

# Binary Representation

Major: All Engineering Majors

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<http://numericalmethods.eng.usf.edu>

Transforming Numerical Methods Education for STEM  
Undergraduates

# Binary Representation

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# How a Decimal Number is Represented

$$257.76 = 2 \times 10^2 + 5 \times 10^1 + 7 \times 10^0 + 7 \times 10^{-1} + 6 \times 10^{-2}$$

# Base 2

$$(1011.0011)_2 = \left( \begin{array}{l} (1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0) \\ + (0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}) \end{array} \right)_{10}$$
$$= 11.1875$$

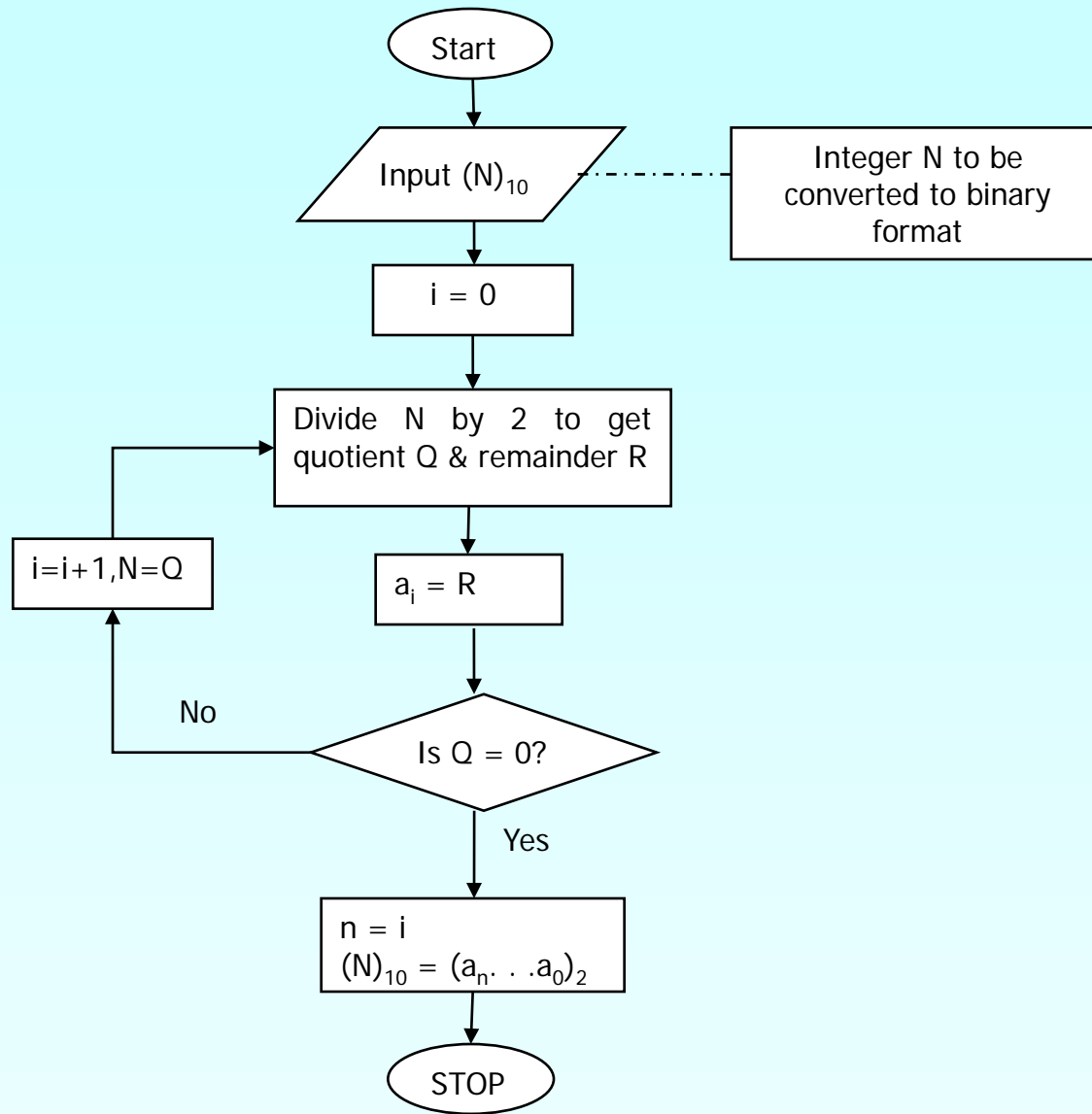
# Convert Base 10 Integer to binary representation

**Table 1** Converting a base-10 integer to binary representation.

|      | <b>Quotient</b> | <b>Remainder</b> |
|------|-----------------|------------------|
| 11/2 | 5               | $1 = a_0$        |
| 5/2  | 2               | $1 = a_1$        |
| 2/2  | 1               | $0 = a_2$        |
| 1/2  | 0               | $1 = a_3$        |

Hence

$$\begin{aligned}(11)_{10} &= (a_3 a_2 a_1 a_0)_2 \\ &= (1011)_2\end{aligned}$$



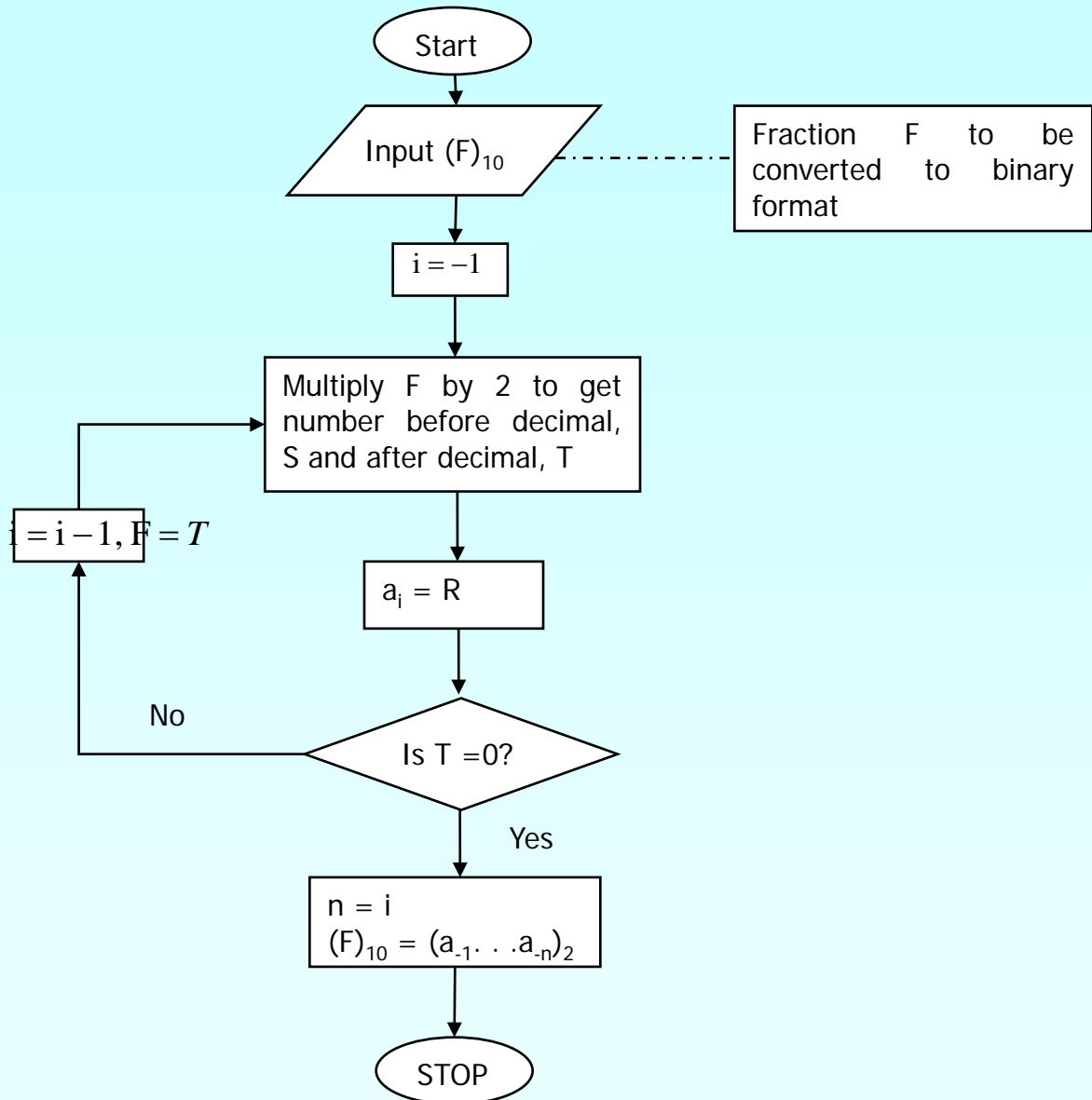
# Fractional Decimal Number to Binary

**Table 2.** Converting a base-10 fraction to binary representation.

|                   | Number | Number after<br>decimal | Number before<br>decimal |
|-------------------|--------|-------------------------|--------------------------|
| $0.1875 \times 2$ | 0.375  | 0.375                   | $0 = a_{-1}$             |
| $0.375 \times 2$  | 0.75   | 0.75                    | $0 = a_{-2}$             |
| $0.75 \times 2$   | 1.5    | 0.5                     | $1 = a_{-3}$             |
| $0.5 \times 2$    | 1.0    | 0.0                     | $1 = a_{-4}$             |

Hence

$$\begin{aligned}(0.1875)_{10} &= (a_{-1}a_{-2}a_{-3}a_{-4})_2 \\ &= (0.0011)_2\end{aligned}$$





# Decimal Number to Binary

$$(11.1875)_{10} = ( \quad ?.\? \quad )_2$$

Since

$$(11)_{10} = (1011)_2$$

and

$$(0.1875)_{10} = (0.0011)_2$$

we have

$$(11.1875)_{10} = (1011.0011)_2$$

# All Fractional Decimal Numbers Cannot be Represented Exactly

**Table 3.** Converting a base-10 fraction to approximate binary representation.

|                | <b>Number</b> | <b>Number after decimal</b> | <b>Number before Decimal</b> |
|----------------|---------------|-----------------------------|------------------------------|
| $0.3 \times 2$ | 0.6           | 0.6                         | $0 = a_{-1}$                 |
| $0.6 \times 2$ | 1.2           | 0.2                         | $1 = a_{-2}$                 |
| $0.2 \times 2$ | 0.4           | 0.4                         | $0 = a_{-3}$                 |
| $0.4 \times 2$ | 0.8           | 0.8                         | $0 = a_{-4}$                 |
| $0.8 \times 2$ | 1.6           | 0.6                         | $1 = a_{-5}$                 |

$$(0.3)_{10} \approx (a_{-1}a_{-2}a_{-3}a_{-4}a_{-5})_2 = (0.01001)_2 = 0.28125$$

# Another Way to Look at Conversion

Convert  $(11.1875)_{10}$  to base 2

$$\begin{aligned}(11)_{10} &= 2^3 + 3 \\ &= 2^3 + 2^1 + 1 \\ &= 2^3 + 2^1 + 2^0 \\ &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= (1011)_2\end{aligned}$$

$$\begin{aligned} (0.1875)_{10} &= 2^{-3} + 0.0625 \\ &= 2^{-3} + 2^{-4} \\ &= 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} \\ &= (.0011)_2 \end{aligned}$$

$$(11.1875)_{10} = (1011.0011)_2$$

# Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

[http://numericalmethods.eng.usf.edu/topics/binary\\_representation.html](http://numericalmethods.eng.usf.edu/topics/binary_representation.html)

**THE END**

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