Chapter 09.00G

Physical Problem for Optimization Mechanical Engineering

Problem Statement

During a baseball game, the center-fielder catches a baseball hit by a batter, close to the outer perimeter of the field. If she wishes to make a double-play, she must throw the baseball to home plate in order to get an advancing base-runner out. As an engineering student, she is aware that that while this distance will tax her limits of athleticism, she can optimize the range of the thrown ball by throwing it at a certain angle.

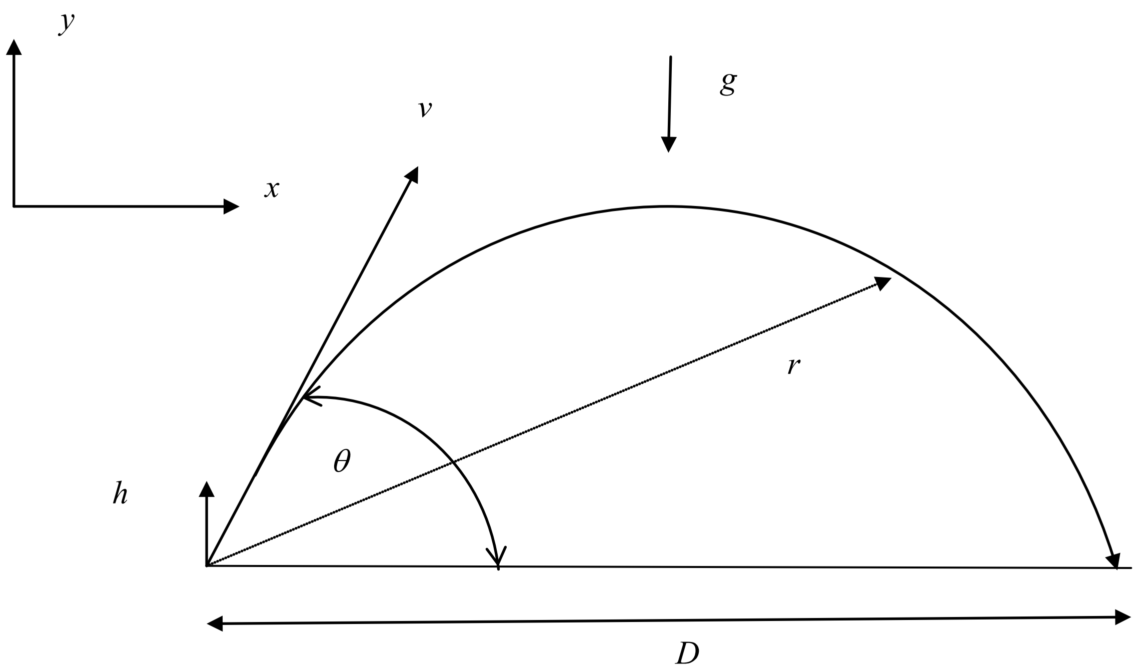

Figure 1: Depiction of the projectile motion of a baseball

Given the following information $v = 24.5\ m/s, g = 9.81 m/s^2\ and\ h = 1.96 m$

In order to maximize the distance D traveled, at what angle relative to the horizontal should she throw the ball? What is this maximum distance?

Solution

Starting with a general position vector for projectile motion disregarding non-conservative forces; the position vector is modeled by

$$\begin{aligned} r &= \left(x_0 + v_0 \cos(\theta)t + \frac{1}{2}at^2 \right)i + \left(y_0 + v_0 \sin(\theta)t + \frac{1}{2}at^2 \right)j \\ &= \left(v\cos(\theta)t\right)i + \left(h + v\sin(\theta)t - \frac{1}{2}gt^2 \right)j \end{aligned} \quad (1)$$

Where

v=velocity at which the ball is thrown
g=acceleration due to gravity
h=height from the ground at which the ball is thrown.
θ=angle at which the baseball is thrown.

When $y = 0$ at the landing spot of the projectile, the j component of the velocity vector equals zero

$$h + v\sin(\theta)t - \frac{g}{2}t^2 = 0$$

$$0 = \frac{g}{2}t^2 - v\sin(\theta)t - h \quad (2)$$

This is a quadratic equation of t . Solving for t using the quadratic formula yields

$$t = \frac{v\sin(\theta) \pm \sqrt{v^2 \sin^2(\theta) - 4(-h)\left(\frac{g}{2}\right)}}{g} \quad (3)$$

Using the positive root and plugging the result into the i component of Equation (1) gives

$$D = v\cos(\theta)\left(\frac{v\sin(\theta)}{g} + \frac{\sqrt{v^2 \sin^2(\theta) + 2hg}}{g} \right) \quad (4)$$

Substituting the known values into the equation gives us a one-dimensional optimization problem as follows:

$$D = 61.25\cos(\theta)\sin(\theta) + 2.45\cos(\theta)\sqrt{625\sin^2(\theta) + 40} \quad (5)$$

OPTIMIZATION	
Topic	Physical problem
Summary	A physical problem of finding the maximum distance of a projectile.
Major	Mechanical Engineering
Authors	Russell Aveney, Autar Kaw
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Web Site	http://numericalmethods.eng.usf.edu