

Chapter 02.00D

Physical Problem for Differentiation Computer Engineering

Human vision is an intriguing ability. The fact that we can effortlessly recognize objects, people, and read this document belies the complexity of the task that a significant portion of our brain is devoted to solving. You might be surprised to learn that this almost photographic perception of the world that we have is not what our eyes send to our brain. What we perceive is a reconstruction from the noisy, shaky output of elementary “things” or “features” detected in the retina. There is strong evidence that the first level of processing done in the retina involves detecting something called “edges” or positions of transitions from dark to bright or bright to dark points in the images. These points usually coincide with boundaries of things.



Figure 1 On left is an image, with the corresponding edge image shown on the right.

In Figure 1 above we see an example of an edge image. This process of “detecting” edges in an image is called edge detection and can be modeled as a differentiation process. In this module we will look into this differentiation process.

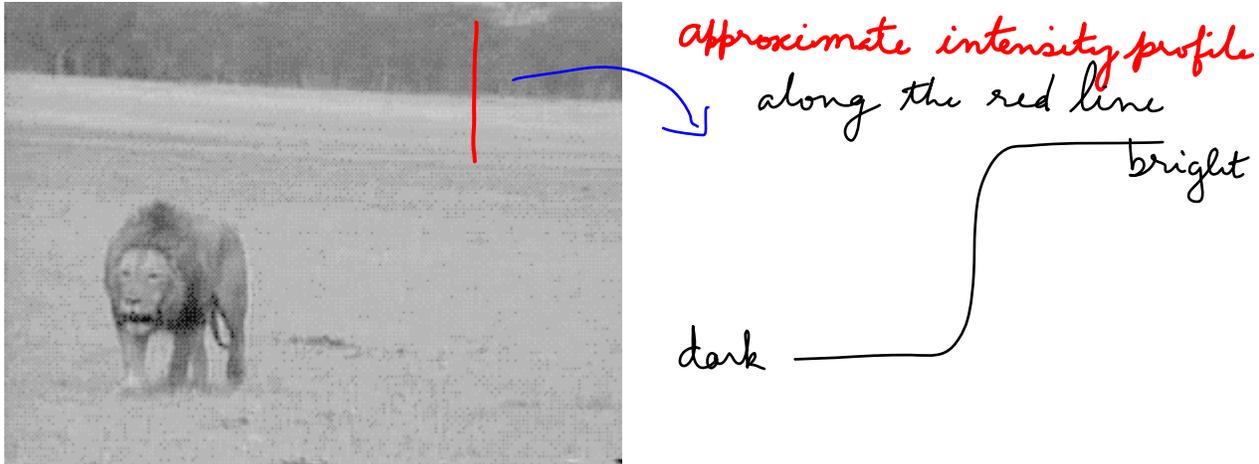


Figure 2 Edges corresponds to points in the image where there are intensity changes. In the above Figure we see the profile of the intensity value along a line (shown in red) cutting an edge in the image. We represent small values to represent dark points in the image and large values to detect bright regions in the image. The value increases and saturates to the value of the bright region. The edges can be marked at point of the maximum change in intensity, or where the derivative is a maximum. We will study the behavior of this derivative for a class of models of the edges.

Example

One way to model the edge, which is denote by e^x , where x is the direction perpendicular to the edge, is using exponential functions.

$$e(x) = \begin{cases} 1 - e^{-ax} & \forall x \geq 0 \\ e^{(ax)} - 1 & \forall x \leq 0 \end{cases}$$

Plots of this function for various values of a are shown in the Figure 3 below. Ignore the fact that we have used negative values to represent intensity. We can always add a constant to shift the function “up”, but the constant will not have any impact on the final derivative.

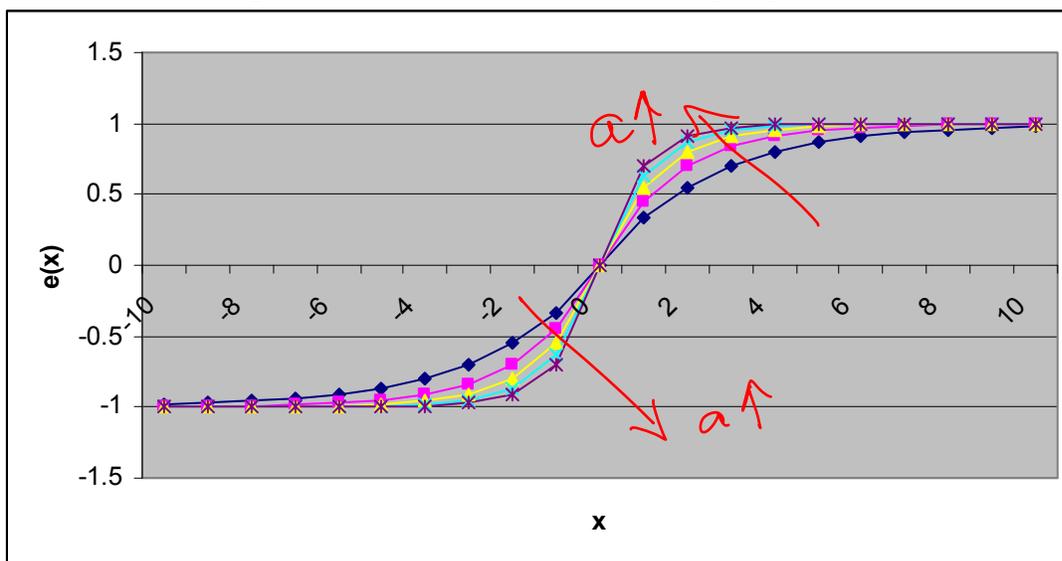


Figure 3 Plot of the exponential edge profile model for various values of the modeling parameter a .

The derivative of this profile model is given by

$$e(x) = \begin{cases} 1 - e^{-ax} & \forall x \geq 0 \\ e^{ax} - 1 & \forall x \leq 0 \end{cases}$$

(Note that at $x=0$ both the derivatives have the same value of a ; this shows that the function we used to model the edge profile is continuous in value and first derivative. This is just an aside fact that you might store in your brain for future use!)

What happens to the derivative as the underlying edge gets sharper? We see from Figure 4 that the value of the derivative gets larger and the peak gets sharper. Thus, strong edges, i.e. edges with sharp intensity transitions will result in stronger response.

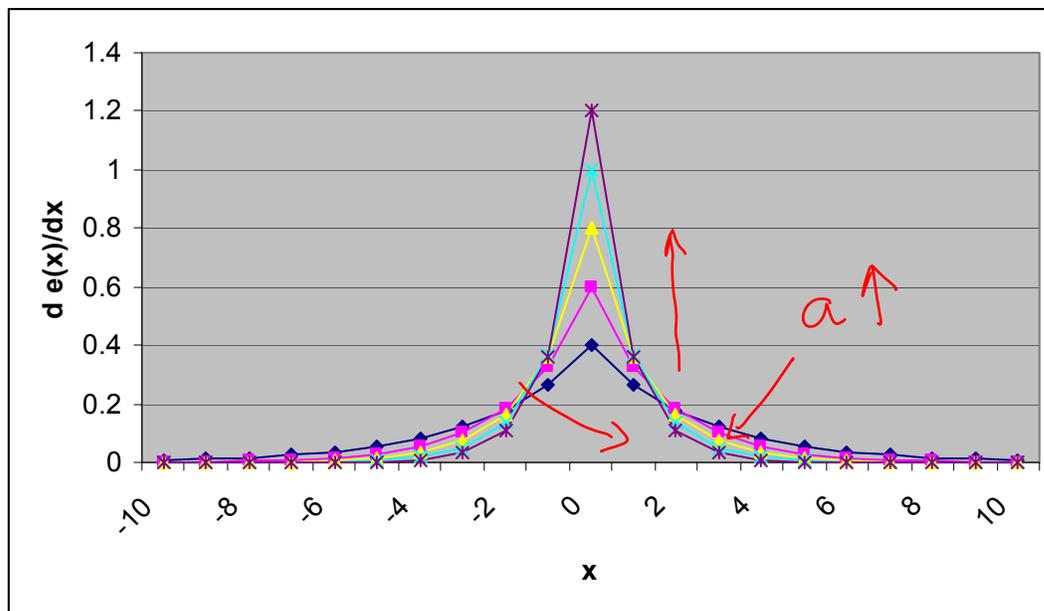


Figure 4 Plot of the derivative of the edge profile as the parameter a is increased.

QUESTIONS

1. What kind of “edge” is represented by the following edge profile model?

$$e(x) = e^{-ax^2}$$

- Answer: Line edge: the edge will appear as a white line in a dark background. What happens to the width of this line as you change a .
2. Compute the first derivative of the above function given in #1.
 3. Where, that is for what values of x , does this derivative reach a maximum or a minimum values? What happens to the distance between the maximum and minimum value locations as a is increased?
 4. Where is the value of the derivative zero?

Topic	DIFFERENTIATION
Sub Topic	Physical Problem
Summary	To detect edges (boundaries) in images
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