INTRODUCING AND ASSESSING LABORATORY EXPERIENCE IN A NUMERICAL METHODS COURSE FOR ENGINEERS

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Abstract

Several low cost, low space, low setup time experiments were developed and implemented in an undergraduate course in Numerical Methods for Engineers. The analysis and interpretation of the collected experimental data encompassed most of the mathematical procedures covered in the course. This paper describes these experiments and shows how they were used throughout the course. The effect of introducing experiments in the course was quantitatively and qualitatively surveyed via student satisfaction surveys over a two-semester period. The results of the student surveys indicate high student satisfaction, especially in the areas of applying programming concepts, problem formulation, and relevance to their engineering major.

Introduction

Since 2000, the Accreditation Board of Engineering and Technology (ABET) [1] that accredits undergraduate engineering degrees in USA requires implementation of feedback received from current and past students about their undergraduate experience. Until several years ago, one of the major and common themes during our graduating seniors exit interviews and alumni surveys was that they would like more hands-on and more real-life applications in their mechanical engineering courses.

In response to such requests, several lecture courses in our department have now incorporated experiments that include class demonstrations, collection of data in a laboratory, building of simple experiments, and application of numerical methods using numerical computing and programming software packages.

This request of including more hands-on experience is supported by considerable current research exploring how to enhance student learning in science, mathematics, engineering, and technology (SMET) courses. One such source of research is the outstanding text *How People Learn* [2]. In the book, it states, "A major goal of schooling is to prepare students for flexible adaptation to new problems and settings [2, p. 65]" and that "knowledge that is taught in only a single context is less likely to support flexible knowledge transfer than is knowledge that is taught in multiple contexts [2, p. 66]." The use of experiments gives the student yet another context to learn the course material and hence maximizes the likelihood of lasting and flexible learning transfer of essential numerical methods course content.

The National Science Education Standards [3] are underlining once again the importance of the laboratory experience in gaining fundamental knowledge and skills in science. The addition of experiments in a course also addresses inclusiveness of different learning styles such as for those students who prefer active learning over reflective learning [4, 5]. The laboratory experience also brings students together in small groups, and hence creates an atmosphere for social interaction, cooperative learning, and cognitive growth [6, 7].

The Experiments

Developing experiments for the Numerical Methods course required special consideration, especially when we had limited budget, space, and class time. We developed experiments that

1) are low cost so that other universities can develop them with minimal material cost (some experiments need use of a university machine shop and already available basic/common instrumentation),
2) require low space so that they can be carried to the classroom or set up in the laboratory that has limited space,
3) need low set-up time so that nominal amount of classroom or laboratory time is used. This is especially important in the Numerical Methods course at University of South Florida (USF) where other educational components such as problem-centered approach, programming, and real-life project assignments are also incorporated.

Five experiments are incorporated in the course. The first two experiments are demonstrated in the classroom with student volunteers collecting the data, while the next three experiments are conducted in a 60-minute laboratory in groups of five students. The background of the experiments and assigned problems are available at the course website [8]. Data obtained from experiments is assigned for analysis as homework projects. For most experiments, we are also providing extensive information on the material costs and drawings needed to set up the experiments. The experiments are described below.

Experiment#1. Cooling an aluminum cylinder

In this experiment, an aluminum cylinder that has two inserted thermocouples is immersed in an iced-water bath (Figure 1). The thermocouples placed in the cylinder are connected to a digital temperature recorder that measures the temperature vs. time data. Taking the data every ten seconds takes just a couple of minutes. The data is used for several homework exercises such as

- finding the rate of change of temperature via *numerical differentiation*,
- extracting the coefficient of convection of iced water using *regression* based on theoretical models,
- reduction in diameter of the cylinder via *integration* where the coefficient of thermal expansion is a function of temperature,
- comparing the experimental temperature profile with one obtained from the solution of the *ordinary differential equation* that governs the system.

With convection coefficients depending on temperature, the theoretical model is the solution of a nonlinear ordinary differential equation that is solvable only by numerical methods. The theoretical model for the problem [9] is given by

$$mC\frac{d\theta}{dt} = -hA(\theta - \theta_a) \qquad (1)$$

where

$h(\theta)$ =the convective coefficient, $W/(m^2\text{-}{}^oC)$

A = surface area, m^2

θ_a = ambient temperature of iced water, ${}^oC$

m = mass of the aluminum cylinder, kg

C = specific heat of aluminum, $J/(kg\text{-}K)$

The ordinary differential equation is subjected to

$$\theta(0) = \theta_0 \qquad (2)$$

where

$\theta_0 =$ initial temperature of aluminum cylinder, ^{o}C

In case of assuming the convection coefficient to be a constant, the exact solution to the ordinary differential equation (1) is

$$\frac{\theta - \theta_a}{\theta_0 - \theta_a} = e^{-\frac{hAt}{mC}} \qquad (3)$$

The nonlinear model given in Equation (3) is used as a regression model to find the average convection coefficient.

Figure 1. Immersing aluminum cylinder in iced-water experiment

Experiment#2. Loading a Truss

A second experiment is that of an aluminum truss (Figure 2) that is loaded in the center. Strain gages are placed on three of the truss members. Balance of forces and moments in the truss result in a set of *simultaneous linear equations* with the member forces and reactions as the unknowns. Students set up these equations using the force balance method. They then use any of the mathematical packages [10-13] to find the force in the members on which strain gages are placed. The strain in a member is calculated from these forces by

$$\varepsilon = \frac{F}{AE} \qquad (4)$$

where

F= force in member, N

A = Cross-sectional area of member, m^2

E= Young's modulus of member, Pa

and then compared with the strains measured by the strain gages.

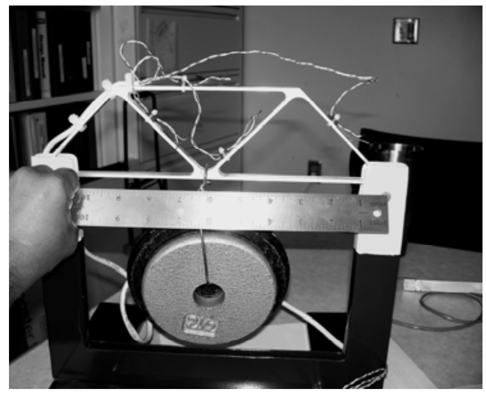

Figure 2: Loading a truss experiment

Experiment#3. Estimating the volume of a champagne glass

A third experiment takes several odd-shaped champagne glasses (Figure 3) that are measured for their outer radius at different locations along their height. Subtracting the thickness of the glass from the outer radius and using *spline interpolation* and *integration*, students estimate the volume of water these champagne glasses can hold.

The spline interpolation develops the spline interpolants for the inner radius as a function of height. Then the volume of the champagne glass can be calculated as

$$V = \int_0^H \pi r^2 dh \qquad (5)$$

where

r is the varying radius of the champagne glass as a function of height, h,

H is the height of the champagne glass.

This value is then compared with the actual volume of water that the champagne glass can hold by pouring a fully filled champagne glass into a graduated cylinder.

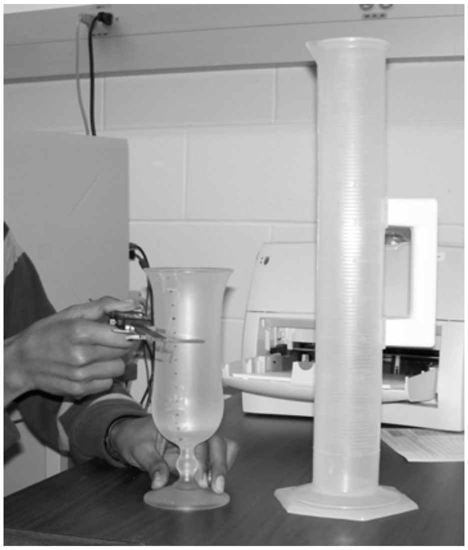

Figure 3. Finding the volume capacity of a champagne glass

Experiment#4. Choosing the best mousetrap

The fourth experiment is to choose the best mousetrap for powering a mousetrap-car. To do so, we want to pick the mousetrap that can store the most amount of torsional energy. We take several mousetraps from the local hardware store and measure the force required to twist the spring as a function of angle of rotation (Figure 4). Torque T is calculated using the measured lever moment arm, that is,

$$T = F\,L \qquad (6)$$

where,

F= Force applied (N)

L = Moment arm (m)

The relationship between the torque T applied and the angle θ of the rotation of the spring rotation is assumed to be a straight line

$$T = k_0 + k_1\,\theta \qquad (7)$$

Figure 4. Twisting the mousetrap spring experiment

Using *regression*, the constants of the linear model in Equation (7) are found. The torsional energy stored U is then given by

$$U = \int_{\theta_{low}}^{\theta_{high}} T d\theta = \int_{\theta_{low}}^{\theta_{high}} (k_0 + k_1\,\theta) d\theta \qquad (8)$$

Knowing that in our case, $\theta_{low} = 0$ and $\theta_{high} = \pi$, the maximum potential energy stored is given as

$$U_{\max} = \int_0^{\pi} (k_0 + k_1\,\theta) d\theta \;=\; k_0\pi + k_1\frac{\pi^2}{2} \qquad (9)$$

This number is calculated for each of the mousetrap springs, and the one with the highest value is the one that stores the most amount of torsional energy.

Experiment#5. Finding the length of a curve

In this experiment, a flexible curve (Figure 5) of length 12" made of lead-core construction with graduations in both millimeters and inches is used to draw a curve on a graphing paper as shown. Students are required to draw a curve similar in shape to the classical Runge curve of $y = 1/(1+25x^2)$, $-1 \le x \le 1$. This function was used by Runge [14] to show that higher order *interpolation* is a bad idea.

Once the student has drawn the 12" long curve, he/she is asked to choose several points along the curve. The student can now take the data pairs and find the interpolants by using *polynomial interpolation* and *spline interpolation*. One clearly notices the oscillatory behavior (Figure 6) of the polynomial interpolant and the smooth nature of the spline interpolant. The length of the two interpolants is found by using *numerical integration* by the formula

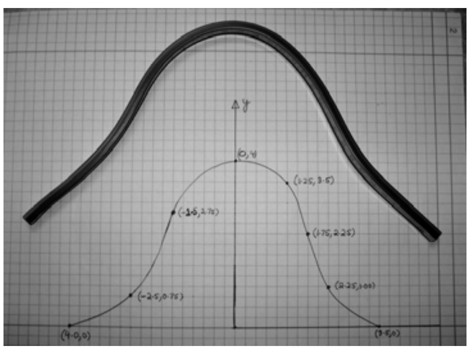

Figure 5. Using a flexible curve to draw a curve of known length

$$S = \int_a^b \sqrt{1 + \left(\frac{df}{dx}\right)^2}\, dx \qquad (10)$$

where

S is the length of a path,
f is the interpolant *f(x),*
a is the starting *x*-point, and
b is the end *x*-point.

Now the length of the interpolants is compared with the actual length of the original curve drawn by the flexible curve. This exercise is also then related to a real-life problem of finding the shortest (but smoothest) path of a robot that needs to traverse consecutively through several discrete data points [14].

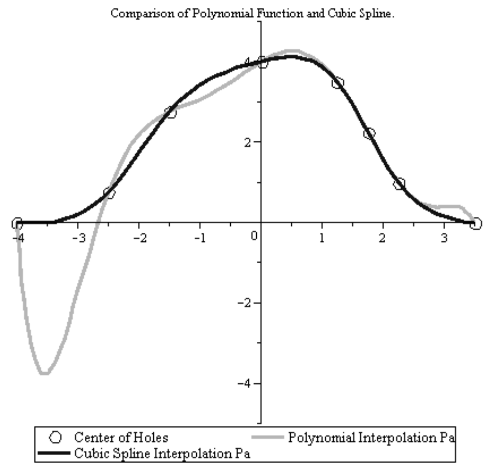

Figure 6. Comparison of Polynomial and Spline Interpolants

Assessment

To measure student satisfaction, a survey was developed to gather information on students' perceptions of the experiments, and on how the inclusion of experiments affected their learning of the course material. The survey included both quantitative and qualitative questions, thus permitting exploration of the reasons behind student ratings. The instrument consisted of six questions (see Table 1) using Likert [15] scale from 1 (far below average) to 5 (outstanding), and three open-ended questions as follows:

1. What did you like most about the experiments? Please be specific.
2. What did you like least about the experiments? Please be specific.
3. What would you like to change about the experiments? Please be specific.

The survey was administered in Spring 2008 and Summer 2008 to two classes of 42 and 55 students, respectively.

Table 1. Summary results of quantitative student survey questions

	Mean* (Standard Deviation) [95% confidence interval]	
Questions	**Spring 2008 (n=37)**	**Summer 2008 (n=51)**
In terms of the value of helping me acquire basic knowledge and skills, I'd say that the experiments were	3.84 (0.69) [3.6-4.1]	3.92 (0.86) [3.7-4.2]
In terms of their value in reinforcing information presented in class, reading assignments and problem sets, I'd say that the experiments were	3.84 (0.93) [3.5-4.1]	3.85(0.87) [3.6-4.1]
In terms of their value in helping me learn to clearly formulate a specific problem and then work it through to completion, I'd say that the experiments were	3.84 (0.93) [3.5-4.1]	3.90(0.91) [3.7-4.2]
In terms of the value of helping me develop generic higher-order thinking (e.g. analysis, synthesis and evaluation from Bloom's taxonomy [16] and problem solving skills, I'd say that the experiments were	3.70 (0.91) [3.4-4.0]	3.71(0.91) [3.5-4.0]
In terms of their value of helping me develop a sense of competence and confidence, I'd say that the experiments were	3.65 (0.98) [3.3-4.0]	3.75 (0.95) [3.5-4.0]
In terms of helping me see the relevance of the course material to my major, I'd say the experiments were	4.03 (1.04) [3.7-4.4]	4.15 (0.85) [3.9-4.4]

* 1=Far Below Average, 2=Below Average, 3=Average, 4= Very Good, 5=Outstanding

Quantitative Analysis

The percentage of students responding to the survey was 88% for the spring and 93% for the summer terms. These response rates are well above the recommended 75% rate for classes of this size [17].

Using the two-sample t-test [18], the student evaluations showed no significant difference (at the α=0.05 level) for the six quantitative questions between the summer and the spring semesters. This was expected, as there were no major changes to the course material between these semesters. Table 1 shows the mean, standard deviation and the 95% confidence intervals of the student evaluations for the quantitative questions.

Based on the responses to the quantitative questions, the students felt that the experiments were "very good" in helping them: 1) acquire basic knowledge and skills; 2) reinforce information presented in class, reading assignments and problem sets; 3) learn to clearly formulate a specific problem and then work it through to completion; 4) develop generic higher-order thinking and problem solving skills; 5) develop a sense of competence and confidence; and 6) see relevance of the course material to their major.

Qualitative Analysis

The responses to the three open-ended questions are reviewed and thematically analyzed.

Question 1. What did you like most about the experiments? Please be specific.
Of the 85 total numbers of responses to this question, four distinct themes emerged regarding what the students most liked about the experiments. 27 students indicated application of course material to real-life problems, 19 students indicated improving their programming skills using a programming language (e.g. MATLAB, Maple) in the experiments, 18 students indicated being able to relate the lecture materials to the experiments and 15 students indicated the hands-on nature of the experiments as what they most like about the experiments.

Question 2. What did you like least about the experiments? Please be specific.
Of the 86 responses to this question, 22 students indicated that they had difficulty with the programming aspects related to the experiments. 11 students indicated that the experiments were not explained in sufficient detail and finally 8 students disliked working alone and preferred groups.

Question 3. What would you like to change about the experiments? Please be specific.
Of the 85 responses, 17 indicated that they would change nothing associated with the experiments. The most frequent change indicated in 14 responses was related to the programming aspects of the experiments. The comments ranged from removing the programming aspect all together to providing some form of assistance such as reference documents to aid the students. Finally, 6 students expressed the desire to work in groups. These topics were also prevalent in the comments to the previous open-ended questions.

Based on the above comments, we are increasing the programming review sessions (the pre-requisite to the course is a 1-credit hour programming concepts course) from two hours to four hours for the course. We have revised the handouts for each experiment by having separate sections on background, laboratory instructions, and assignments. Regarding the comments on working in groups, three of the experiments are conducted in groups of five, but individual reports are required of each student. At this time, the individual projects are too short to justify group work.

Conclusions

The open-ended questions in the student satisfaction surveys clearly show that the goal of providing our students with more hands-on and more real-life applications in their mechanical engineering courses has been achieved through the incorporation of the experiments into the numerical analysis course. These two themes emerged as two of the top four most liked attributes of the experiments.

Coupled with other improvements in the course such as the problem-centered approach [19], the effect of the experiments resulted in high student satisfaction and learning. The most prevalent criticism associated with the experiments was the difficulty associated with using the programming software packages. Interestingly enough, this topic was also among the most liked aspects of the experiments.

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