# Direct Method of Interpolation

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# What is Interpolation ?

Given $(x_0,y_0)$, $(x_1,y_1)$, ...... $(x_n,y_n)$, find the value of ‘y’ at a value of ‘x’ that is not given.

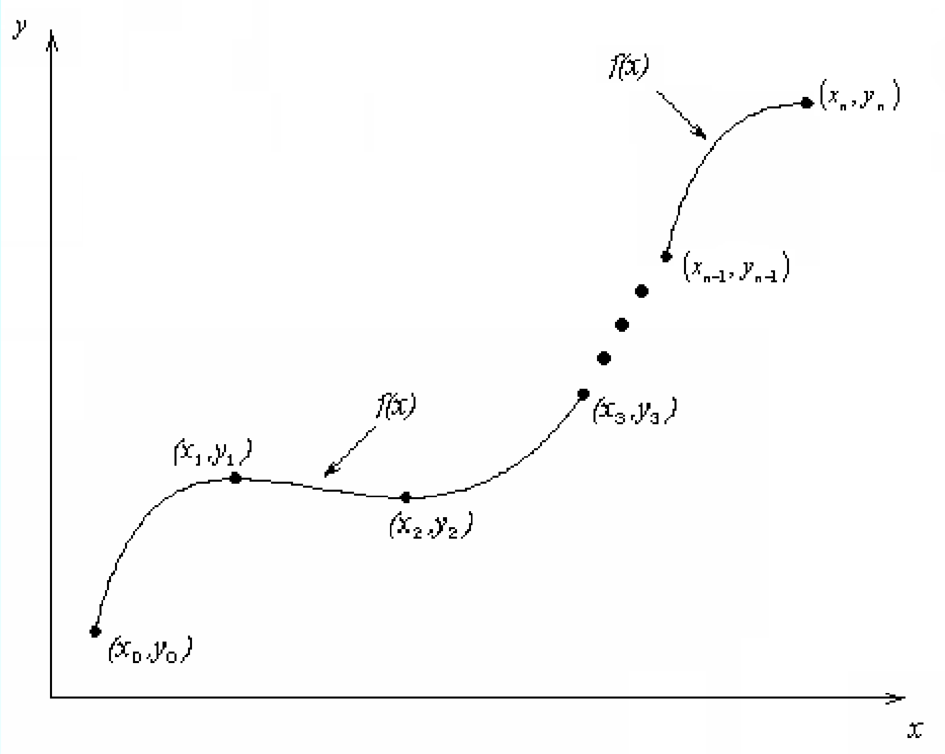



**Figure 1** Interpolation of discrete.

# Interpolants

Polynomials are the most common choice of interpolants because they are easy to:

- Evaluate
- Differentiate, and
- Integrate

# Direct Method

Given ‘n+1’ data points $(x_0,y_0)$, $(x_1,y_1)$,………….. $(x_n,y_n)$, pass a polynomial of order ‘n’ through the data as given below:

$$y = a_0 + a_1 x + \dots\dots\dots\dots\dots\dots\dots + a_n x^n.$$

where $a_0$, $a_1$,………………. $a_n$ are real constants.

- Set up ‘n+1’ equations to find ‘n+1’ constants.
- To find the value ‘y’ at a given value of ‘x’, simply substitute the value of ‘x’ in the above polynomial.

# Example

A robot arm with a rapid laser scanner is doing a quick quality check on holes drilled in a rectangular plate. The hole centers in the plate that describe the path the arm needs to take are given below.

If the laser is traversing from x = 2 to x = 4.25 in a linear path, find the value of y at x = 4 using the direct method for linear interpolation.

| x (m) | y (m) |
|---|---|
| 2 | 7.2 |
| 4.25 | 7.1 |
| 5.25 | 6.0 |
| 7.81 | 5.0 |
| 9.2 | 3.5 |
| 10.6 | 5.0 |

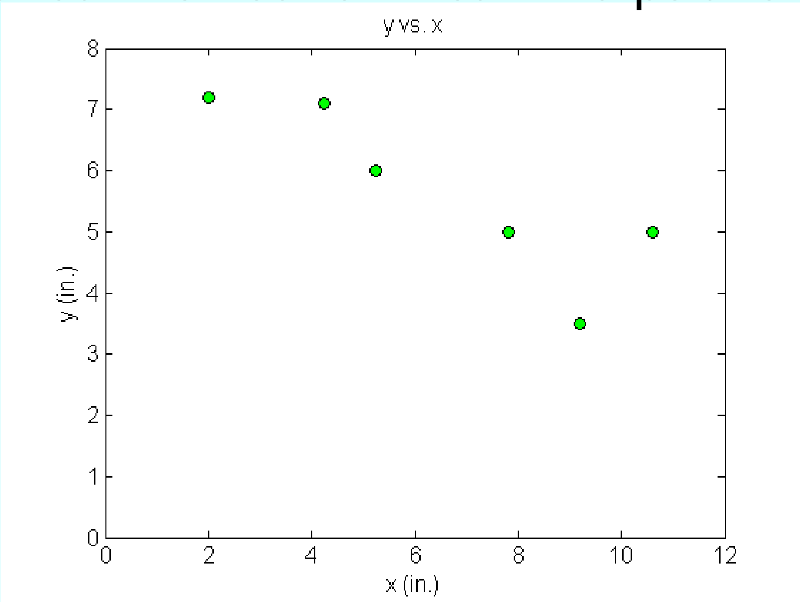


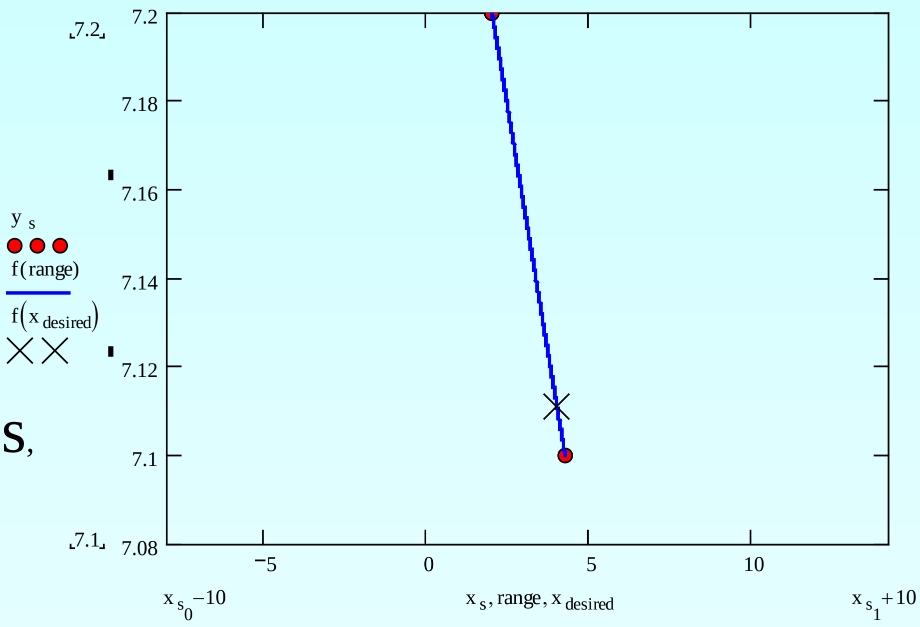

**Figure 2** Location of holes on the rectangular plate.

# Linear Interpolation

$$y(x) = a_0 + a_1 x$$

$$y(2.00) = a_0 + a_1(2.00) = 7.2$$

$$y(4.25) = a_0 + a_1(4.25) = 7.1$$




Solving the above two equations gives,

$$a_0 = 7.2889 \qquad a_1 = -0.044444$$

Hence

$$y(x) = 7.2889 - 0.044444x, \quad 2.00 \le x \le 4.25$$

$$y(4.00) = 7.2889 - 0.044444(4.00) = 7.1111 \text{ in.}$$

# Example

A robot arm with a rapid laser scanner is doing a quick quality check on holes drilled in a rectangular plate. The hole centers in the plate that describe the path the arm needs to take are given below.

If the laser is traversing from x = 2 to x = 4.25 in a linear path, find the value of y at x = 4 using the direct method for quadratic interpolation.

| x (m) | y (m) |
|---|---|
| 2 | 7.2 |
| 4.25 | 7.1 |
| 5.25 | 6.0 |
| 7.81 | 5.0 |
| 9.2 | 3.5 |
| 10.6 | 5.0 |

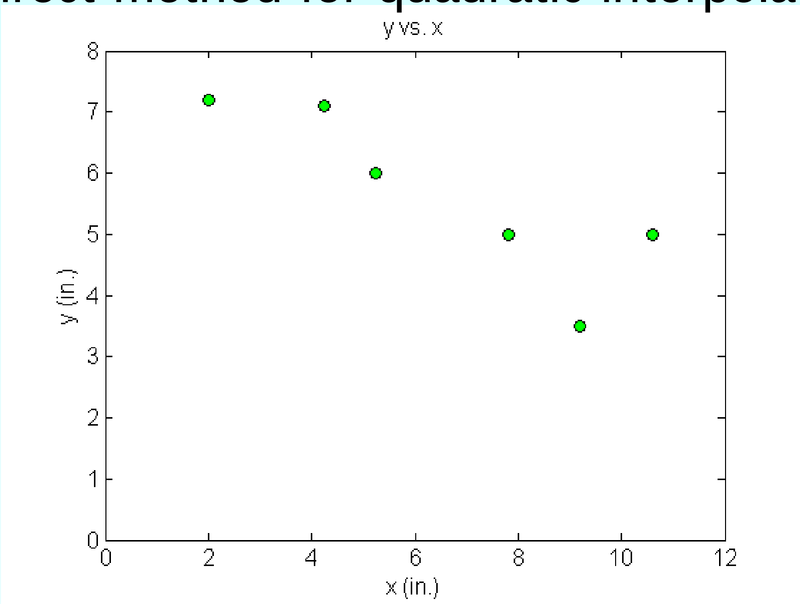



**Figure 2** Location of holes on the rectangular plate.

# Quadratic Interpolation

$$y(x) = a_0 + a_1 x + a_2 x^2$$

$$y(2.00) = a_0 + a_1(2.00) + a_2(2.00)^2 = 7.2$$

$$y(4.25) = a_0 + a_1(4.25) + a_2(4.25)^2 = 7.1$$

$$y(5.25) = a_0 + a_1(5.25) + a_2(5.25)^2 = 6.0$$

Solving the above three equations gives

$$a_0 = 4.5282 \qquad a_1 = 1.9855 \qquad a_2 = -0.32479$$

# Quadratic Interpolation (contd)

$$y(x) = 4.5282 + 1.9855x - 0.32479x^2, \quad 2.00 \le x \le 5.25$$

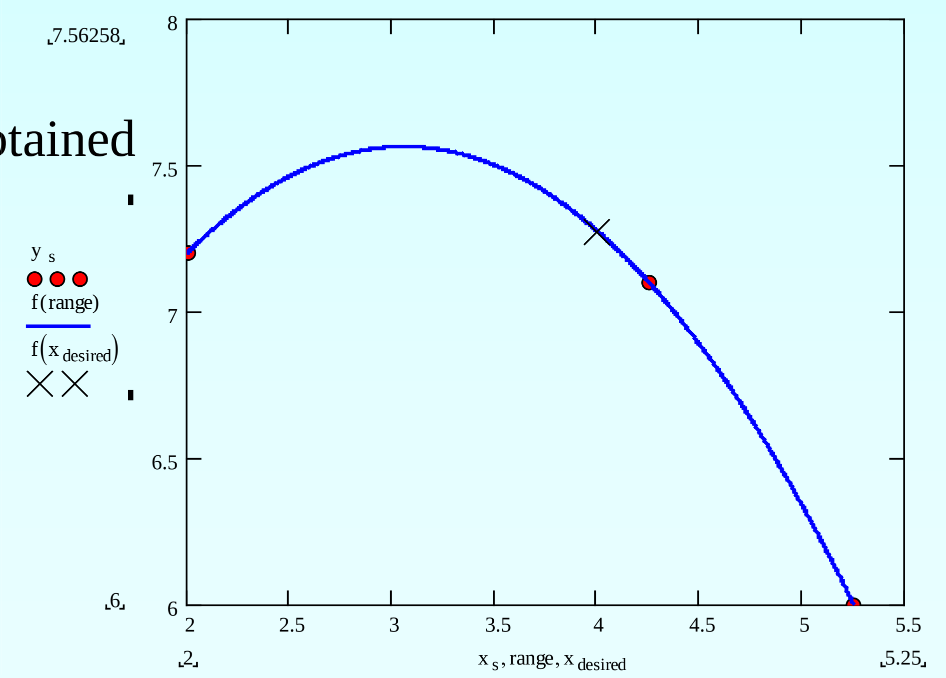
$$y(4.00) = 4.5282 + 1.9855(4.00) - 0.32479(4.00)^2$$

$$= 7.2735 \; in.$$

The absolute relative approximate error $|\in_a|$ obtained between first and second order polynomial is

$$|\in_a| = \left|\frac{7.2735 - 7.1111}{7.2735}\right| \times 100$$

$$= 2.2327\%$$



# Comparison Table

| Order of Polynomial | 1 | 2 |
|---|---|---|
| Location (in.) | 7.1111 | 7.2735 |
| Absolute Relative Approximate Error | ---------- | 2.2327% |

# Example

A robot arm with a rapid laser scanner is doing a quick quality check on holes drilled in a rectangular plate. The hole centers in the plate that describe the path the arm needs to take are given below.

If the laser is traversing from x = 2 to x = 4.25 in a linear path, find the value of y at x = 4 using the direct method using a fifth order polynomial.

| x (m) | y (m) |
|---|---|
| 2 | 7.2 |
| 4.25 | 7.1 |
| 5.25 | 6.0 |
| 7.81 | 5.0 |
| 9.2 | 3.5 |
| 10.6 | 5.0 |

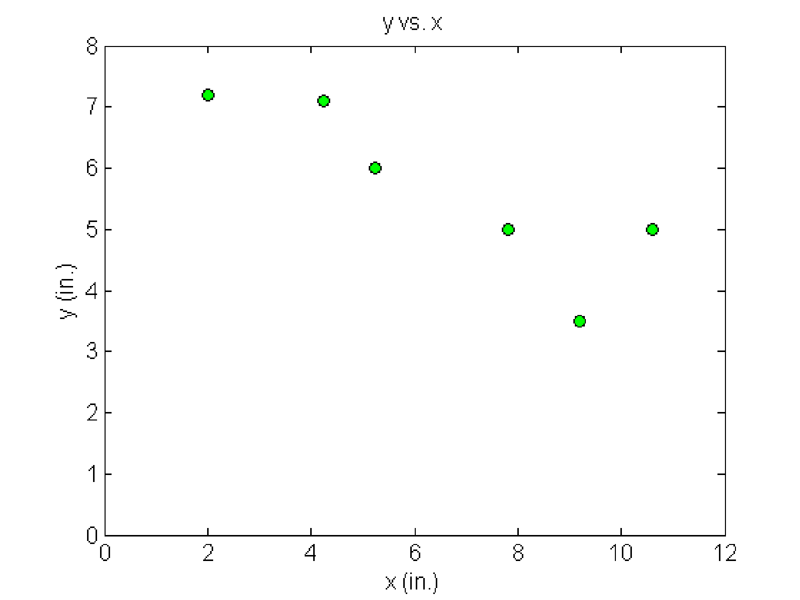



**Figure 2** Location of holes on the rectangular plate.

# Fifth Order Interpolation

$$y(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$$

$$y(2.00) = 7.2 = a_0 + a_1(2.00) + a_2(2.00)^2 + a_3(2.00)^3 + a_4(2.00)^4 + a_5(2.00)^5$$

$$y(4.25) = 7.1 = a_0 + a_1(4.25) + a_2(4.25)^2 + a_3(4.25)^3 + a_4(4.25)^4 + a_5(4.25)^5$$

$$y(5.25) = 6.0 = a_0 + a_1(5.25) + a_2(5.25)^2 + a_3(5.25)^3 + a_4(5.25)^4 + a_5(5.25)^5$$

$$y(7.81) = 5.0 = a_0 + a_1(7.81) + a_2(7.81)^2 + a_3(7.81)^3 + a_4(7.81)^4 + a_5(7.81)^5$$

$$y(9.20) = 3.5 = a_0 + a_1(9.20) + a_2(9.20)^2 + a_3(9.20)^3 + a_4(9.20)^4 + a_5(9.20)^5$$

$$y(10.60) = 5.0 = a_0 + a_1(10.60) + a_2(10.60)^2 + a_3(10.60)^3 + a_4(10.60)^4 + a_5(10.60)^5$$

# Fifth Order Interpolation (contd)

Writing the six equations in matrix form, we have

$$\begin{bmatrix} 1 & 2.00 & 4.00 & 8.00 & 16.00 & 32 \\ 1 & 4.25 & 18.063 & 76.766 & 326.25 & 1386.6 \\ 1 & 5.25 & 27.563 & 144.70 & 759.69 & 3988.4 \\ 1 & 7.81 & 60.996 & 476.38 & 3720.5 & 29057 \\ 1 & 9.20 & 84.640 & 778.69 & 7163.9 & 65908 \\ 1 & 10.6 & 112.36 & 1191.0 & 12625 & 133820 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{bmatrix} = \begin{bmatrix} 7.2 \\ 7.1 \\ 6.0 \\ 5.0 \\ 3.5 \\ 5.0 \end{bmatrix}$$

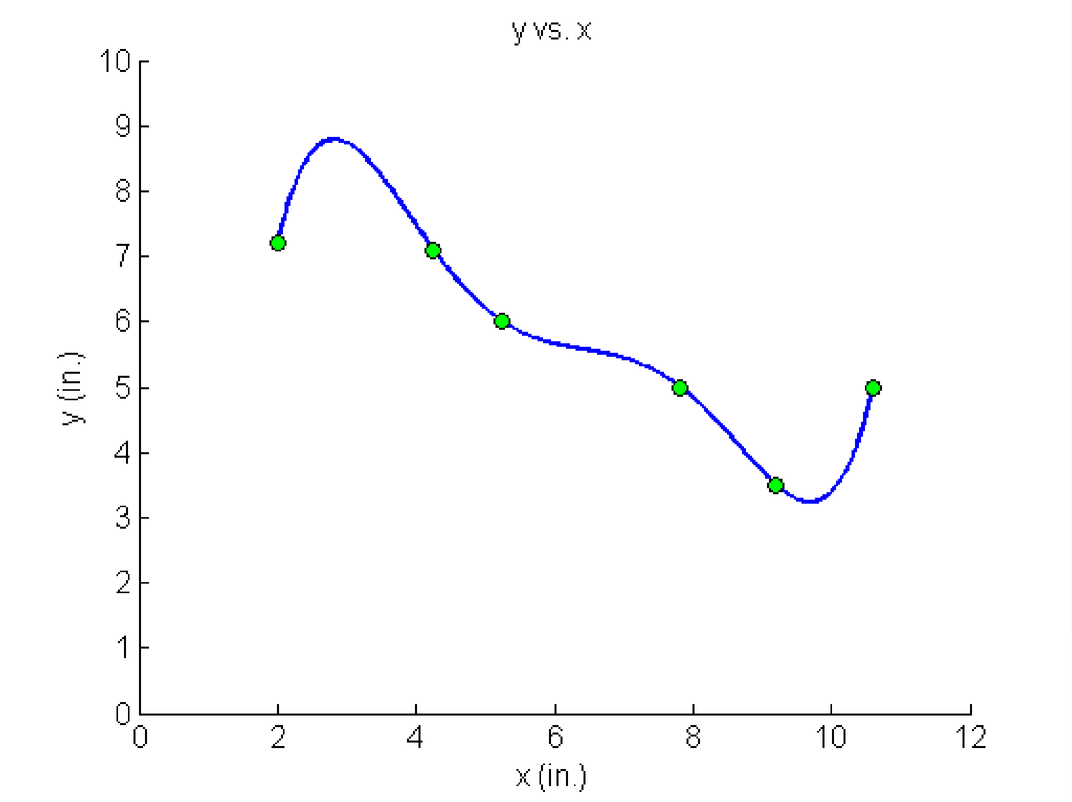
$$a_0 = -30.898 \quad a_1 = 41.344 \quad a_2 = -15.855$$

$$a_3 = 2.7862 \quad a_4 = -0.23091 \quad a_5 = 0.0072923$$

$$y(x) = -30.898 + 41.344x - 15.855x^2 + 2.7862x^3 - 0.23091x^4 + 0.0072923x^5, 2 \le x \le 10.6$$

# Fifth Order Interpolation (contd)

$$y(x) = -30.898 + 41.344x - 15.855x^2 + 2.7862x^3 - 0.23091x^4 + 0.0072923x^5, 2 \le x \le 10.6$$



# Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/direct_method.html

# THE END

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