Forward Divided Difference

Topic: Differentiation

Major: General Engineering

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Definition

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

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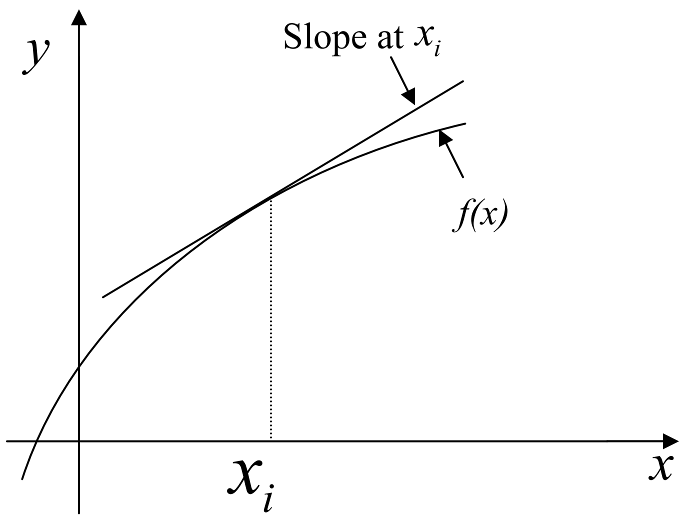
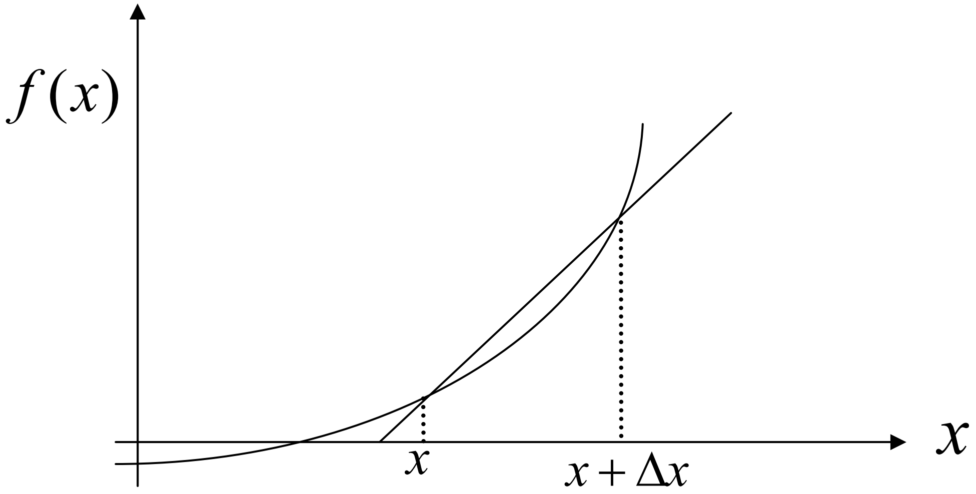


Forward Divided Difference

$$f'(x) \cong \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



$$f'(x_i) \cong \frac{f(x_i+\Delta x) - f(x_i)}{\Delta x}$$

Example

Example:

The velocity of a rocket is given by

$$v(t) = 2000 \ln\left[\frac{14\times10^4}{14\times10^4 - 2100t}\right] - 9.8t, 0 \le t \le 30$$

where v given in m/s and t is given in seconds. Use forward difference approximation of the first derivative of $v(t)$ to calculate the acceleration at $t = 16s$. Use a step size of $\Delta t = 2s$.

Solution:

$$a(t_i) \cong \frac{v(t_{i+1}) - v(t_i)}{\Delta t}$$

$$t_i = 16$$

Example (contd.)

$$\Delta t = 2$$

$$t_{i+1} = t_i + \Delta t \quad = 16 + 2 = 18$$

$$a(16) = \frac{v(18) - v(16)}{2}$$

$$v(18) = 2000\ln\left[\frac{14\times10^4}{14\times10^4 - 2100(18)}\right] - 9.8(18) \quad = 453.02 m/s$$

$$v(16) = 2000\ln\left[\frac{14\times10^4}{14\times10^4 - 2100(16)}\right] - 9.8(16) \quad = 392.07 m/s$$

Example (contd.)

Hence

$$a(16) = \frac{\nu(18) - \nu(16)}{2} = 453.02 - 392.07 = 30.475 m/s^2$$

The exact value of $a(16)$ can be calculated by differentiating

$$\nu(t) = 2000 \ln\left[\frac{14 \times 10^4}{14 \times 10^4 - 2100t}\right] - 9.8t$$

as

$$a(t) = \frac{d}{dt}[\nu(t)] = \frac{-4040 - 29.4t}{-200 + 3t}$$

$$a(16) = 29.674 m/s^2$$

Example (contd.)

The absolute relative true error is

$$|\varepsilon_t| = \left| \frac{TrueValue - ApproximateValue}{TrueValue} \right| \times 100$$

$$= \left| \frac{29.674 - 30.475}{29.674} \right| \times 100$$

$$= 2.6993\%$$

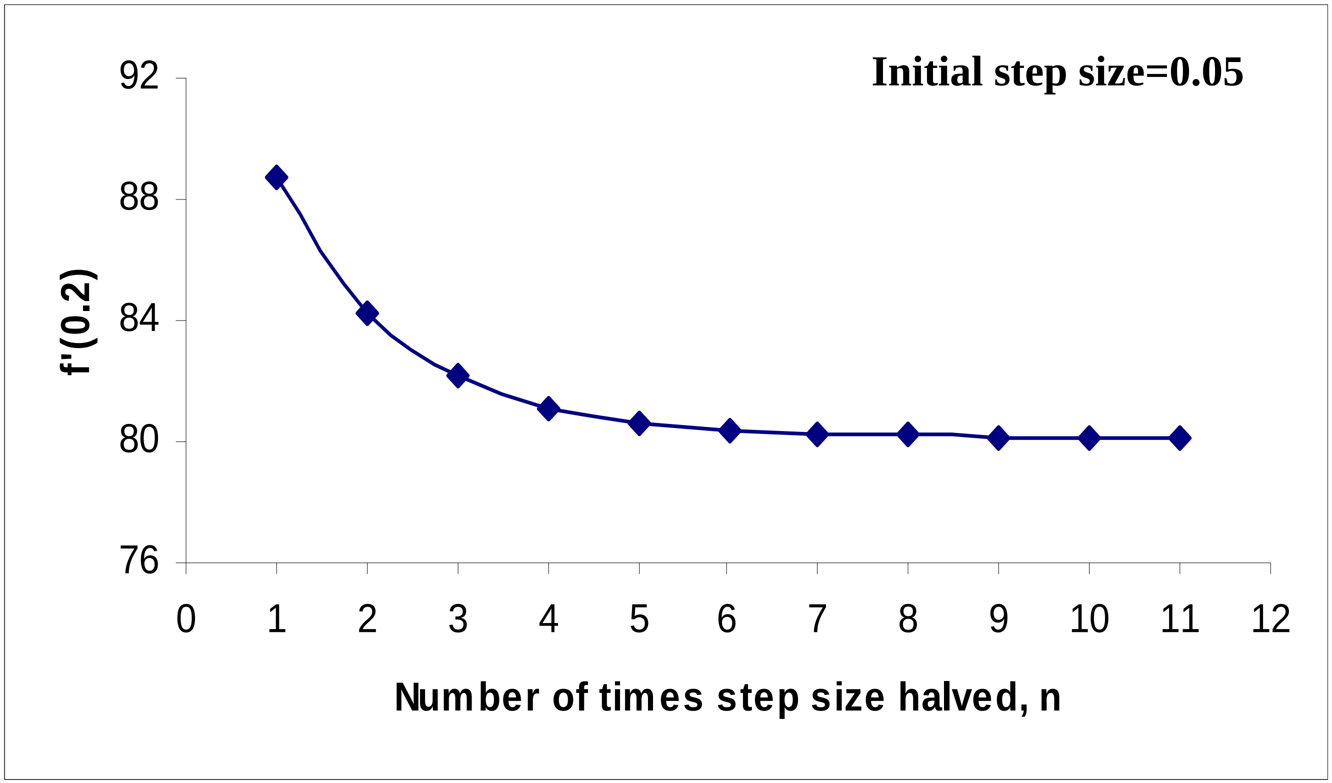
Effect Of Step Size

$$f(x) = 9e^{4x}$$

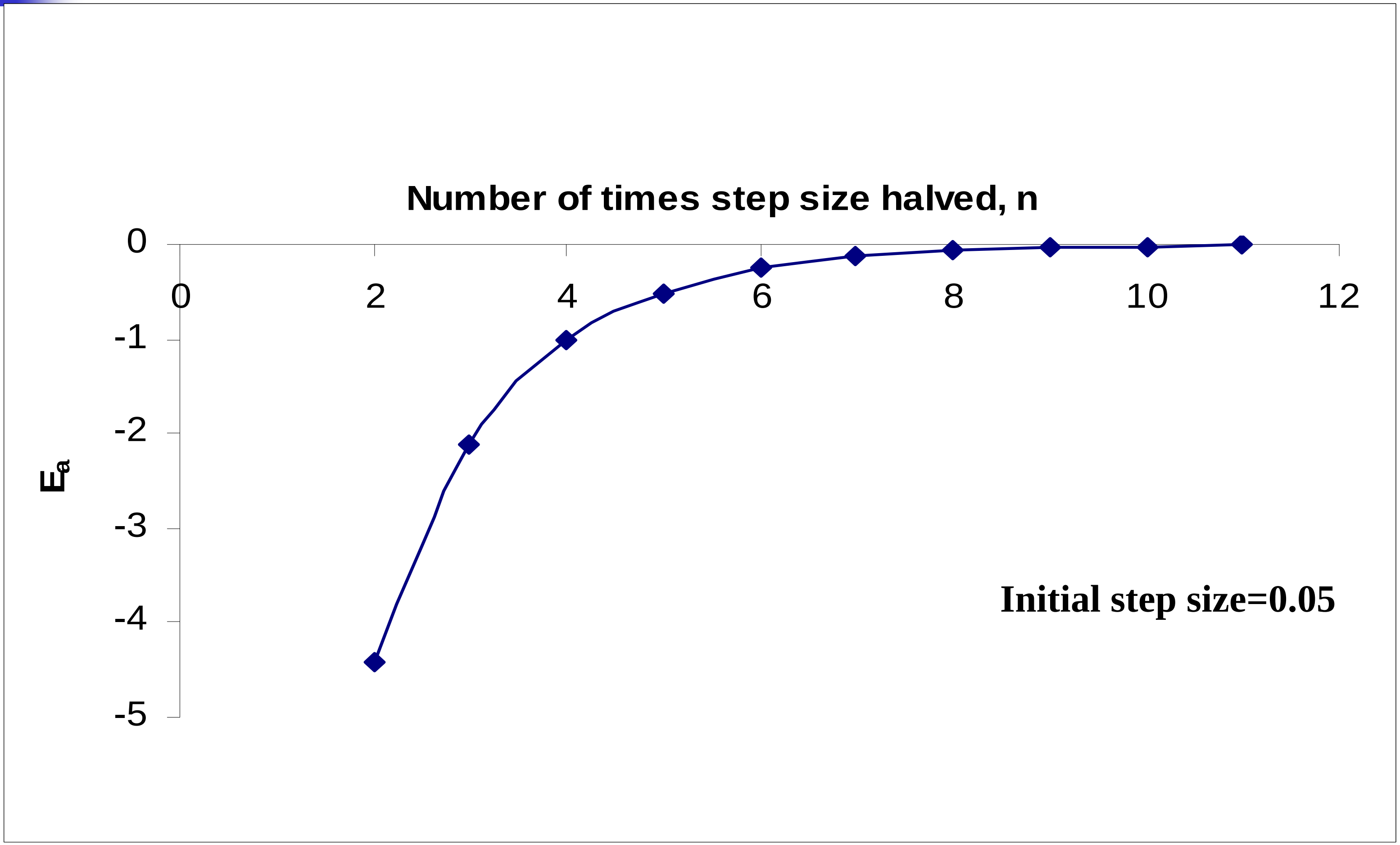
Value of $f'(0.2)$ Using forward difference method.

h	$f'(0.2)$	E_a	$\|\varepsilon_a\|\%$	Significant digits	E_t	$\|\varepsilon_t\|\%$
0.05	88.69336				-8.57389	10.70138
0.025	84.26239	-4.430976	5.258546	0	-4.14291	5.170918
0.0125	82.15626	-2.106121	2.563555	1	-2.03679	2.542193
0.00625	81.12937	-1.0269	1.265756	1	-1.00989	1.260482
0.003125	80.62231	-0.507052	0.628923	1	-0.50284	0.627612
0.001563	80.37037	-0.251944	0.313479	2	-0.25090	0.313152
0.000781	80.24479	-0.125579	0.156494	2	-0.12532	0.156413
0.000391	80.18210	-0.062691	0.078186	2	-0.06263	0.078166
0.000195	80.15078	-0.031321	0.039078	3	-0.03130	0.039073
9.77E-05	80.13512	-0.015654	0.019535	3	-0.01565	0.019534
4.88E-05	80.12730	-0.007826	0.009767	3	-0.00782	0.009766

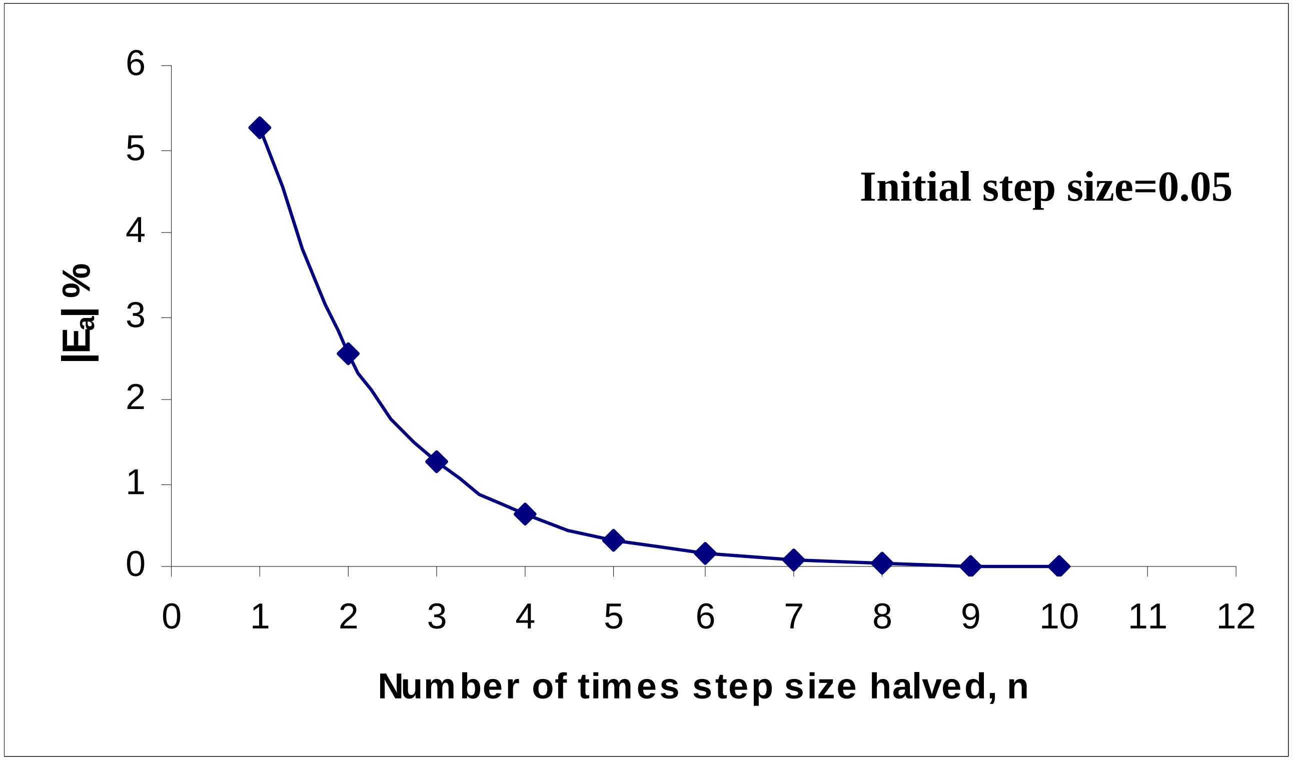
Effect of Step Size in Forward Divided Difference Method



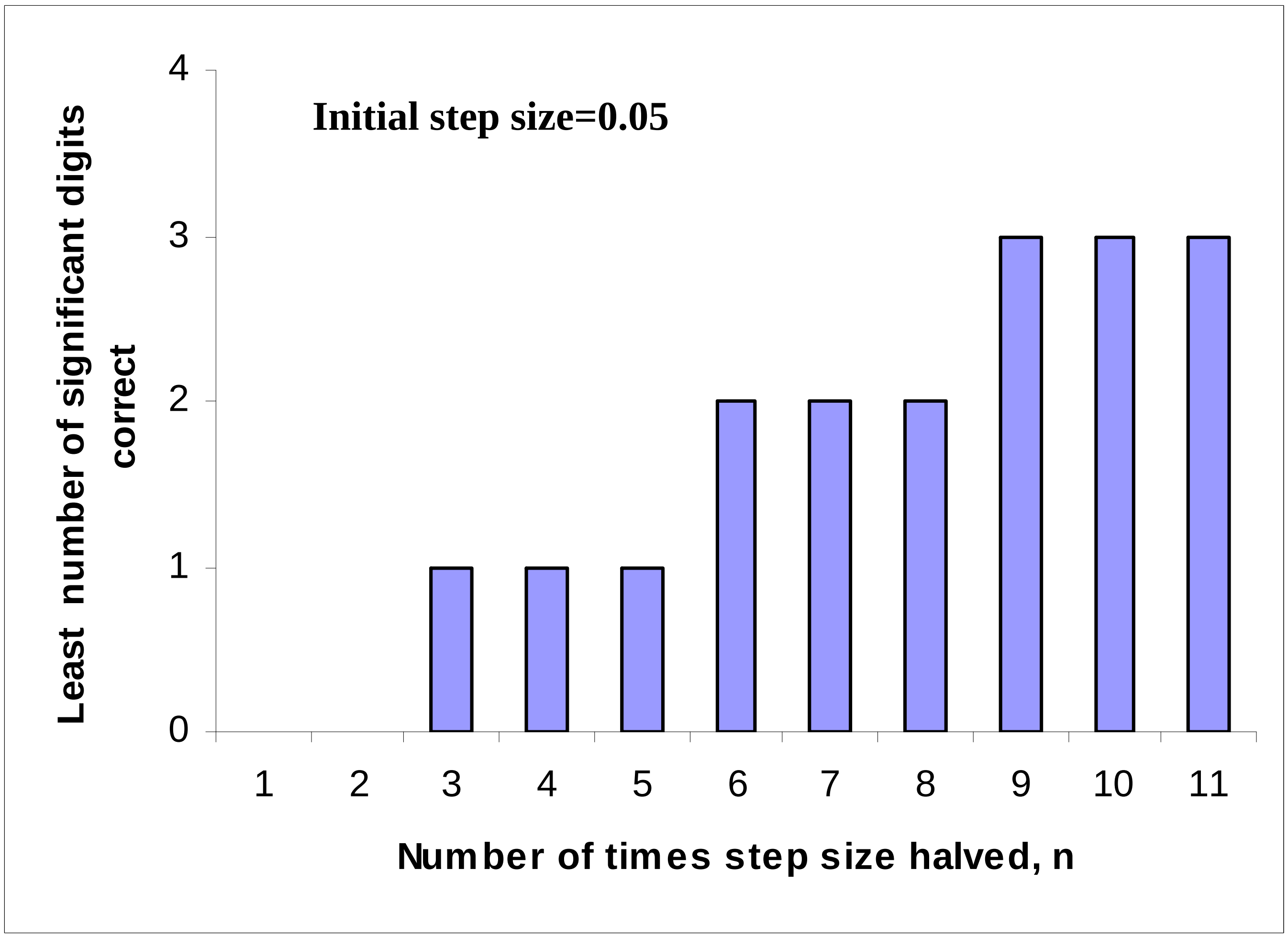
Effect of Step Size on Approximate Error



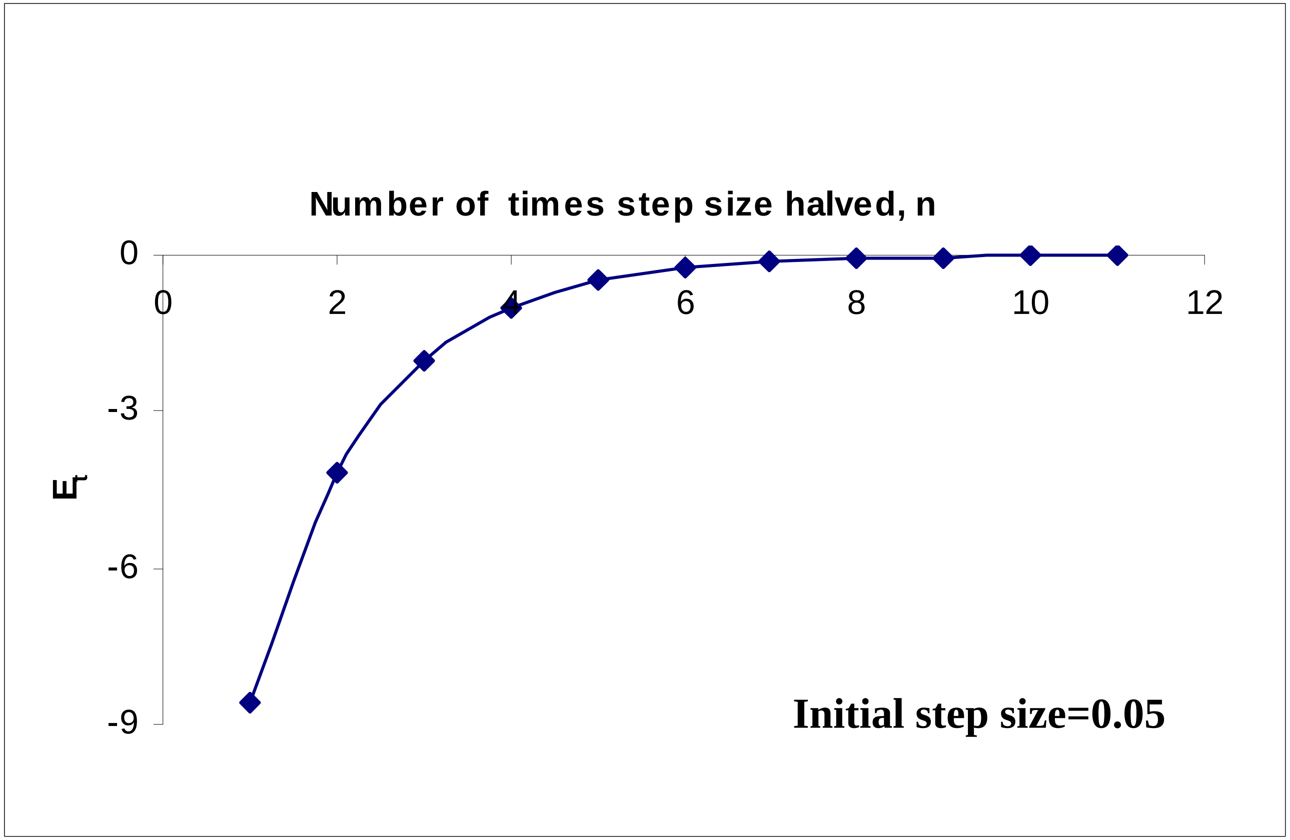
Effect of Step Size on Absolute Relative Approximate Error



Effect of Step Size on Least Number of Significant Digits Correct



Effect of Step Size on True Error



Effect of Step Size on Absolute Relative True Error