Central Divided Difference

Topic: Differentiation

Major: General Engineering

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Definition

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x) - f(x - \Delta x)}{\Delta x}$$

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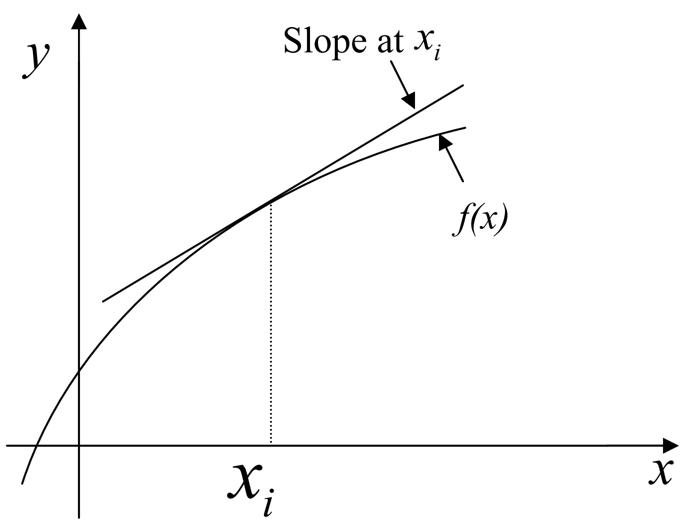
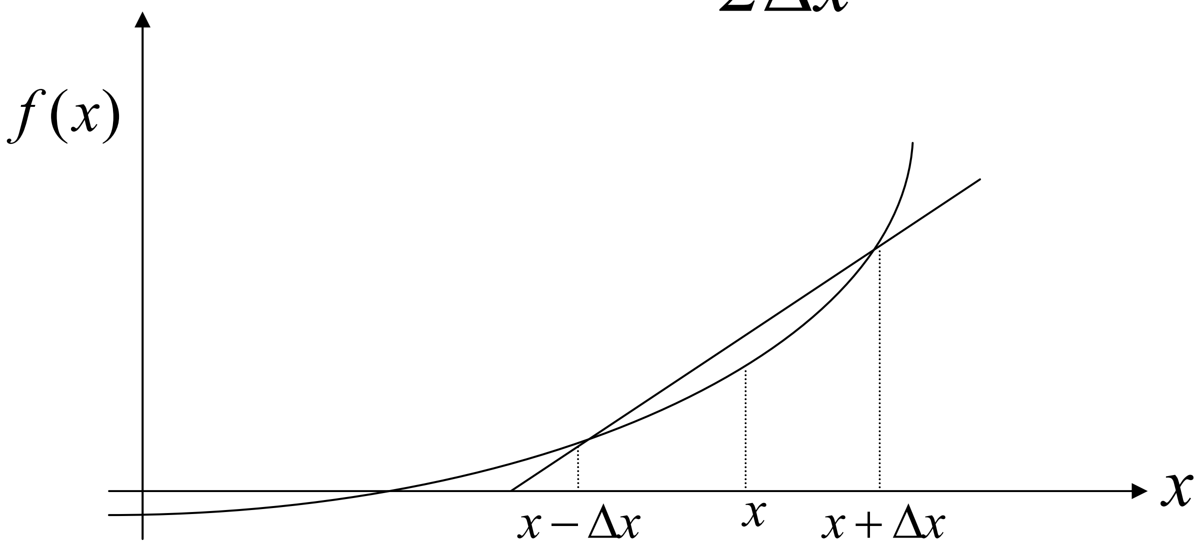


Central Divided Difference

$$f'(x) \cong \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$



$$f'(x_i) \cong \frac{f(x_i+\Delta x) - f(x_i-\Delta x)}{2\Delta x}$$

Example

Example:

The velocity of a rocket is given by

$$v(t) = 2000 \ln\left[\frac{14\times 10^4}{14\times 10^4 - 2100t}\right] - 9.8t, 0 \le t \le 30$$

where v given in m/s and t is given in seconds. Use central difference approximation of the first derivative of $v(t)$ to calculate the acceleration at $t = 16s$. Use a step size of $\Delta t = 2s$.

Solution:

$$a(t_i) \cong \frac{v(t_{i+1}) - v(t_i - 1)}{2\Delta t}$$

$$t_i = 16$$

Example (contd.)

$$\Delta t = 2$$

$$t_{i+1} = t_i + \Delta t \quad = 16 + 2 = 18$$

$$t_{i-1} = t_i - \Delta t \quad = 16 - 2 = 14$$

$$a(16) = \frac{v(18) - v(14)}{2(2)} = \frac{v(18) - v(14)}{4}$$

$$v(18) = 2000 \ln\left[\frac{14 \times 10^4}{14 \times 10^4 - 2100(18)}\right] - 9.8(18) = 453.02 m/s$$

$$v(14) = 2000 \ln\left[\frac{14 \times 10^4}{14 \times 10^4 - 2100(14)}\right] - 9.8(14) = 334.24 m/s$$

Example (contd.)

Hence

$$a(16) = \frac{v(18) - v(14)}{4} = 453.02 - 334.24 = 29.695 m/s^2$$

The exact value of $a(16)$ can be calculated by differentiating

$$v(t) = 2000 \ln\left[\frac{14 \times 10^4}{14 \times 10^4 - 2100t}\right] - 9.8t$$

as

$$a(t) = \frac{d}{dt}[v(t)] = \frac{-4040 - 29.4t}{-200 + 3t}$$

$$a(16) = 29.674 m/s^2$$

Example (contd.)

The absolute relative true error is

$$|\varepsilon_t| = \left|\frac{True Value - Approximate Value}{True Value}\right| \times 100$$

$$= \left|\frac{29.674 - 29.695}{29.674}\right| \times 100$$

$$= 0.070769\,\%$$

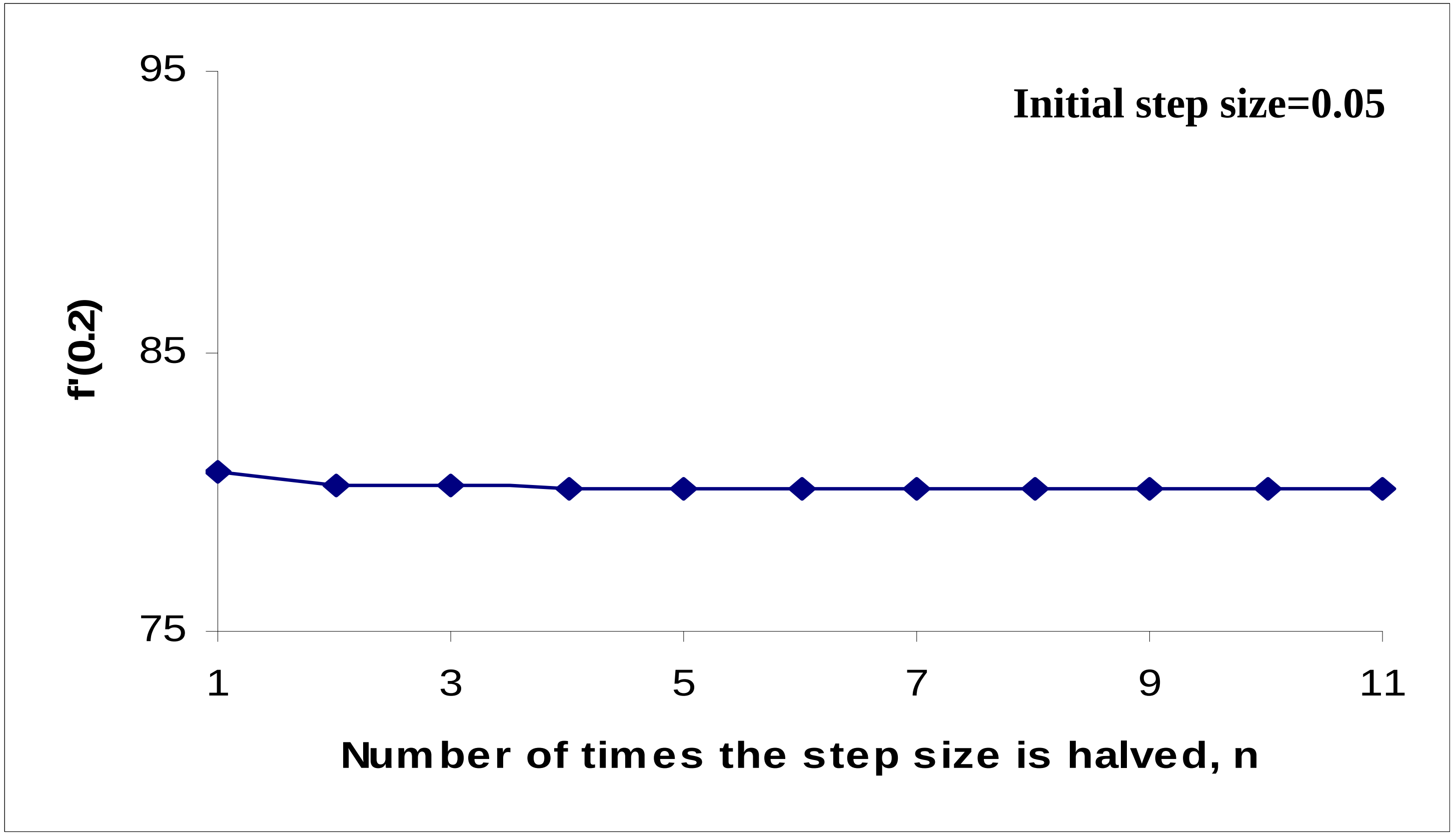
Effect Of Step Size

$$f(x) = 9e^{4x}$$

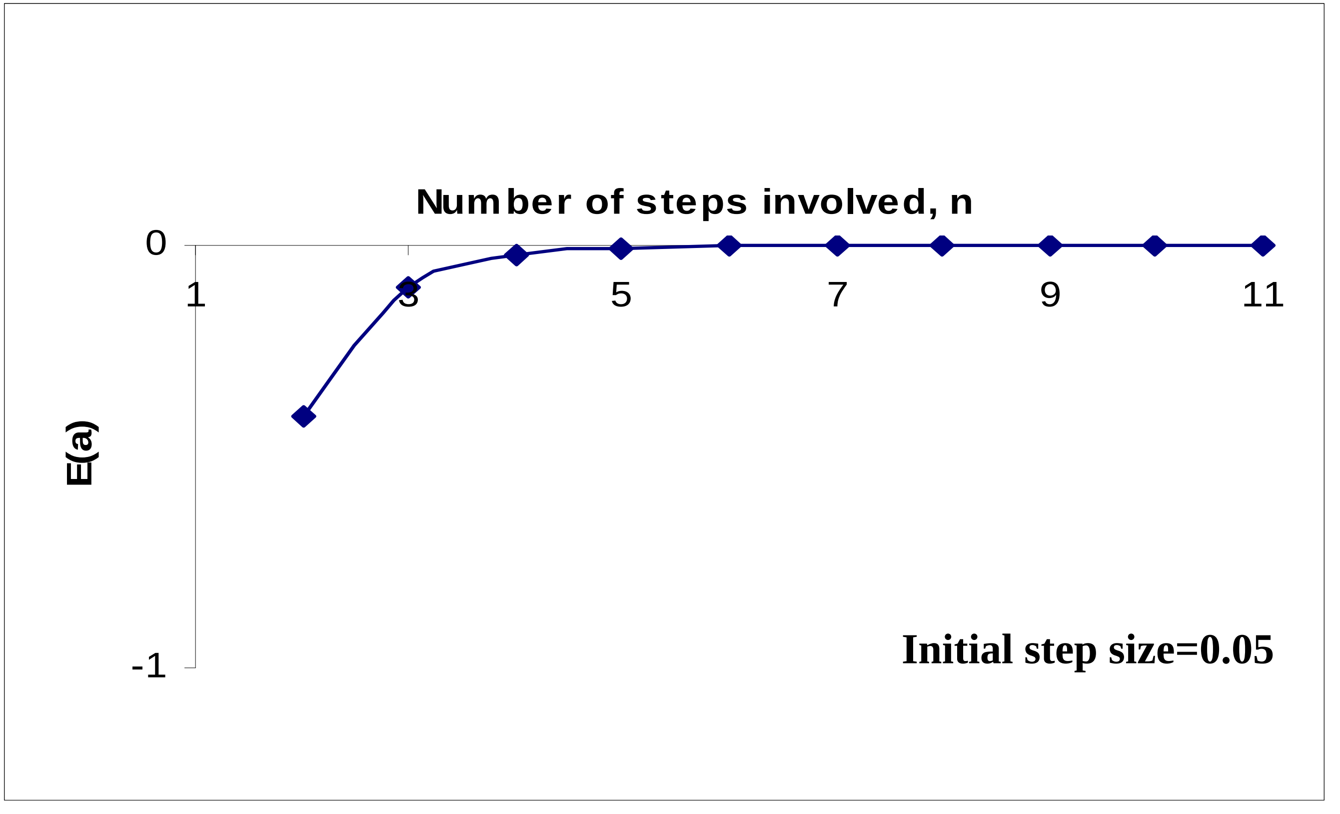
Value of $f'(0.2)$ Using Central Divided Difference difference method.

h	$f'(0.2)$	E_a	$\lvert\varepsilon_a\rvert\%$	Significant digits	E_t	$\lvert\varepsilon_t\rvert\%$
0.05	80.65467				-0.53520	0.668001
0.025	80.25307	-0.4016	0.500417	1	-0.13360	0.16675
0.0125	80.15286	-0.100212	0.125026	2	-0.03339	0.041672
0.00625	80.12782	-0.025041	0.031252	3	-0.00835	0.010417
0.003125	80.12156	-0.00626	0.007813	3	-0.00209	0.002604
0.001563	80.12000	-0.001565	0.001953	4	-0.00052	0.000651
0.000781	80.11960	-0.000391	0.000488	5	-0.00013	0.000163
0.000391	80.11951	-9.78E-05	0.000122	5	-0.00003	4.07E-05
0.000195	80.11948	-2.45E-05	3.05E-05	6	-0.00001	1.02E-05
9.77E-05	80.11948	-6.11E-06	7.63E-06	6	0.00000	2.54E-06
4.88E-05	80.11947	-1.53E-06	1.91E-06	7	0.00000	6.36E-07

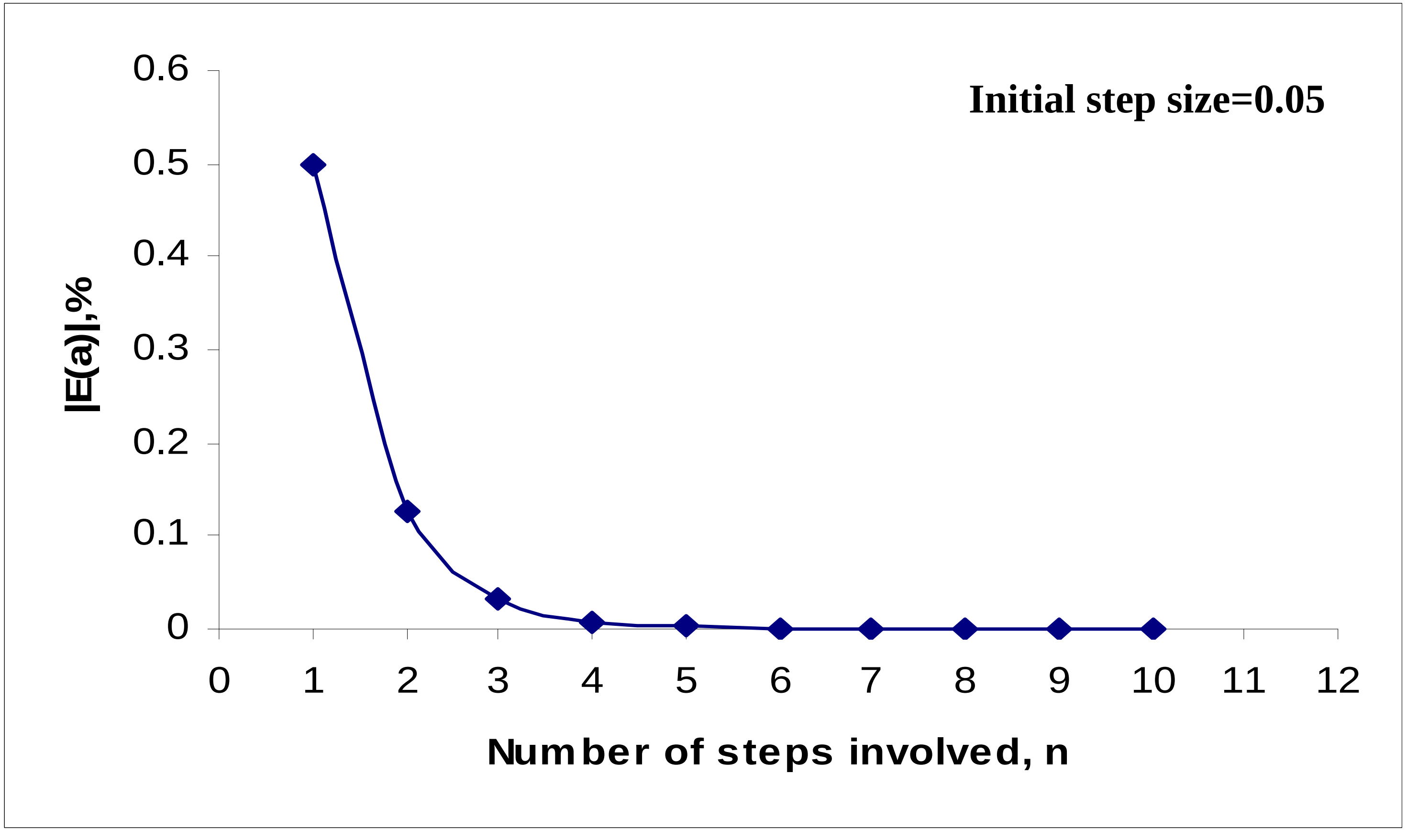
Effect of Step Size in Central Divided Difference Method



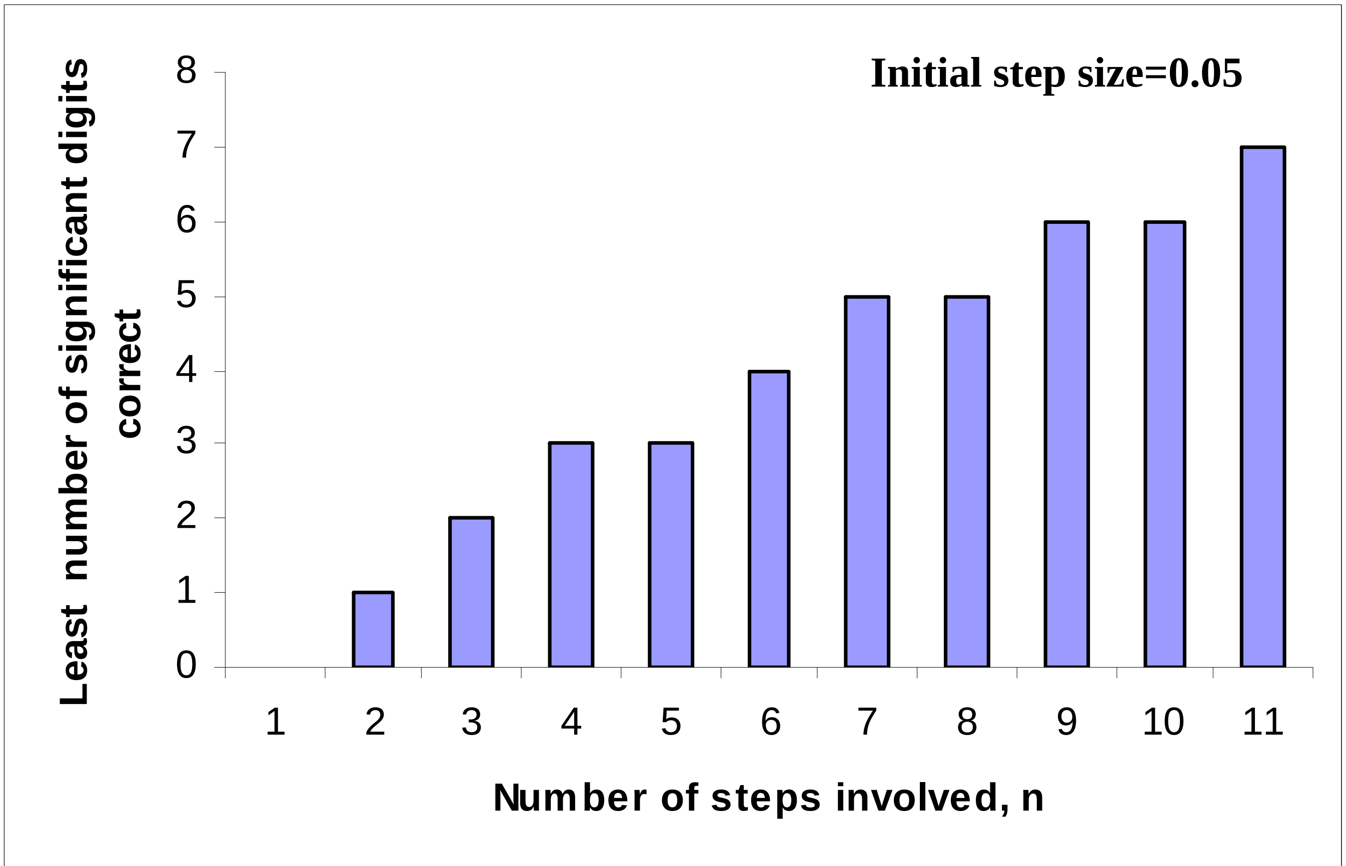
Effect of Step Size on Approximate Error



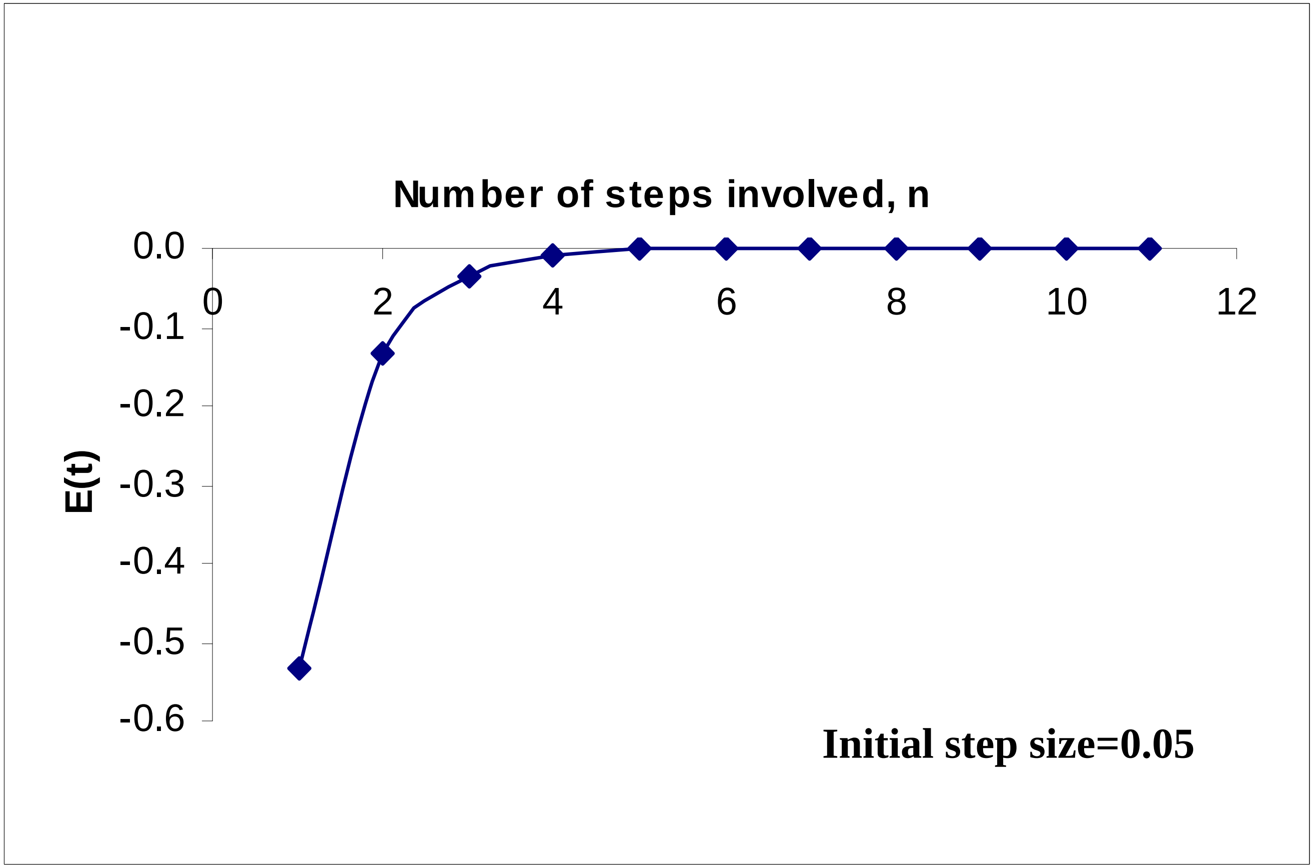
Effect of Step Size on Absolute Relative Approximate Error



Effect of Step Size on Least Number of Significant Digits Correct



Effect of Step Size on True Error



Effect of Step Size on Absolute Relative True Error