CONCEPTS OF ERROR

# Error Propagation in a Simple Function Evaluation

**Sean Rodby, Luke Snyder, Autar Kaw**
**University of South Florida**
**United States of America**
**kaw@eng.usf.edu**
**Website: http://numericalmethods.eng.usf.edu**
**Version: Mathcad 14; Last Revised: August 28, 2008**

## Introduction

The following worksheet illustrates how error propagates in a simple function evaluation. This is due largely to the fact that relative error calculations are based on round-off errors and/or measurement errors that exist inherently in simple devices used to take measurements. The user will enter force, length or width of the cross section, and Young's Modulus data in the *Input* section of the program, as well as the corresponding relative measurement error for each input. From here, Mathcad will be used to demonstrate how this error propagates. The formula used for illustration is Strain = $\frac{F}{h^2 E}$.

## Section 1: Input

This is the only section where the user interacts with the program.

Enter inputs to calculate strain based on the formula Strain = $\frac{F}{h^2 E}$.

- Enter Force (in N)

  $Force := 72$

- Enter relative measurement error in percentage for force:

  $RME\_Force := 2.5\%$

- Enter length or width of cross section (in m):

  $h := 4 \times 10^{-3}$

- Enter relative measurement error in percentage for h:

  $RME\_h := 2.5\%$

- Enter Young's Modulus (in Pa):

  $E := 70 \cdot 10^{9}$

- Enter relative measurement error in percentage for Modulus:

  RME_Young := 2.5%

### Section 2: Procedure

Calculating the measurement error for each input.

$$\text{Meas\_Error\_Force} := \text{Force}\cdot\text{RME\_Force} = 1.8$$

$$\text{Meas\_Error\_h} := \text{h}\cdot\text{RME\_h} = 1\times 10^{-4}$$

$$\text{Meas\_Error\_Young} := \text{E}\cdot\text{RME\_Young} = 1.75\times 10^{9}$$

First we must calculate the strain in the worksheet given by the equation $\varepsilon = \frac{F}{h^2 E}$.

$$\text{strain} := \frac{\text{Force}}{h^2 E} = 6.429\times 10^{-5}$$

Using the formula for maximum possible error to derive the maximum possible error in the measured strain. Note that different variable names are used because the previous variable names represented actual numbers meaning that if the partial differential is taken, in all cases it will be zero.

$$\text{partial\_wrt\_Force}(F, h1, E1) := \left|\frac{d}{dF}\left(\frac{F}{h1^2\cdot E1}\right)\right|$$

$$\text{partial\_wrt\_h}(F, h1, E1) := \left|\frac{d}{dh1}\left(\frac{F}{h1^2\cdot E1}\right)\right|$$

$$\text{partial\_wrt\_Young}(F, h1, E1) := \left|\frac{d}{dE1}\left(\frac{F}{h1^2\cdot E1}\right)\right|$$

Calculating the range of value for each measured quantity based in the RME percentages given.

$$Range\_Force := RME\_Force \cdot Force = 1.8$$

$$Range\_h := RME\_h \cdot h = 1 \times 10^{-4}$$

$$Range\_Young := RME\_Young \cdot E = 1.75 \times 10^{9}$$

Calculating the maximum possible error by adding the absolute value of the partial differential equations multiplied by their respective range of variation in the measured value. Then calculating the individual error contributions due to Force, Area, and the length or width dimension respectively.

With respect to force:

$$Error\_Force := partial\_wrt\_Force(Force, h, E) \cdot Range\_Force = 1.607 \times 10^{-6}$$

With respect to h:

$$Error\_h := partial\_wrt\_h(Force, h, E) \cdot Range\_h = 3.214 \times 10^{-6}$$

With respect to Young's Modulus:

$$Error\_Young := partial\_wrt\_Young(Force, h, E) \cdot Range\_Young = 1.607 \times 10^{-6}$$

Calculating total error by adding all the individual contributions given by the user defined inputs.

$$Tot\_Error := Error\_Force + Error\_h + Error\_Young = 6.429 \times 10^{-6}$$

Calculating the percent contribution of force error to the total error.

$$Rel\_Force\_Error := \frac{Error\_Force}{Tot\_Error} \cdot 100 = 25$$

Calculating the percent contribution of h error to the total error.

$$Rel\_h\_Error := \frac{Error\_h}{Tot\_Error} \cdot 100 = 50$$

Calculating the percent contribution of Young's Modulus error to the total error.

$$Rel\_Young\_Error := \frac{Error\_Young}{Tot\_Error} \cdot 100 = 25$$

Calculating the effective range that the axial strain could be within using the total error calculation.

$$Low\_strain := strain - Tot\_Error = 5.786 \times 10^{-5}$$

$$High\_strain := strain + Tot\_Error = 7.071 \times 10^{-5}$$

Calculating the maximum percent relative measurement error for strain.

$$RME\_strain := \frac{Tot\_Error}{strain} \cdot 100 = 10$$

## Conclusion

If a calculation is made with numbers that are not exact, then the calculation itself will have an error. Since the final results of an experiment are not usually directly measured but are some function of one or more of the measured quantities, it is important to understand and utilize propagation of error concepts to better interpret and represent experimental results.

## References

Propagation of Errors.
See:
http://numericalmethods.eng.usf.edu/mcd/gen/01aae/mcd_gen_aae_txt_propagationoferrors.pdf