

Chapter 07.03

Simpson's 1/3 Rule of Integration

After reading this chapter, you should be able to

1. *derive the formula for Simpson's 1/3 rule of integration,*
2. *use Simpson's 1/3 rule to solve integrals,*
3. *develop the formula for multiple-segment Simpson's 1/3 rule of integration,*
4. *use multiple-segment Simpson's 1/3 rule of integration to solve integrals, and*
5. *derive the true error formula for multiple-segment Simpson's 1/3 rule.*

What is integration?

Integration is the process of measuring the area under a function plotted on a graph. Why would we want to integrate a function? Among the most common examples are finding the velocity of a body from an acceleration function, and displacement of a body from a velocity function. Throughout many engineering fields, there are (what sometimes seems like) countless applications for integral calculus. You can read about some of these applications in Chapters 07.00A-07.00G.

Sometimes, the evaluation of expressions involving these integrals can become daunting, if not indeterminate. For this reason, a wide variety of numerical methods has been developed to simplify the integral. Here, we will discuss Simpson's 1/3 rule of integral approximation, which improves upon the accuracy of the trapezoidal rule.

Here, we will discuss the Simpson's 1/3 rule of approximating integrals of the form

$$I = \int_a^b f(x)dx$$

where

$f(x)$ is called the integrand,

a = lower limit of integration

b = upper limit of integration

Simpson's 1/3 Rule

The trapezoidal rule was based on approximating the integrand by a first order polynomial, and then integrating the polynomial over interval of integration. Simpson's 1/3 rule is an

extension of Trapezoidal rule where the integrand is approximated by a second order polynomial.

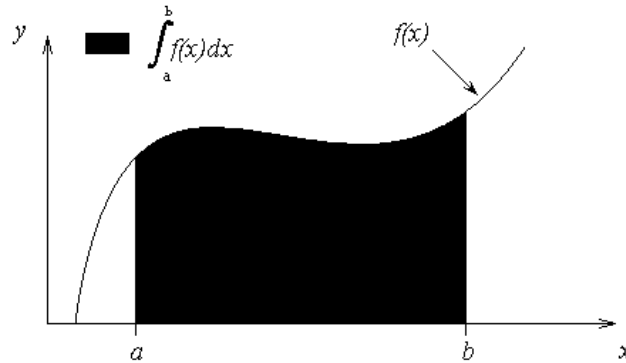


Figure 1 Integration of a function

Method 1:

Hence

$$I = \int_a^b f(x) dx \approx \int_a^b f_2(x) dx$$

where $f_2(x)$ is a second order polynomial given by

$$f_2(x) = a_0 + a_1x + a_2x^2$$

Choose

$$(a, f(a)), \left(\frac{a+b}{2}, f\left(\frac{a+b}{2}\right) \right), \text{ and } (b, f(b))$$

as the three points of the function to evaluate a_0 , a_1 and a_2 .

$$f(a) = f_2(a) = a_0 + a_1a + a_2a^2$$

$$f\left(\frac{a+b}{2}\right) = f_2\left(\frac{a+b}{2}\right) = a_0 + a_1\left(\frac{a+b}{2}\right) + a_2\left(\frac{a+b}{2}\right)^2$$

$$f(b) = f_2(b) = a_0 + a_1b + a_2b^2$$

Solving the above three equations for unknowns, a_0 , a_1 and a_2 give

$$a_0 = \frac{a^2 f(b) + abf(b) - 4abf\left(\frac{a+b}{2}\right) + abf(a) + b^2 f(a)}{a^2 - 2ab + b^2}$$

$$a_1 = -\frac{af(a) - 4af\left(\frac{a+b}{2}\right) + 3af(b) + 3bf(a) - 4bf\left(\frac{a+b}{2}\right) + bf(b)}{a^2 - 2ab + b^2}$$

$$a_2 = \frac{2\left(f(a) - 2f\left(\frac{a+b}{2}\right) + f(b)\right)}{a^2 - 2ab + b^2}$$

Then

$$\begin{aligned} I &\approx \int_a^b f_2(x) dx \\ &= \int_a^b (a_0 + a_1x + a_2x^2) dx \\ &= \left[a_0x + a_1 \frac{x^2}{2} + a_2 \frac{x^3}{3} \right]_a^b \\ &= a_0(b-a) + a_1 \frac{b^2 - a^2}{2} + a_2 \frac{b^3 - a^3}{3} \end{aligned}$$

Substituting values of a_0 , a_1 and a_2 give

$$\int_a^b f_2(x) dx = \frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

Since for Simpson 1/3 rule, the interval $[a, b]$ is broken into 2 segments, the segment width

$$h = \frac{b-a}{2}$$

Hence the Simpson's 1/3 rule is given by

$$\int_a^b f(x) dx \approx \frac{h}{3} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

Since the above form has 1/3 in its formula, it is called Simpson's 1/3 rule.

Method 2:

Simpson's 1/3 rule can also be derived by approximating $f(x)$ by a second order polynomial using Newton's divided difference polynomial as

$$f_2(x) = b_0 + b_1(x-a) + b_2(x-a)\left(x - \frac{a+b}{2}\right)$$

where

$$\begin{aligned} b_0 &= f(a) \\ b_1 &= \frac{f\left(\frac{a+b}{2}\right) - f(a)}{\frac{a+b}{2} - a} \end{aligned}$$

$$b_2 = \frac{\frac{f(b) - f\left(\frac{a+b}{2}\right)}{b - \frac{a+b}{2}} - \frac{f\left(\frac{a+b}{2}\right) - f(a)}{\frac{a+b}{2} - a}}{b - a}$$

Integrating Newton's divided difference polynomial gives us

$$\begin{aligned} \int_a^b f(x) dx &\approx \int_a^b f_2(x) dx \\ &= \int_a^b \left[b_0 + b_1(x-a) + b_2(x-a)\left(x - \frac{a+b}{2}\right) \right] dx \\ &= \left[b_0 x + b_1 \left(\frac{x^2}{2} - ax \right) + b_2 \left(\frac{x^3}{3} - \frac{(3a+b)x^2}{4} + \frac{a(a+b)x}{2} \right) \right]_a^b \\ &= b_0(b-a) + b_1 \left(\frac{b^2 - a^2}{2} - a(b-a) \right) \\ &\quad + b_2 \left(\frac{b^3 - a^3}{3} - \frac{(3a+b)(b^2 - a^2)}{4} + \frac{a(a+b)(b-a)}{2} \right) \end{aligned}$$

Substituting values of b_0 , b_1 , and b_2 into this equation yields the same result as before

$$\begin{aligned} \int_a^b f(x) dx &\approx \frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \\ &= \frac{h}{3} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \end{aligned}$$

Method 3:

One could even use the Lagrange polynomial to derive Simpson's formula. Notice any method of three-point quadratic interpolation can be used to accomplish this task. In this case, the interpolating function becomes

$$f_2(x) = \frac{\left(x - \frac{a+b}{2}\right)(x-b)}{\left(a - \frac{a+b}{2}\right)(a-b)} f(a) + \frac{(x-a)(x-b)}{\left(\frac{a+b}{2} - a\right)\left(\frac{a+b}{2} - b\right)} f\left(\frac{a+b}{2}\right) + \frac{(x-a)\left(x - \frac{a+b}{2}\right)}{(b-a)\left(b - \frac{a+b}{2}\right)} f(b)$$

Integrating this function gets

$$\begin{aligned}
 \int_a^b f_2(x) dx &= \left[\frac{x^3 - \frac{(a+3b)x^2}{4} + \frac{b(a+b)x}{2}}{\left(a - \frac{a+b}{2}\right)(a-b)} f(a) + \frac{x^3 - \frac{(a+b)x^2}{2} + abx}{\left(\frac{a+b}{2} - a\right)\left(\frac{a+b}{2} - b\right)} f\left(\frac{a+b}{2}\right) \right]_a^b \\
 &+ \left[\frac{x^3 - \frac{(3a+b)x^2}{4} + \frac{a(a+b)x}{2}}{(b-a)\left(b - \frac{a+b}{2}\right)} f(b) \right]_a^b \\
 &= \frac{b^3 - a^3}{3} - \frac{(a+3b)(b^2 - a^2)}{4} + \frac{b(a+b)(b-a)}{2} f(a) \\
 &+ \frac{b^3 - a^3}{3} - \frac{(a+b)(b^2 - a^2)}{2} + ab(b-a) f\left(\frac{a+b}{2}\right) \\
 &+ \frac{b^3 - a^3}{3} - \frac{(3a+b)(b^2 - a^2)}{4} + \frac{a(a+b)(b-a)}{2} f(b)
 \end{aligned}$$

Believe it or not, simplifying and factoring this large expression yields you the same result as before

$$\begin{aligned}
 \int_a^b f(x) dx &\approx \frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \\
 &= \frac{h}{3} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right].
 \end{aligned}$$

Method 4:

Simpson's 1/3 rule can also be derived by the method of coefficients. Assume

$$\int_a^b f(x) dx \approx c_1 f(a) + c_2 f\left(\frac{a+b}{2}\right) + c_3 f(b)$$

Let the right-hand side be an exact expression for the integrals $\int_a^b 1 dx$, $\int_a^b x dx$, and $\int_a^b x^2 dx$. This implies that the right hand side will be exact expressions for integrals of any linear combination of the three integrals for a general second order polynomial. Now

$$\int_a^b 1 dx = b - a = c_1 + c_2 + c_3$$

$$\int_a^b x dx = \frac{b^2 - a^2}{2} = c_1 a + c_2 \frac{a+b}{2} + c_3 b$$

$$\int_a^b x^2 dx = \frac{b^3 - a^3}{3} = c_1 a^2 + c_2 \left(\frac{a+b}{2}\right)^2 + c_3 b^2$$

Solving the above three equations for c_0 , c_1 and c_2 give

$$c_1 = \frac{b-a}{6}$$

$$c_2 = \frac{2(b-a)}{3}$$

$$c_3 = \frac{b-a}{6}$$

This gives

$$\begin{aligned} \int_a^b f(x) dx &\approx \frac{b-a}{6} f(a) + \frac{2(b-a)}{3} f\left(\frac{a+b}{2}\right) + \frac{b-a}{6} f(b) \\ &= \frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \\ &= \frac{h}{3} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right] \end{aligned}$$

The integral from the first method

$$\int_a^b f(x) dx \approx \int_a^b (a_0 + a_1 x + a_2 x^2) dx$$

can be viewed as the area under the second order polynomial, while the equation from Method 4

$$\int_a^b f(x) dx \approx \frac{b-a}{6} f(a) + \frac{2(b-a)}{3} f\left(\frac{a+b}{2}\right) + \frac{b-a}{6} f(b)$$

can be viewed as the sum of the areas of three rectangles.

Example 1

A trunnion of diameter 12.363" has to be cooled from a room temperature of 80°F before it is shrink fit into a steel hub (Figure 2).

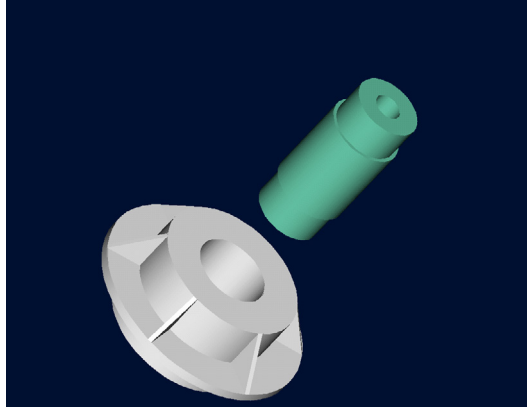


Figure 2 Trunnion to be slid through the hub after contracting.

The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is -108°F) is given by:

$$\Delta D = 12.363 \int_{80}^{-108} (-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}) dT$$

- Use Simpson's 1/3 Rule to find the approximate value of ΔD .
- Find the true error, E_t , for part (a).
- Find the absolute relative true error for part (a).

Solution

$$\text{a) } \Delta D \approx \frac{b-a}{6} \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

$$a = 80$$

$$b = -108$$

$$\frac{a+b}{2} = -14$$

$$f(T) = 12.363(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6})$$

$$f(80) = 12.363(-1.2278 \times 10^{-11}(80)^2 + 6.1946 \times 10^{-9}(80) + 6.015 \times 10^{-6}) \\ = 7.9519 \times 10^{-5}$$

$$f(-108) = 12.363(-1.2278 \times 10^{-11}(-108)^2 + 6.1946 \times 10^{-9}(-108) + 6.015 \times 10^{-6}) \\ = 6.4322 \times 10^{-5}$$

$$f(-14) = 12.363(-1.2278 \times 10^{-11}(-14)^2 + 6.1946 \times 10^{-9}(-14) + 6.015 \times 10^{-6}) \\ = 7.3262 \times 10^{-5}$$

$$\Delta D \approx \left(\frac{b-a}{6}\right) \left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b) \right]$$

$$\approx \left(\frac{-108-80}{6}\right) [f(80) + 4f(-14) + f(-108)]$$

$$\approx \left(\frac{-188}{6}\right) [7.9519 \times 10^{-5} + 4(7.3262 \times 10^{-5}) + 6.4322 \times 10^{-5}]$$

$$\approx -0.013689 \text{ in}$$

b) The exact value of the above integral is

$$\begin{aligned}\Delta D &= 12.363 \int_{80}^{-108} (-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}) dT \\ &= -0.013689 \text{ in}\end{aligned}$$

so the true error is

$$\begin{aligned}E_t &= \text{True Value} - \text{Approximate Value} \\ &= -0.013689 - (-0.013689) \\ &= 0.0000\end{aligned}$$

c) The absolute relative true error, $|\epsilon_t|$, would then be

$$\begin{aligned}|\epsilon_t| &= \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100 \% \\ &= \left| \frac{0.0000}{-0.013689} \right| \times 100 \% \\ &= 0.0000 \%\end{aligned}$$

Multiple-segment Simpson's 1/3 Rule

Just like in multiple-segment trapezoidal rule, one can subdivide the interval $[a, b]$ into n segments and apply Simpson's 1/3 rule repeatedly over every two segments. Note that n needs to be even. Divide interval $[a, b]$ into n equal segments, so that the segment width is given by

$$h = \frac{b - a}{n}.$$

Now

$$\int_a^b f(x) dx = \int_{x_0}^{x_n} f(x) dx$$

where

$$x_0 = a$$

$$x_n = b$$

$$\int_a^b f(x) dx = \int_{x_0}^{x_2} f(x) dx + \int_{x_2}^{x_4} f(x) dx + \dots + \int_{x_{n-4}}^{x_{n-2}} f(x) dx + \int_{x_{n-2}}^{x_n} f(x) dx$$

Apply Simpson's 1/3rd Rule over each interval,

$$\int_a^b f(x) dx \cong (x_2 - x_0) \left[\frac{f(x_0) + 4f(x_1) + f(x_2)}{6} \right] + (x_4 - x_2) \left[\frac{f(x_2) + 4f(x_3) + f(x_4)}{6} \right] + \dots$$

$$+ (x_{n-2} - x_{n-4}) \left[\frac{f(x_{n-4}) + 4f(x_{n-3}) + f(x_{n-2})}{6} \right] + (x_n - x_{n-2}) \left[\frac{f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)}{6} \right]$$

Since

$$\begin{aligned} x_i - x_{i-2} &= 2h \\ i &= 2, 4, \dots, n \end{aligned}$$

then

$$\begin{aligned} \int_a^b f(x) dx &\cong 2h \left[\frac{f(x_0) + 4f(x_1) + f(x_2)}{6} \right] + 2h \left[\frac{f(x_2) + 4f(x_3) + f(x_4)}{6} \right] + \dots \\ &\quad + 2h \left[\frac{f(x_{n-4}) + 4f(x_{n-3}) + f(x_{n-2})}{6} \right] + 2h \left[\frac{f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)}{6} \right] \\ &= \frac{h}{3} [f(x_0) + 4\{f(x_1) + f(x_3) + \dots + f(x_{n-1})\} + 2\{f(x_2) + f(x_4) + \dots + f(x_{n-2})\} + f(x_n)] \end{aligned}$$

$$= \frac{h}{3} \left[f(x_0) + 4 \sum_{i=1, \text{odd}}^{n-1} f(x_i) + 2 \sum_{i=2, \text{even}}^{n-2} f(x_i) + f(x_n) \right]$$

$$\boxed{\int_a^b f(x) dx \cong \frac{b-a}{3n} \left[f(x_0) + 4 \sum_{i=1, \text{odd}}^{n-1} f(x_i) + 2 \sum_{i=2, \text{even}}^{n-2} f(x_i) + f(x_n) \right]}$$

Example 2

A trunnion of diameter 12.363" has to be cooled from a room temperature of 80°F before it is shrink fit into a steel hub (Figure 2). The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is -108°F) is given by:

$$\Delta D = 12.363 \int_{80}^{-108} \left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6} \right) dT$$

- Use four segment Simpson's 1/3 Rule to find the contraction.
- Find the true error, E_t , for part (a).
- Find the absolute relative true error for part (a).

Solution

$$\text{a) } \Delta D = \frac{b-a}{3n} \left[f(T_0) + 4 \sum_{i=1, \text{odd}}^{n-1} f(T_i) + 2 \sum_{i=2, \text{even}}^{n-2} f(T_i) + f(T_n) \right]$$

$$n = 4$$

$$a = 80$$

$$b = -108$$

$$\begin{aligned}
h &= \frac{b-a}{n} \\
&= \frac{-108-80}{4} \\
&= -47 \\
f(T) &= 12.363(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}) \\
\Delta D &\approx \frac{b-a}{3n} \left[f(T_0) + 4 \sum_{\substack{i=1 \\ i=\text{odd}}}^{n-1} f(T_i) + 2 \sum_{\substack{i=2 \\ i=\text{even}}}^{n-2} f(T_i) + f(T_n) \right] \\
&\approx \frac{-108-80}{3(4)} \left[f(80) + 4 \sum_{\substack{i=1 \\ i=\text{odd}}}^3 f(T_i) + 2 \sum_{\substack{i=2 \\ i=\text{even}}}^2 f(T_i) + f(-108) \right] \\
&\approx \frac{-188}{12} [f(80) + 4f(T_1) + 4f(T_3) + 2f(T_2) + f(-108)] \\
&\approx \frac{-188}{12} [f(80) + 4f(33) + 4f(-61) + 2f(-14) + f(-108)] \\
&\approx -15.667 \left[7.9519 \times 10^{-5} + 4(7.6725 \times 10^{-5}) + 4(7.0257 \times 10^{-5}) \right. \\
&\quad \left. + 2(7.3321 \times 10^{-5}) + 6.4322 \times 10^{-5} \right] \\
&\approx -0.013689 \text{ in}
\end{aligned}$$

Since

$$\begin{aligned}
f(T_0) &= f(80) \\
&= 12.363(-1.2278 \times 10^{-11} (80)^2 + 6.1946 \times 10^{-9} (80) + 6.015 \times 10^{-6}) \\
&= 7.9519 \times 10^{-5}
\end{aligned}$$

$$\begin{aligned}
f(T_1) &= f(80 - 47) \\
&= f(33) \\
&= 12.363(-1.2278 \times 10^{-11} (33)^2 + 6.1946 \times 10^{-9} (33) + 6.015 \times 10^{-6}) \\
&= 7.6725 \times 10^{-5}
\end{aligned}$$

$$\begin{aligned}
f(T_2) &= f(33 - 47) \\
&= f(-14) \\
&= 12.363(-1.2278 \times 10^{-11} (-14)^2 + 6.1946 \times 10^{-9} (-14) + 6.015 \times 10^{-6}) \\
&= 7.3262 \times 10^{-5}
\end{aligned}$$

$$\begin{aligned}
f(T_3) &= f(-14 - 47) \\
&= f(-61) \\
&= 12.363(-1.2278 \times 10^{-11} (-61)^2 + 6.1946 \times 10^{-9} (-61) + 6.015 \times 10^{-6}) \\
&= 7.0257 \times 10^{-5}
\end{aligned}$$

$$\begin{aligned}
 f(T_4) &= f(T_n) \\
 &= f(-108) \\
 &= 12.363 \left(-1.2278 \times 10^{-11} (-108)^2 + 6.1946 \times 10^{-9} (-108) + 6.015 \times 10^{-6} \right) \\
 &= 6.4322 \times 10^{-5}
 \end{aligned}$$

b) The exact value of the above integral is

$$\begin{aligned}
 \Delta D &= 12.363 \int_{80}^{-108} \left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6} \right) dT \\
 &= -0.013689 \text{ in}
 \end{aligned}$$

so the true error is

$$\begin{aligned}
 E_t &= \text{True Value} - \text{Approximate Value} \\
 &= -0.013689 - (-0.013689) \\
 &= 0.0000
 \end{aligned}$$

c) The absolute relative true error, $|\epsilon_t|$, would then be

$$\begin{aligned}
 |\epsilon_t| &= \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100 \% \\
 &= \left| \frac{0.0000}{-0.013689} \right| \times 100 \% \\
 &= 0.0000 \%
 \end{aligned}$$

Table 1 Values of Simpson's 1/3 Rule for Example 2 with multiple segments.

n	Approximate Value	E_t	$ \epsilon_t $ %
2	-0.013689	0.0000	0.0000
4	-0.013689	0.0000	0.0000
6	-0.013689	0.0000	0.0000
8	-0.013689	0.0000	0.0000
10	-0.013689	0.0000	0.0000

Error in Multiple-segment Simpson's 1/3 rule

The true error in a single application of Simpson's 1/3rd Rule is given¹ by

¹ The $f^{(4)}$ in the true error expression stands for the fourth derivative of the function $f(x)$.

$$E_t = -\frac{(b-a)^5}{2880} f^{(4)}(\zeta), \quad a < \zeta < b$$

In multiple-segment Simpson's 1/3 rule, the error is the sum of the errors in each application of Simpson's 1/3 rule. The error in the n segments Simpson's 1/3rd Rule is given by

$$\begin{aligned} E_1 &= -\frac{(x_2 - x_0)^5}{2880} f^{(4)}(\zeta_1), \quad x_0 < \zeta_1 < x_2 \\ &= -\frac{h^5}{90} f^{(4)}(\zeta_1) \\ E_2 &= -\frac{(x_4 - x_2)^5}{2880} f^{(4)}(\zeta_2), \quad x_2 < \zeta_2 < x_4 \\ &= -\frac{h^5}{90} f^{(4)}(\zeta_2) \\ &\vdots \\ E_i &= -\frac{(x_{2i} - x_{2(i-1)})^5}{2880} f^{(4)}(\zeta_i), \quad x_{2(i-1)} < \zeta_i < x_{2i} \\ &= -\frac{h^5}{90} f^{(4)}(\zeta_i) \\ &\vdots \\ E_{\frac{n}{2}-1} &= -\frac{(x_{n-2} - x_{n-4})^5}{2880} f^{(4)}\left(\zeta_{\frac{n}{2}-1}\right), \quad x_{n-4} < \zeta_{\frac{n}{2}-1} < x_{n-2} \\ &= -\frac{h^5}{90} f^{(4)}\left(\zeta_{\frac{n}{2}-1}\right) \\ E_{\frac{n}{2}} &= -\frac{(x_n - x_{n-2})^5}{2880} f^{(4)}\left(\zeta_{\frac{n}{2}}\right), \quad x_{n-2} < \zeta_{\frac{n}{2}} < x_n \end{aligned}$$

Hence, the total error in the multiple-segment Simpson's 1/3 rule is

$$\begin{aligned} &= -\frac{h^5}{90} f^{(4)}\left(\zeta_{\frac{n}{2}}\right) \\ E_t &= \sum_{i=1}^{\frac{n}{2}} E_i \\ &= -\frac{h^5}{90} \sum_{i=1}^{\frac{n}{2}} f^{(4)}(\zeta_i) \\ &= -\frac{(b-a)^5}{90n^5} \sum_{i=1}^{\frac{n}{2}} f^{(4)}(\zeta_i) \end{aligned}$$

$$= -\frac{(b-a)^5}{90n^4} \frac{\sum_{i=1}^{\frac{n}{2}} f^{(4)}(\zeta_i)}{n}$$

The term $\frac{\sum_{i=1}^{\frac{n}{2}} f^{(4)}(\zeta_i)}{n}$ is an approximate average value of $f^{(4)}(x)$, $a < x < b$. Hence

$$E_t = -\frac{(b-a)^5}{90n^4} \bar{f}^{(4)}$$

where

$$\bar{f}^{(4)} = \frac{\sum_{i=1}^{\frac{n}{2}} f^{(4)}(\zeta_i)}{n}$$

INTEGRATION

Topic	Simpson's 1/3 rule
Summary	Textbook notes of Simpson's 1/3 rule
Major	Mechanical Engineering
Authors	Autar Kaw, Michael Keteltas
Date	November 17, 2012
Web Site	http://numericalmethods.eng.usf.edu
