Chapter 09.00F

Physical Problem for Optimization
Industrial Engineering

Problem Statement

Fundamental inventory replenishment cost models attempt to describe the trade-off between the fixed ordering costs and the holding costs. If the order quantity Q is too high, average inventory levels and holding costs increase, and if it is too low then fixed ordering costs increase due to too many small orders being placed. The optimal solution to the problem is to determine the order quantity Q which minimizes the inventory replenishment and holding costs. This is traditionally referred to as the **Economic Order Quantity**.

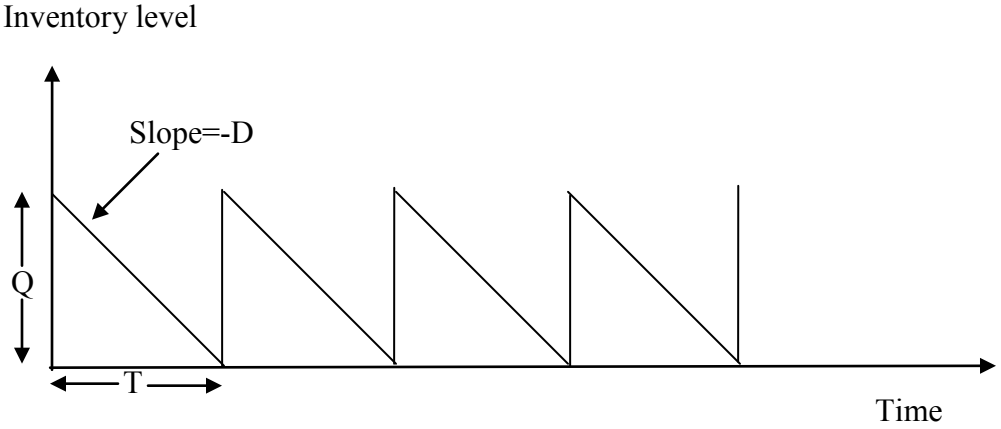

Figure 1: Level of inventory over time for a single product

Solution

Inventory Replenishment Cost Model

The basic inventory replenishment cost model involves a fixed ordering cost denoted by A and a unit purchase price C. For a time period T (usually annual) where Q units are ordered, the average ordering cost is denoted by

$$TC(Q) = \frac{A + CQ}{T}$$

Considering the unit holding costs denoted by h, the above equation can be extended to

$$TC(Q) = \frac{A + CQ}{T} + \frac{hQ}{2} \quad (1)$$

Note that the average level of inventory is $Q/2$ for each cycle as inventory decreases linearly from Q to 0 in each cycle. Considering a constant demand rate D, T can be expressed as Q/D and substituting this expression in Equation (1), the resulting $TC(Q)$ function is

$$TC(Q) = \frac{AD}{Q} + CD + \frac{hQ}{2} \quad (2)$$

where the three terms correspond to ordering cost, purchase cost and holding cost for time period T, respectively. The optimal order quantity is found by taking the derivative of Equation (2), setting it equal to 0, and solving for Q^*, which results in

$$Q^* = \sqrt{\frac{2AD}{h}} \quad (3)$$

where Q^* is the **economic order quantity**.

Example Problem

Consider an Industrial Engineering Department office which has a copy machine used by its faculty and staff for printing and making photocopies. The department maintains an inventory of paper to be used in the copier. Historical data of the orders placed in the last 7 months with office supplies is as follows:

Table 1: Historical data of cases of paper ordered.

Order Date	Number of cases ordered
2/16/2007	4
4/4/2007	3
4/30/2007	7
6/11/2007	2
6/29/2007	3
8/2/2007	2
9/25/2007	3

Questions

1. Based on the data in Table 1, estimate the average number of cases of paper used by the copier annually.

2. The cost of each case of paper is $30.08. An office Supplier charges $7.95 flat rate shipping, and it is estimated that the staff time required to place an order is approximately $20.00. Assuming that holding costs are determined at a rate of 25% annual interest rate, what should be the optimal order quantity (cases)?

3. How often should an order be placed?
4. There are 10 reams in each case, and each ream is sold for $3.15. Calculate the optimal order quantity (reams) and how frequently an order should be placed.

5. What order policy is preferable to order-in, reams or cases? What is the annual savings?

OPTIMIZATION	
Topic	Physical optimization problem for industrial engineering
Summary	A physical problem of finding economic order quantity.
Major	Industrial Engineering
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