin{aligned} y(4.00) &= 0.88-0.41935(1.10-1.28) \\ &= 0.95548 \text{ in.} \end{aligned}$$

If we expand

$$y(x)=0.88-0.41935(x-1.28), \qquad 0.66 \le x \le 1.28$$

we get

$$y(x)=1.4168-0.41935x, \qquad 0.66 \le x \le 1.28$$

This is the same expression that was obtained with the direct method.

Quadratic Interpolation

Given (x_0,y_0), (x_1,y_1), and (x_2,y_2), fit a quadratic interpolant through the data. Noting $y=f(x)$, $y_0=f(x_0)$, $y_1=f(x_1)$, and $y_2=f(x_2)$, assume the quadratic interpolant $f_2(x)$ is given by

$$f_2(x)=b_0+b_1(x-x_0)+b_2(x-x_0)(x-x_1)$$

At $x=x_0$,

$$\begin{aligned} f_2(x_0)=f(x_0) &= b_0+b_1(x_0-x_0)+b_2(x_0-x_0)(x_0-x_1) \\ &= b_0 \end{aligned}$$

$$b_0=f(x_0)$$

At $x=x_1$

$$f_2(x_1) = f(x_1) = b_0 + b_1(x_1 - x_0) + b_2(x_1 - x_0)(x_1 - x_1)$$
$$f(x_1) = f(x_0) + b_1(x_1 - x_0)$$

giving

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

At $x = x_2$

$$f_2(x_2) = f(x_2) = b_0 + b_1(x_2 - x_0) + b_2(x_2 - x_0)(x_2 - x_1)$$
$$f(x_2) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_2 - x_0) + b_2(x_2 - x_0)(x_2 - x_1)$$

Giving

$$b_2 = \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

Hence the quadratic interpolant is given by

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$
$$= f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) + \frac{\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} - \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}(x - x_0)(x - x_1)$$

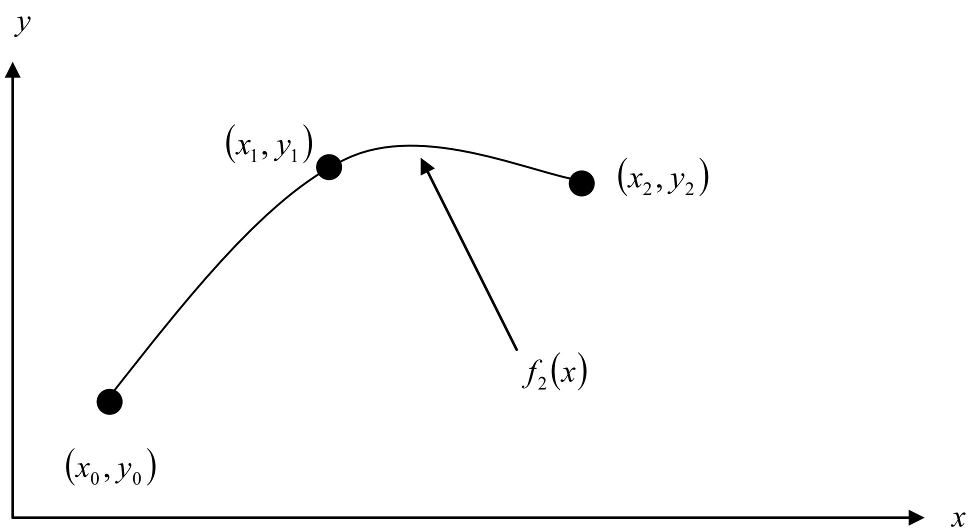

Figure 4 Quadratic interpolation.

Example 2

The geometry of a cam is given in Figure 5. A curve needs to be fit through the seven points given in Table 2 to fabricate the cam.

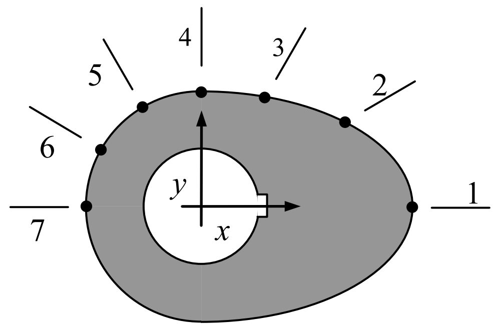

Figure 5 Schematic of cam profile.

Table 2 Geometry of the cam.

Point	x (in.)	y (in.)
1	2.20	0.00
2	1.28	0.88
3	0.66	1.14
4	0.00	1.20
5	–0.60	1.04
6	–1.04	0.60
7	–1.20	0.00

If the cam follows a quadratic profile from $x = 2.20$ to $x = 1.28$ to $x = 0.66$, what is the value of y at $x = 1.10$ using Newton's divided difference method of interpolation and a second order polynomial. Find the absolute relative approximate error for the second order polynomial approximation.

Solution

For quadratic interpolation, the value of y is chosen as

$$y(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

Since we want to find the value of y at $x = 1.10$, using the three points $x_0 = 2.20$, $x_1 = 1.28$ and $x_2 = 0.66$, then

$$x_0 = 2.20, \quad y(x_0) = 0.00$$

$$x_1 = 1.28, \quad y(x_1) = 0.88$$

$$x_2 = 0.66, \quad y(x_2) = 1.14$$

gives

$$b_0 = y(x_0)$$

$$= 0.00$$

$$b_1 = \frac{y(x_1) - y(x_0)}{x_1 - x_0}$$
$$= \frac{0.88 - 0.00}{1.28 - 2.20}$$
$$= -0.95652$$
$$b_2 = \frac{\dfrac{y(x_2) - y(x_1)}{x_2 - x_1} - \dfrac{y(x_1) - y(x_0)}{x_1 - x_0}}{x_2 - x_0}$$
$$= \frac{\dfrac{1.14 - 0.88}{0.66 - 1.28} - \dfrac{0.88 - 0.00}{1.28 - 2.20}}{0.66 - 2.20}$$
$$= \frac{-0.41935 + 0.95652}{-1.54}$$
$$= -0.34881$$

Hence

$$y(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$
$$= 0 - 0.95652(x - 2.20) - 0.34881(x - 2.20)(x - 1.28), \quad 0.66 \le x \le 2.20$$

At $x = 1.10$,

$$y(1.10) = 0 - 0.95652(1.10 - 2.20) - 0.34881(1.10 - 2.20)(1.10 - 1.28)$$
$$= 0.98311 \text{ in.}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the first and second order polynomial is

$$|\in_a| = \left|\frac{0.98311 - 0.95548}{0.98311}\right| \times 100$$
$$= 2.8100\%$$

If we expand

$$y(x) = 0 - 0.95652(x - 2.20) - 0.34881(x - 2.20)(x - 1.28), \quad 0.66 \le x \le 2.20$$

we get

$$y(x) = 1.1221 + 0.25734x - 0.34881x^2, \quad 0.66 \le x \le 2.20$$

This is the same expression that was obtained with the direct method.

General Form of Newton's Divided Difference Polynomial

In the two previous cases, we found linear and quadratic interpolants for Newton's divided difference method. Let us revisit the quadratic polynomial interpolant formula

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

where

$$b_0 = f(x_0)$$
$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\dfrac{f(x_2)-f(x_1)}{x_2-x_1} - \dfrac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0}$$

Note that b_0, b_1, and b_2 are finite divided differences. b_0, b_1, and b_2 are the first, second, and third finite divided differences, respectively. We denote the first divided difference by

$$f[x_0] = f(x_0)$$

the second divided difference by

$$f[x_1, x_0] = \frac{f(x_1)-f(x_0)}{x_1-x_0}$$

and the third divided difference by

$$f[x_2, x_1, x_0] = \frac{f[x_2, x_1]-f[x_1, x_0]}{x_2-x_0}$$

$$= \frac{\dfrac{f(x_2)-f(x_1)}{x_2-x_1} - \dfrac{f(x_1)-f(x_0)}{x_1-x_0}}{x_2-x_0}$$

where $f[x_0]$, $f[x_1, x_0]$, and $f[x_2, x_1, x_0]$ are called bracketed functions of their variables enclosed in square brackets.

Rewriting,

$$f_2(x) = f[x_0] + f[x_1, x_0](x-x_0) + f[x_2, x_1, x_0](x-x_0)(x-x_1)$$

This leads us to writing the general form of the Newton's divided difference polynomial for $n+1$ data points, $(x_0, y_0), (x_1, y_1), ..., (x_{n-1}, y_{n-1}), (x_n, y_n)$, as

$$f_n(x) = b_0 + b_1(x-x_0) + ... + b_n(x-x_0)(x-x_1)...(x-x_{n-1})$$

where

$$b_0 = f[x_0]$$
$$b_1 = f[x_1, x_0]$$
$$b_2 = f[x_2, x_1, x_0]$$
$$\vdots$$
$$b_{n-1} = f[x_{n-1}, x_{n-2}, ..., x_0]$$
$$b_n = f[x_n, x_{n-1}, ..., x_0]$$

where the definition of the m^{th} divided difference is

$$b_m = f[x_m, ..., x_0]$$
$$= \frac{f[x_m, ..., x_1] - f[x_{m-1}, ..., x_0]}{x_m - x_0}$$

From the above definition, it can be seen that the divided differences are calculated recursively.

For an example of a third order polynomial, given (x_0, y_0), (x_1, y_1), (x_2, y_2), and (x_3, y_3),

$$f_3(x) = f[x_0] + f[x_1, x_0](x-x_0) + f[x_2, x_1, x_0](x-x_0)(x-x_1)$$
$$+ f[x_3, x_2, x_1, x_0](x-x_0)(x-x_1)(x-x_2)$$

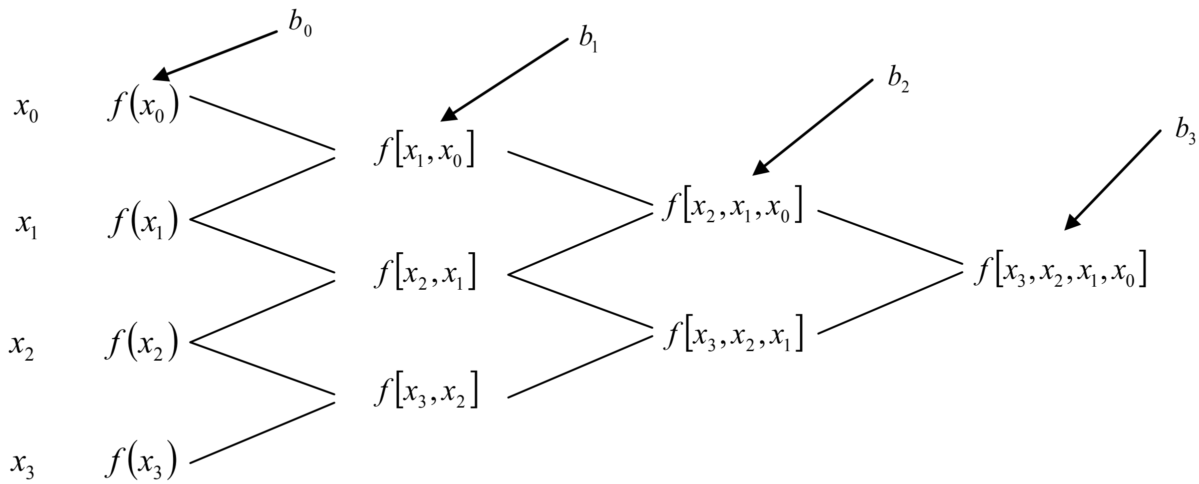

Figure 6 Table of divided differences for a cubic polynomial.

Example 3

The geometry of a cam is given in Figure 7. A curve needs to be fit through the seven points given in Table 3 to fabricate the cam.

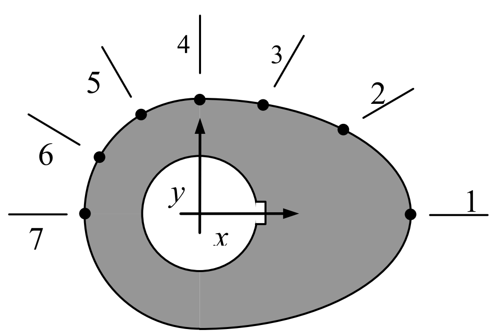

Figure 7 Schematic of cam profile.

Table 3 Geometry of the cam.

Point	x (in.)	y (in.)
1	2.20	0.00
2	1.28	0.88
3	0.66	1.14
4	0.00	1.20
5	–0.60	1.04
6	–1.04	0.60
7	–1.20	0.00

Find the cam profile using all seven points in Table 3, Newton's divided difference method of interpolation and a sixth order polynomial.

Solution

For 6th order interpolation, the value of y is given by

$$y(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2)$$
$$+ b_4(x - x_0)(x - x_1)(x - x_2)(x - x_3) + b_5(x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4)$$
$$+ b_6(x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4)(x - x_5)$$

Using the seven points,

$$x_0 = 2.20, \quad y(x_0) = 0.00$$
$$x_1 = 1.28, \quad y(x_1) = 0.88$$
$$x_2 = 0.66, \quad y(x_2) = 1.14$$
$$x_3 = 0.00, \quad y(x_3) = 1.20$$
$$x_4 = -.60, \quad y(x_4) = 1.04$$
$$x_5 = -1.04, \quad y(x_5) = 0.60$$
$$x_6 = -1.20, \quad y(x_6) = 0.00$$

gives

$$b_0 = y[x_0]$$
$$= y(x_0)$$
$$= 0.00$$
$$b_1 = y[x_1, x_0]$$
$$= \frac{y(x_1) - y(x_0)}{x_1 - x_0}$$
$$= \frac{0.88 - 0.00}{1.28 - 2.20}$$
$$= -0.95652$$
$$b_2 = y[x_2, x_1, x_0]$$
$$= \frac{y[x_2, x_1] - y[x_1, x_0]}{x_2 - x_0}$$
$$y[x_2, x_1] = \frac{y(x_2) - y(x_1)}{x_2 - x_1}$$
$$= \frac{1.14 - 0.88}{0.66 - 1.28}$$
$$= -0.41935$$
$$y[x_1, x_0] = -0.95652$$
$$b_2 = \frac{y[x_2, x_1] - y[x_1, x_0]}{x_2 - x_0}$$

$$= \frac{-0.41935 + 0.95652}{0.66 - 2.20}$$
$$= -0.34881$$

$$b_3 = y[x_3, x_2, x_1, x_0]$$
$$= \frac{y[x_3, x_2, x_1] - y[x_2, x_1, x_0]}{x_3 - x_0}$$

$$y[x_3, x_2, x_1] = \frac{y[x_3, x_2] - y[x_2, x_1]}{x_3 - x_1}$$

$$y[x_3, x_2] = \frac{y(x_3) - y(x_2)}{x_3 - x_2}$$
$$= \frac{1.20 - 1.14}{0.00 - 0.66}$$
$$= -0.090909$$

$$y[x_2, x_1] = -0.41935$$

$$y[x_3, x_2, x_1] = \frac{y[x_3, x_2] - y[x_2, x_1]}{x_3 - x_1}$$
$$= \frac{-0.090909 + 0.41935}{0.00 - 1.28}$$
$$= -0.25660$$

$$y[x_2, x_1, x_0] = -0.34881$$

$$b_3 = y[x_3, x_2, x_1, x_0]$$
$$= \frac{y[x_3, x_2, x_1] - y[x_2, x_1, x_0]}{x_3 - x_0}$$
$$= \frac{-0.25660 + 0.34881}{0.00 - 2.20}$$
$$= -0.041914$$

$$b_4 = y[x_4, x_3, x_2, x_1, x_0]$$
$$= \frac{y[x_4, x_3, x_2, x_1] - y[x_3, x_2, x_1, x_0]}{x_4 - x_0}$$

$$y[x_4, x_3, x_2, x_1] = \frac{y[x_4, x_3, x_2] - y[x_3, x_2, x_1]}{x_4 - x_1}$$

$$y[x_4, x_3, x_2] = \frac{y[x_4, x_3] - y[x_3, x_2]}{x_4 - x_2}$$

$$y[x_4, x_3] = \frac{y(x_4) - y(x_3)}{x_4 - x_3}$$
$$= \frac{1.04 - 1.20}{-0.60 - 0}$$

$$= 0.26667$$

$$y[x_3, x_2] = -0.090909$$

$$y[x_4, x_3, x_2] = \frac{y[x_4, x_3] - y[x_3, x_2]}{x_4 - x_2}$$

$$= \frac{0.26667 + 0.090909}{-0.60 - 0.66}$$

$$= -0.28379$$

$$y[x_3, x_2, x_1] = -0.25660$$

$$y[x_4, x_3, x_2, x_1] = \frac{y[x_4, x_3, x_2] - y[x_3, x_2, x_1]}{x_4 - x_1}$$

$$= \frac{-0.28379 + 0.25660}{-0.60 - 1.28}$$

$$= 0.014464$$

$$y[x_3, x_2, x_1, x_0] = -0.041914$$

$$b_4 = \frac{y[x_4, x_3, x_2, x_1] - y[x_3, x_2, x_1, x_0]}{x_4 - x_0}$$

$$= \frac{0.014464 + 0.041914}{-0.60 - 2.20}$$

$$= -0.020135$$

$$b_5 = y[x_5, x_4, x_3, x_2, x_1, x_0]$$

$$= \frac{y[x_5, x_4, x_3, x_2, x_1] - y[x_4, x_3, x_2, x_1, x_0]}{x_5 - x_0}$$

$$y[x_5, x_4, x_3, x_2, x_1] = \frac{y[x_5, x_4, x_3, x_2] - y[x_4, x_3, x_2, x_1]}{x_5 - x_1}$$

$$y[x_5, x_4, x_3, x_2] = \frac{y[x_5, x_4, x_3] - y[x_4, x_3, x_2]}{x_5 - x_2}$$

$$y[x_5, x_4, x_3] = \frac{y[x_5, x_4] - y[x_4, x_3]}{x_5 - x_3}$$

$$y[x_5, x_4] = \frac{y(x_5) - y(x_4)}{x_5 - x_4}$$

$$= \frac{0.60 - 1.04}{-1.04 + 0.60}$$

$$= 1$$

$$y[x_4, x_3] = 0.26667$$

$$y[x_5, x_4, x_3] = \frac{y[x_5, x_4] - y[x_4, x_3]}{x_5 - x_3}$$

$$= \frac{1-0.26667}{-1.04-0}$$
$$= -0.70513$$
$$y[x_4, x_3, x_2] = -0.28379$$
$$y[x_5, x_4, x_3, x_2] = \frac{y[x_5, x_4, x_3] - y[x_4, x_3, x_2]}{x_5 - x_2}$$
$$= \frac{-0.70513 + 0.28379}{-1.04 - 0.66}$$
$$= 0.24785$$
$$y[x_4, x_3, x_2, x_1] = 0.014464$$
$$y[x_5, x_4, x_3, x_2, x_1] = \frac{y[x_5, x_4, x_3, x_2] - y[x_4, x_3, x_2, x_1]}{x_5 - x_1}$$
$$= \frac{0.24785 - 0.014464}{-1.04 - 1.28}$$
$$= -0.10060$$
$$y[x_4, x_3, x_2, x_1, x_0] = -0.020135$$
$$b_5 = y[x_5, x_4, x_3, x_2, x_1, x_0]$$
$$= \frac{y[x_5, x_4, x_3, x_2, x_1] - y[x_4, x_3, x_2, x_1, x_0]}{x_5 - x_0}$$
$$= \frac{-0.10060 + 0.020135}{-1.04 - 2.20}$$
$$= 0.024834$$

$$b_6 = y[x_6, x_5, x_4, x_3, x_2, x_1, x_0]$$
$$= \frac{y[x_6, x_5, x_4, x_3, x_2, x_1] - y[x_5, x_4, x_3, x_2, x_1, x_0]}{x_6 - x_0}$$
$$y[x_6, x_5, x_4, x_3, x_2, x_1] = \frac{y[x_6, x_5, x_4, x_3, x_2] - y[x_5, x_4, x_3, x_2, x_1]}{x_6 - x_1}$$
$$y[x_6, x_5, x_4, x_3, x_2] = \frac{y[x_6, x_5, x_4, x_3] - y[x_5, x_4, x_3, x_2]}{x_6 - x_2}$$
$$y[x_6, x_5, x_4, x_3] = \frac{y[x_6, x_5, x_4] - y[x_5, x_4, x_3]}{x_6 - x_3}$$
$$y[x_6, x_5, x_4] = \frac{y[x_6, x_5] - y[x_5, x_4]}{x_6 - x_4}$$
$$y[x_6, x_5] = \frac{y(x_6) - y(x_5)}{x_6 - x_5}$$
$$= \frac{0.00 - 0.60}{-1.20 + 1.04}$$
$$= 3.75$$

$$y[x_5, x_4] = 1$$

$$y[x_6, x_5, x_4] = \frac{y[x_6, x_5] - y[x_5, x_4]}{x_6 - x_4}$$

$$= \frac{3.75 - 1}{-1.20 + 0.60}$$

$$= -4.5833$$

$$y[x_5, x_4, x_3] = -0.70513$$

$$y[x_6, x_5, x_4, x_3] = \frac{y[x_6, x_5, x_4] - y[x_5, x_4, x_3]}{x_6 - x_3}$$

$$= \frac{-4.5833 + 0.70513}{-1.20 - 0}$$

$$= 3.2318$$

$$y[x_5, x_4, x_3, x_2] = 0.24785$$

$$y[x_6, x_5, x_4, x_3, x_2] = \frac{y[x_6, x_5, x_4, x_3] - y[x_5, x_4, x_3, x_2]}{x_6 - x_2}$$

$$= \frac{3.2318 - 0.24785}{-1.20 - 0.66}$$

$$= -1.6043$$

$$y[x_5, x_4, x_3, x_2, x_1] = -0.10060$$

$$y[x_6, x_5, x_4, x_3, x_2, x_1] = \frac{y[x_6, x_5, x_4, x_3, x_2] - y[x_5, x_4, x_3, x_2, x_1]}{x_6 - x_1}$$

$$= \frac{-1.6043 + 0.100596}{-1.20 - 1.28}$$

$$= 0.60633$$

$$y[x_5, x_4, x_3, x_2, x_1, x_0] = 0.024834$$

$$b_6 = \frac{y[x_6, x_5, x_4, x_3, x_2, x_1] - y[x_5, x_4, x_3, x_2, x_1, x_0]}{x_6 - x_0}$$

$$= \frac{0.60633 - 0.024834}{-1.20 - 2.20}$$

$$= -0.17103$$

Hence

$$y(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2)$$
$$+ b_4(x - x_0)(x - x_1)(x - x_2)(x - x_3) + b_5(x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4)$$
$$+ b_6(x - x_0)(x - x_1)(x - x_2)(x - x_3)(x - x_4)(x - x_5)$$

$$
\begin{aligned}
&= 0 - 0.95652(x-2.2) - 0.34881(x-2.2)(x-1.28) \\
&\quad - 0.041914(x-2.2)(x-1.28)(x-0.66) \\
&\quad - 0.020135(x-2.2)(x-1.28)(x-0.66)(x-0) \\
&\quad + 0.024834(x-2.2)(x-1.28)(x-0.66)(x-0)(x+0.6) \\
&\quad - 0.17103(x-2.2)(x-1.28)(x-0.66)(x-0)(x+0.6)(x+1.04)
\end{aligned}
$$

Expanding this formula, we get

$$
\begin{aligned}
y(x) &= 1.2 + 0.25112x - 0.27255x^2 - 0.56765x^3 \\
&\quad + 0.072013x^4 + 0.45241x^5 - 0.17103x^6, \quad -1.20 \le x \le 2.20
\end{aligned}
$$

This is the same expression that was obtained with the direct method.

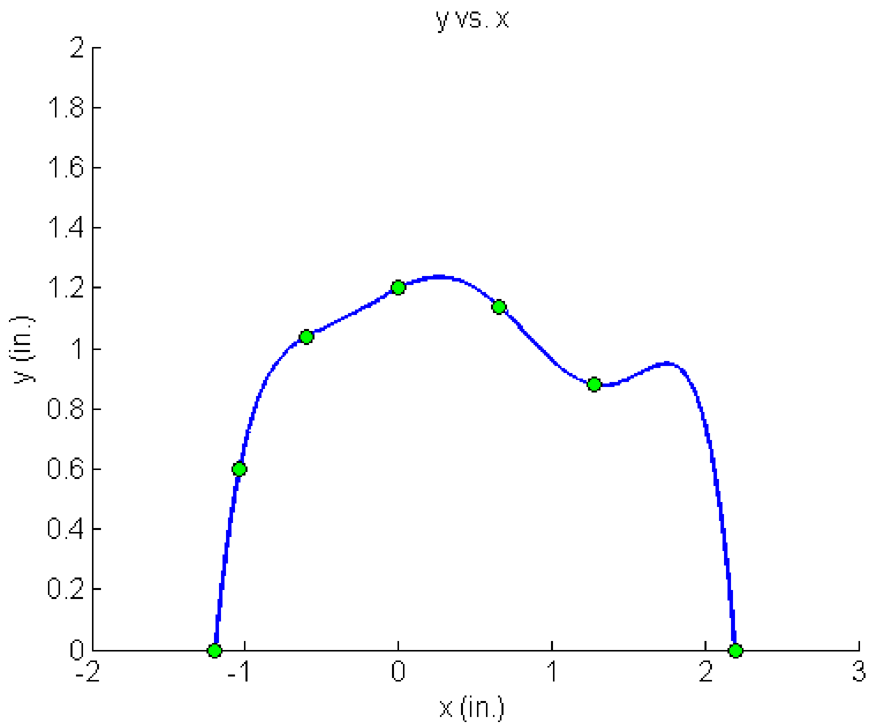

Figure 8 Plot of the cam profile as defined by a 6th order interpolating polynomial (using Newton's divided difference method of interpolation).

INTERPOLATION	
Topic	Newton's Divided Difference Interpolation
Summary	Textbook notes on Newton's divided difference interpolation.
Major	Industrial Engineering
Authors	Autar Kaw
Date	November 15, 2012
Web Site	http://numericalmethods.eng.usf.edu