Secant Method

Mechanical Engineering Majors

Authors: Autar Kaw, Jai Paul

http://numericalmethods.eng.usf.edu

Transforming Numerical Methods Education for STEM Undergraduates

Secant Method

http://numericalmethods.eng.usf.edu

Secant Method – Derivation

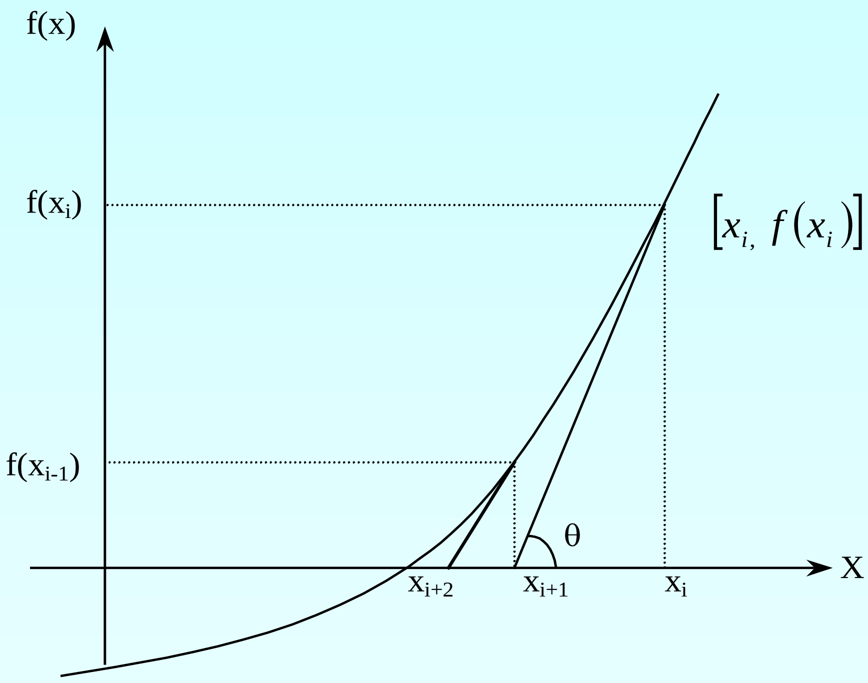


Figure 1 Geometrical illustration of the Newton-Raphson method.

Newton's Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \qquad (1)$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \qquad (2)$$

Substituting Equation (2) into Equation (1) gives the Secant method

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method – Derivation

The secant method can also be derived from geometry:

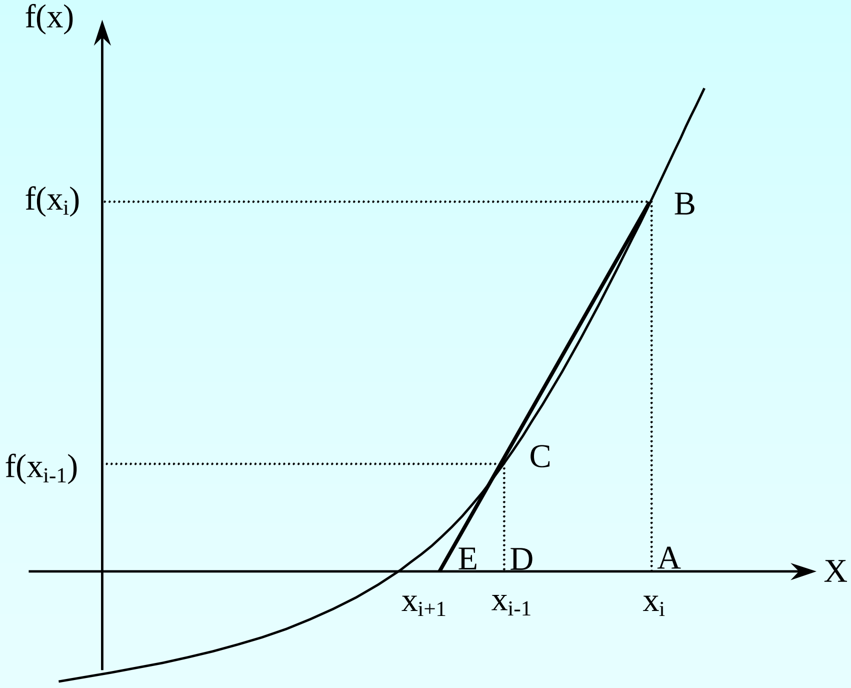


Figure 2 Geometrical representation of the Secant method.

The Geometric Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

can be written as

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

On rearranging, the secant method is given as

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$|\in_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

Step 2

Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example 1

A trunnion has to be cooled before it is shrink fitted into a steel hub

The equation that gives the temperature x to which the trunnion has to be cooled to obtain the desired contraction is given by the following equation.

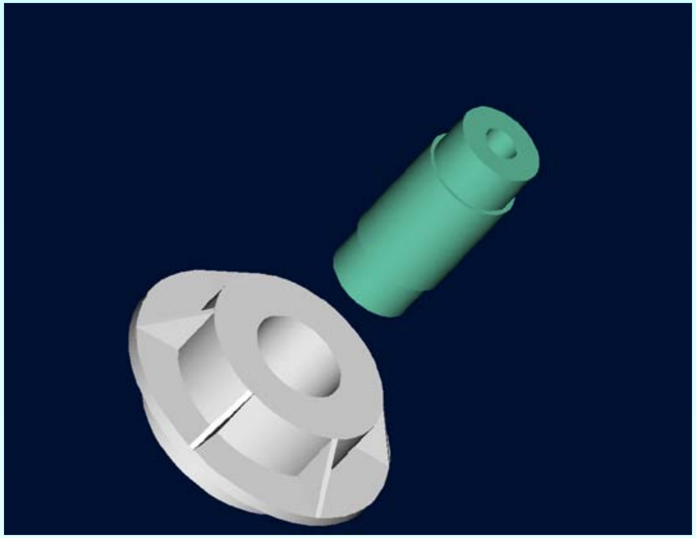
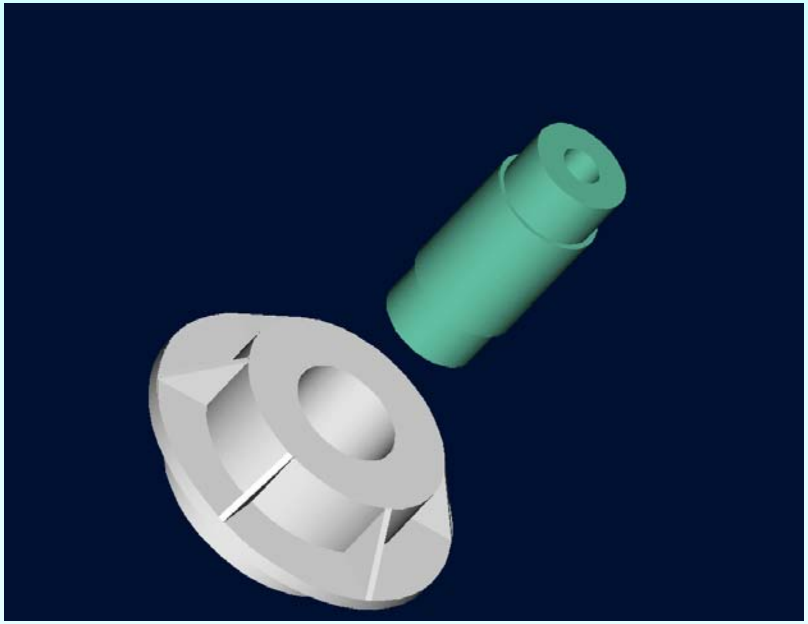


Figure 3 Trunnion to be slid through the hub after contracting.

$$f(x) = -0.50598\times10^{-10}x^3 + 0.38292\times10^{-7}x^2 + 0.74363\times10^{-4}x + 0.88318\times10^{-2} = 0$$

Example 1 Cont.



Use the secant method of finding roots of equations

a) To find the temperature *x* to which the trunnion has to be cooled. Conduct three iterations to estimate the root of the above equation.

b) Find the absolute relative approximate error at the end of each iteration, and

c) the number of significant digits at least correct at the end of each iteration.

Example 1 Cont.

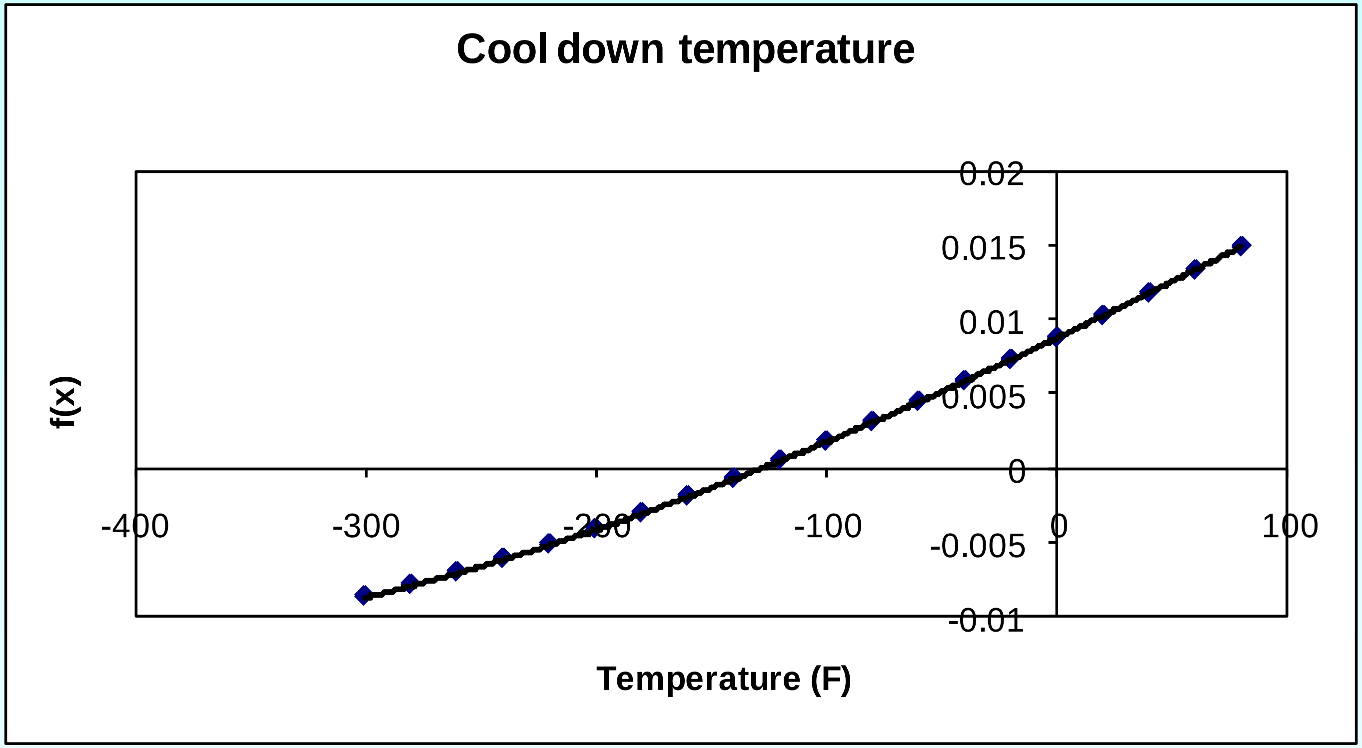


Figure 4 Graph of the function f(x).

$$f(x) = -0.50598 \times 10^{-10} x^3 - 0.38292 \times 10^{-7} x^2 + 0.74363 \times 10^{-4} x + 0.88318 \times 10^{-2} = 0$$

Example 1 Cont.

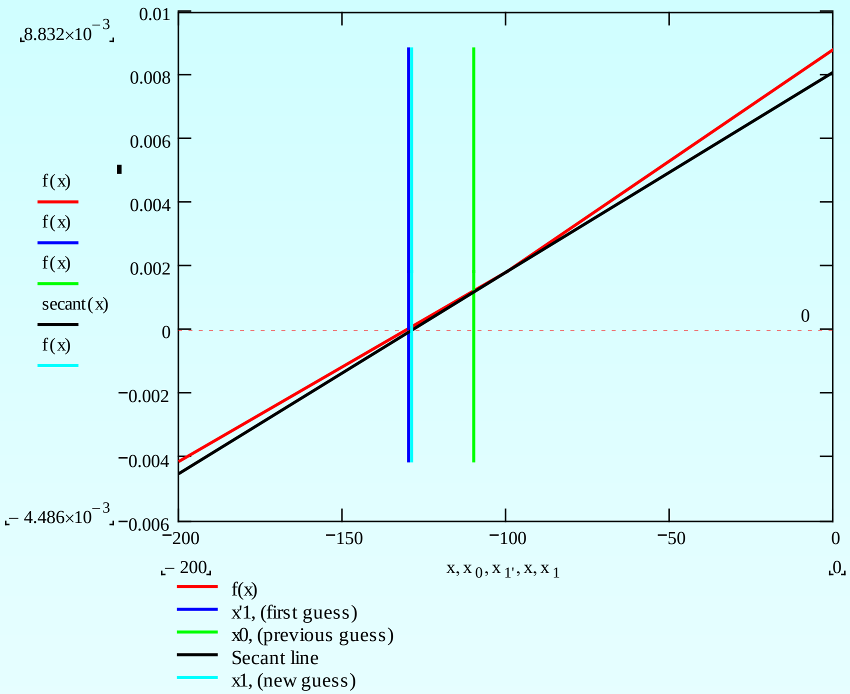


Figure 5 Graph of the estimated root after Iteration 1.

Initial guesses: $x_{-1} = -110, x_0 = -130$

<u>Iteration 1</u>

The estimate of the root is

$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}$$

$$x_1 = -130 - \frac{(7.7091\times10^{-5})(-130-(-110))}{(7.7091\times10^{-5})-(1.1825\times10^{-3})}$$

$$= -128.78$$

The absolute relative approximate error is

$$|\epsilon_a| = \left|\frac{x_1 - x_0}{x_1}\right| \times 100 = 0.95051\%$$

The number of significant digits at least correct is 1.

Example 1 Cont.

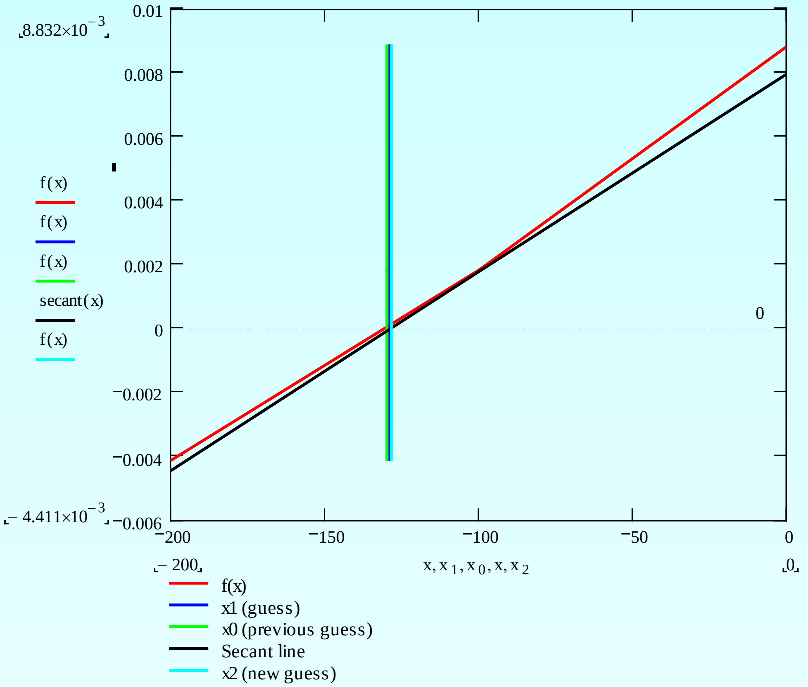


Figure 6 Graph of the estimated root after Iteration 2.

<u>Iteration 2</u>

The estimate of the root is

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = -128.78 - \frac{\left(-1.3089\times10^{-6}\right)\left(-128.78-(-130)\right)}{\left(-1.3089\times10^{-6}\right)-\left(-7.7091\times10^{-5}\right)}$$

$$= -128.75$$

The absolute relative approximate error is

$$\left|\in_a\right| = \left|\frac{x_2 - x_1}{x_2}\right| \times 100 = 0.016419\%$$

The number of significant digits at least correct is 3.

Example 1 Cont.

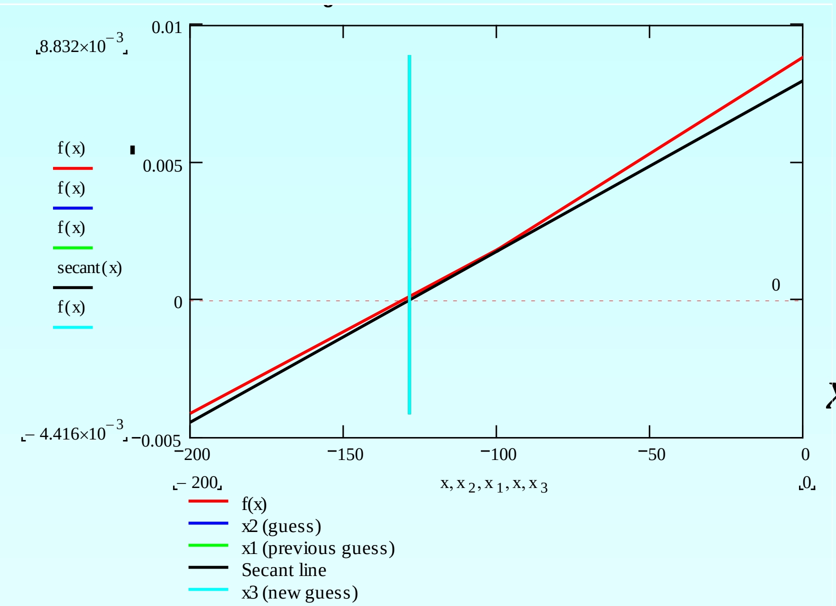


Figure 7 Graph of the estimated root after Iteration 3.

<u>Iteration 3</u>

The estimate of the root is

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = -128.7548 - \frac{(1.5241\times10^{-9})(-128.75-(-128.78))}{(1.5241\times10^{-9})-(-1.3089\times10^{-6})}$$

$$= -128.75$$

The absolute relative approximate error is

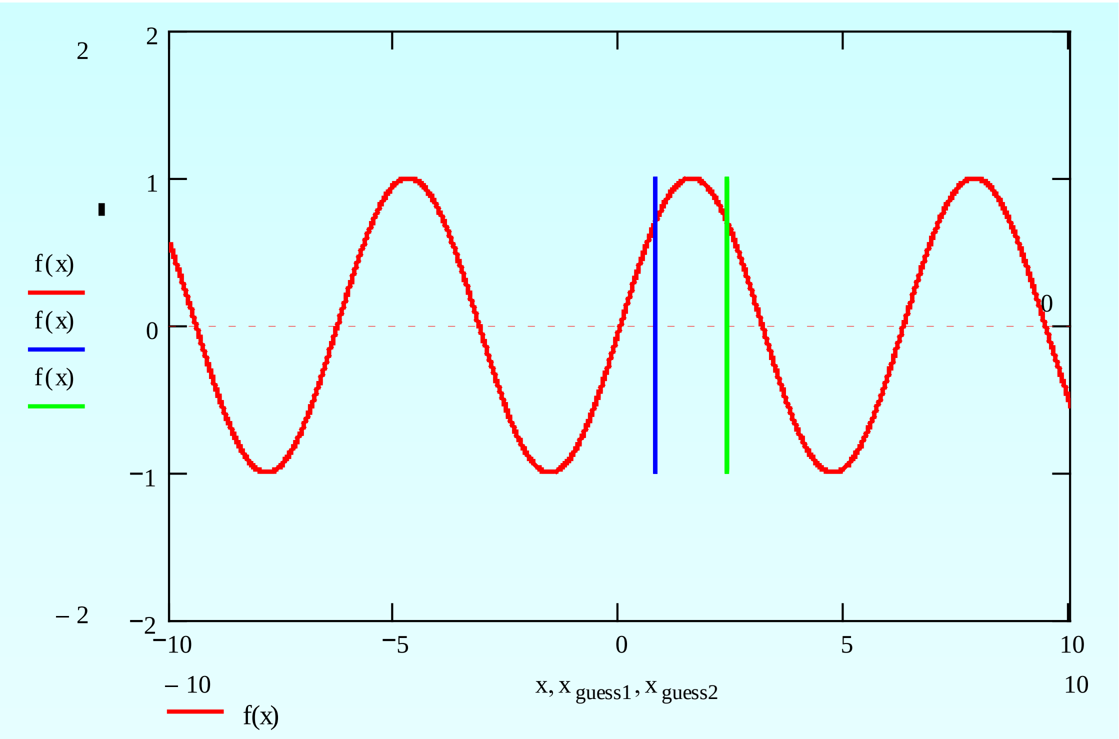
$$|\epsilon_a| = \left|\frac{x_3 - x_2}{x_3}\right|\times100 = 1.9097\times10^{-5}\%$$

The number of significant digits at least correct is 6.

Advantages

- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

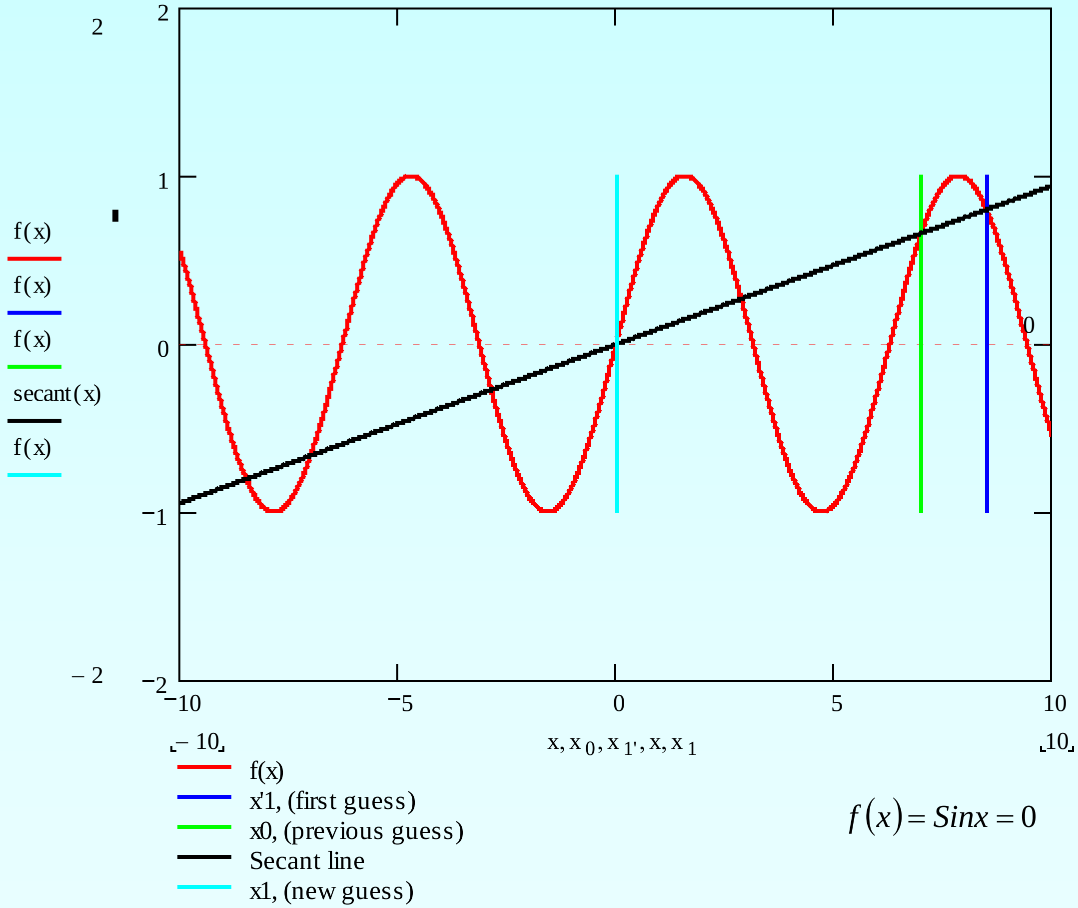
Drawbacks



$f(x) = Sin(x) = 0$

Division by zero

Drawbacks (continued)



$f(x) = Sinx = 0$

Root Jumping

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/secant_method.html

THE END

http://numericalmethods.eng.usf.edu