Chapter 03.00G

Physical Problem for Nonlinear Equations Mechanical Engineering

Problem Statement

To make the fulcrum (Figure 1) of a bascule bridge, a long hollow steel shaft called the trunnion is shrink fit into a steel hub. The resulting steel trunnion-hub assembly is then shrink fit into the girder of the bridge.

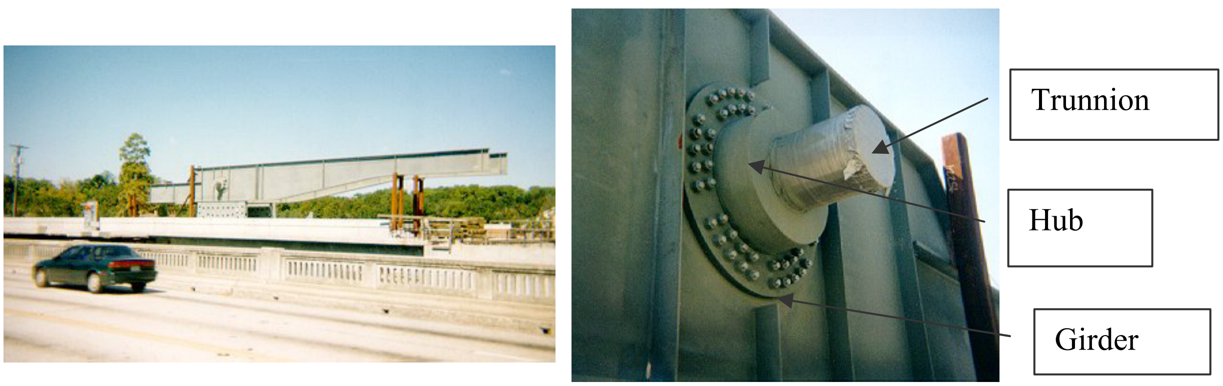

Figure 1 Trunnion-Hub-Girder (THG) assembly.

This is done by first immersing the trunnion in a cold medium such as dry-ice/alcohol mixture. After the trunnion reaches the steady state temperature of the cold medium, the trunnion outer diameter contracts. The trunnion is taken out of the medium and slid though the hole of the hub (Figure 2).

When the trunnion heats up, it expands and creates an interference fit with the hub. In 1995, on one of the bridges in Florida, this assembly procedure did not work as designed. Before the trunnion could be inserted fully into the hub, the trunnion got stuck. So a new trunnion and hub had to be ordered at a cost of $50,000. Coupled with construction delays, the total loss was more than hundred thousand dollars.

Why did the trunnion get stuck? This was because the trunnion had not contracted enough to slide through the hole.

Now the same designer is working on making the fulcrum for another bascule bridge. Can you help him/her so that he does not make the same mistake?

For this new bridge, he needs to fit a hollow trunnion of outside diameter 12.363" in a hub of inner diameter 12.358". His plan is to put the trunnion in dry ice/alcohol mixture (temperature of the fluid - dry ice/alcohol mixture is $-108°\text{F}$) to contract the trunnion so that it can be slided through the hole of the hub. To slide the trunnion without sticking, he has also specified a diametrical clearance of at least 0.01" between the trunnion and the hub. Assuming the room temperature is $80°\text{F}$, is immersing it in dry-ice/alcohol mixture a correct decision? What temperature does he need to cool the trunnion to so that he gets the desired contraction?

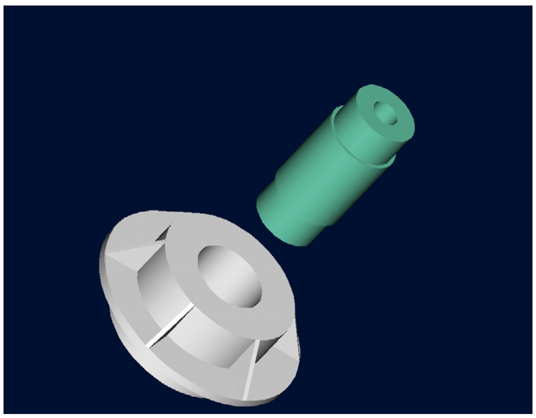

Figure 2 Trunnion slided through the hub after contracting

Solution

Looking at the designer's record for the previous bridge (where the trunnion got stuck in the hub), it was found that he/she used the thermal expansion coefficient at room temperature to calculate the contraction in the trunnion diameter. In that case the reduction, ΔD in the outer diameter of the trunnion is

$$\Delta D = D\alpha\Delta T \quad (1)$$

where

D = outer diameter of the trunnion,

α = coefficient of thermal expansion coefficient at room temperature, and

ΔT = change in temperature,

Given

D = 12.363"

$\alpha = 6.817\times10^{-6}\ \text{in/in/}^0\text{F}$ at 80^0F

$$\begin{aligned}\Delta T &= T_{fluid} - T_{room}\\ &= -108-80\\ &= -188^0\text{F}\end{aligned}$$

where

T_{fluid} = temperature of dry-ice/alcohol mixture

T_{room} = room temperature

the reduction in the trunnion outer diameter is given by

$$\Delta D = (12.363)\left(6.47\times10^{-6}\right)(-188)$$

$= -0.01504''$

So the trunnion is predicted to reduce in diameter by 0.01504”. But, is this enough reduction in diameter? As per specifications, he needs the trunnion to contract by

= trunnion outside diameter - hub inner diameter + diametral clearance

$= 12.363''-12.358''+0.01''$

$= 0.015''$

So according to his calculations, immersing the steel trunnion in dry-ice/alcohol mixture gives the desired contraction of $0.015''$ as he is predicting a contraction of $0.01504''$.

But as shown in Figure 3, the thermal expansion coefficient of steel decreases with temperature and is not constant over the range of temperature the trunnion goes through. Hence the above formula (Equation 1) would overestimate the thermal contraction. This is the mistake he made in the calculations for the earlier bridge.

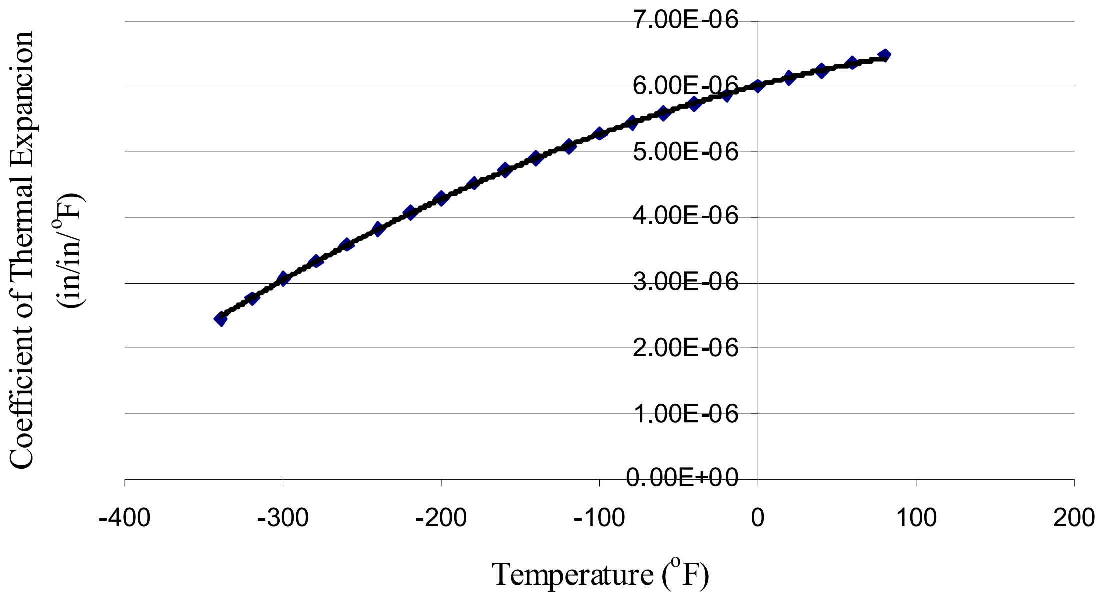

Figure 3 Varying thermal expansion coefficient as a function of temperature for cast steel.

The contraction in the diameter for the trunnion for which the thermal expansion coefficient varies as a function of temperature is given by

$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha dT \qquad (2)$$

Note that Equation (2) reduces to Equation (1) if the coefficient of thermal expansion is assumed to be constant. In Figure 3, the thermal expansion coefficient of a typical cast steel is approximated by a second order polynomial[1] 1as

$$\alpha = -1.2278\times 10^{-11}T^2 + 6.1946\times 10^{-9}T + 6.0150\times 10^{-6}$$

Since the desired contraction is at least $0.015"$, that is, $\Delta D = -0.015"$,

$$-0.015 = 12.363\int_{80}^{T_{fluid}}\left(-1.2278\times 10^{-11}T^2 + 6.1946\times 10^{-9}T + 6.015\times 10^{-6}\right)dT$$

$$-0.015 = 12.363\left[-1.2278\times 10^{-11}\frac{T^3}{3} + 6.1946\times 10^{-9}\frac{T^2}{2} + 6.015\times 10^{-6}T\right]_{80}^{T_{fluid}}$$

$$-0.015 = 12.363(-0.40927\times 10^{-11}T_{fluid}^3 + 0.30973\times 10^{-8}T_{fluid}^2 + 0.60150\times 10^{-5}T_{fluid}$$

$$-0.49893\times 10^{-3})$$

$$\mathrm{f}(T_f) = -0.50598\times 10^{-10}T_f^3 + 0.38292\times 10^{-7}T_f^2 + 0.74363\times 10^{-4}T_f + 0.88318\times 10^{-2} = 0$$

One can solve this nonlinear equation to find the minimum fluid temperature needed to cool down the trunnion and get the desired contraction. Is cooling in dry-ice/alcohol mixture still your recommendation? You will be surprised that it would not be the correct decision to make.

Topic	NONLINEAR EQUATIONS
Sub Topic	Physical Problem
Summary	A physical problem of finding how much cooling a shaft needs to be shrink fit into a hollow hub. The temperature to which the cooling needs to be done is modeled as a nonlinear equation.
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Date	December 7, 2008
Web Site	http://numericalmethods.eng.usf.edu

[1] The second order polynomial is derived using regression analysis which is another mathematical procedure where numerical methods are employed. Regression analysis approximates discrete data such as the thermal expansion coefficient vs. temperature data as a continuous function. This is an excellent example of where one has to use numerical methods of more than one procedure to solve a real life problem.