Concepts of True Error: Absolute True Error, Relative True Error, and Absolute Relative True Error

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Introduction

The following worksheet demonstrates how to calculate different definitions related to true error, such as true error, absolute true error, relative true error, and absolute relative true error. The concept is demonstrated using an example of a Maclaurin series. The user will choose which function to perform the calculation for in the *Input* section of the program. The choices are given as 1 for e^x, 2 for *sin(x)*, and 3 for *cos(x)*. The *true value* of these functions will be assumed as given by the Mathcad commands for these functions.

Section 1: Input

This is the only section where the user interacts with the program.

- Pick the function of your desire by choosing an integer: 1 for e^x; 2 for *sin(x)*; 3 for *cos(x)*

  funcchoice := 1

- Maximum number of terms used in the Maclaurin series

  n := 15

- Value of x at which the function is calculated

  xv := 1.6

Section 2: Procedure

First, determine which function will be used in the calculations, based on the users input. Once the function is determined, the value is calculated using a Maclaurin series in a repetitive loop.

$$f(x) := \left| \begin{array}{l} y \leftarrow e^{x} \ \text{ if } \ funcchoice = 1 \\ y \leftarrow \sin(x) \ \text{ if } \ funcchoice = 2 \\ y \leftarrow \cos(x) \ \text{ if } \ funcchoice = 3 \\ \text{return } \ y \end{array} \right.$$

$$sumprevious := 0$$

$$sumpresent(n) := \left| \begin{array}{l} \text{for } \ i \in 1 .. n \\ \quad \left| \begin{array}{l} Y \leftarrow sumprevious + \dfrac{xv^{i-1}}{(i-1)!} \ \text{ if } \ funcchoice = 1 \\ Y \leftarrow sumprevious + \dfrac{(-1)^{i-1}\, xv^{2 \cdot i-1}}{(2 \cdot i-1)!} \ \text{ if } \ funcchoice = 2 \\ Y \leftarrow sumprevious + \dfrac{(-1)^{i+1}\, xv^{2 \cdot i-2}}{(2 \cdot i-1)!} \ \text{ if } \ funcchoice = 3 \\ sumprevious \leftarrow Y \end{array} \right. \\ Y \end{array} \right.$$

Section 3: Calculation

This loop here calculates the following
N = number of terms used
T = true value
C = calculated value
E_t = true error
$|E_t|$ = absolute true error
ε_t = percentage relative true error
$|\varepsilon_t|$ = percentage absolute relative true error

$$
\begin{aligned}
\text{table1} := \ & \text{for } i \in 0..n \\
& \left| \begin{array}{l}
N_i \leftarrow i + 1 \\
T_i \leftarrow f(xv) \\
C_i \leftarrow \text{sumpresent}(i+1) \\
E_{t_i} \leftarrow T_i - \text{sumpresent}(i+1) \\
E_{ta_i} \leftarrow \left| T_i - \text{sumpresent}(i+1) \right| \\
\text{augment}\left(N, T, C, E_t, E_{ta}\right)
\end{array} \right.
\end{aligned}
$$

$$
\begin{aligned}
\text{table2} := \ & \text{for } i \in 0..n \\
& \left| \begin{array}{l}
N_i \leftarrow i + 1 \\
T_i \leftarrow f(xv) \\
C_i \leftarrow \text{sumpresent}(i+1) \\
E_{t_i} \leftarrow T_i - \text{sumpresent}(i+1) \\
\varepsilon_{t_i} \leftarrow \dfrac{E_{t_i}}{f(xv)} \cdot 100 \\
\varepsilon_{ta_i} \leftarrow \left| \dfrac{E_{t_i}}{f(xv)} \cdot 100 \right| \\
\text{augment}\left(N, T, C, \varepsilon_t, \varepsilon_{ta}\right)
\end{array} \right.
\end{aligned}
$$

Section 4: Table of Values

table1 =

	N (0)	T (1)	C (2)	E_t (3)	$\lvert E_t \rvert$ (4)
0	1	4.953	1	3.953	3.953
1	2	4.953	2.6	2.353	2.353
2	3	4.953	3.88	1.073	1.073
3	4	4.953	4.563	0.39	0.39
4	5	4.953	4.836	0.117	0.117
5	6	4.953	4.923	0.03	0.03
6	7	4.953	4.946	$6.616\cdot10^{-3}$	$6.616\cdot10^{-3}$
7	8	4.953	4.952	$1.29\cdot10^{-3}$	$1.29\cdot10^{-3}$
8	9	4.953	4.953	$2.247\cdot10^{-4}$	$2.247\cdot10^{-4}$
9	10	4.953	4.953	$3.538\cdot10^{-5}$	$3.538\cdot10^{-5}$
10	11	4.953	4.953	$5.076\cdot10^{-6}$	$5.076\cdot10^{-6}$
11	12	4.953	4.953	$6.692\cdot10^{-7}$	$6.692\cdot10^{-7}$
12	13	4.953	4.953	$8.157\cdot10^{-8}$	$8.157\cdot10^{-8}$
13	14	4.953	4.953	$9.244\cdot10^{-9}$	$9.244\cdot10^{-9}$
14	15	4.953	4.953	$9.789\cdot10^{-10}$	$9.789\cdot10^{-10}$
15	16	4.953	4.953	$9.727\cdot10^{-11}$	$9.727\cdot10^{-11}$

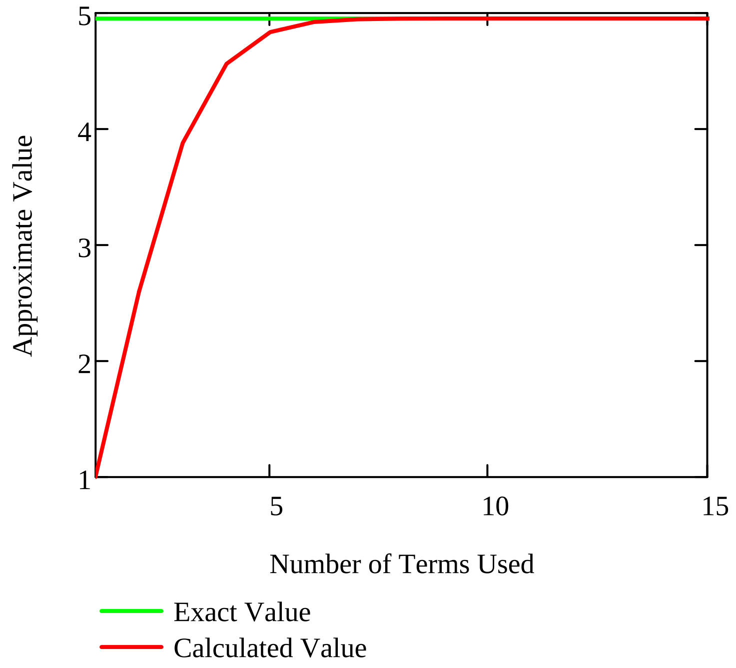
table2 =

	N (0)	T (1)	C (2)	ε_t (3)	$\lvert \varepsilon_t \rvert$ (4)
0	1	4.953	1	79.81	79.81
1	2	4.953	2.6	47.507	47.507
2	3	4.953	3.88	21.664	21.664
3	4	4.953	4.563	7.881	7.881
4	5	4.953	4.836	2.368	2.368
5	6	4.953	4.923	0.604	0.604
6	7	4.953	4.946	0.134	0.134
7	8	4.953	4.952	0.026	0.026
8	9	4.953	4.953	$4.538\cdot10^{-3}$	$4.538\cdot10^{-3}$
9	10	4.953	4.953	$7.142\cdot10^{-4}$	$7.142\cdot10^{-4}$
10	11	4.953	4.953	$1.025\cdot10^{-4}$	$1.025\cdot10^{-4}$
11	12	4.953	4.953	$1.351\cdot10^{-5}$	$1.351\cdot10^{-5}$
12	13	4.953	4.953	$1.647\cdot10^{-6}$	$1.647\cdot10^{-6}$
13	14	4.953	4.953	$1.866\cdot10^{-7}$	$1.866\cdot10^{-7}$
14	15	4.953	4.953	$1.976\cdot10^{-8}$	$1.976\cdot10^{-8}$
15	16	4.953	4.953	$1.964\cdot10^{-9}$	$1.964\cdot10^{-9}$

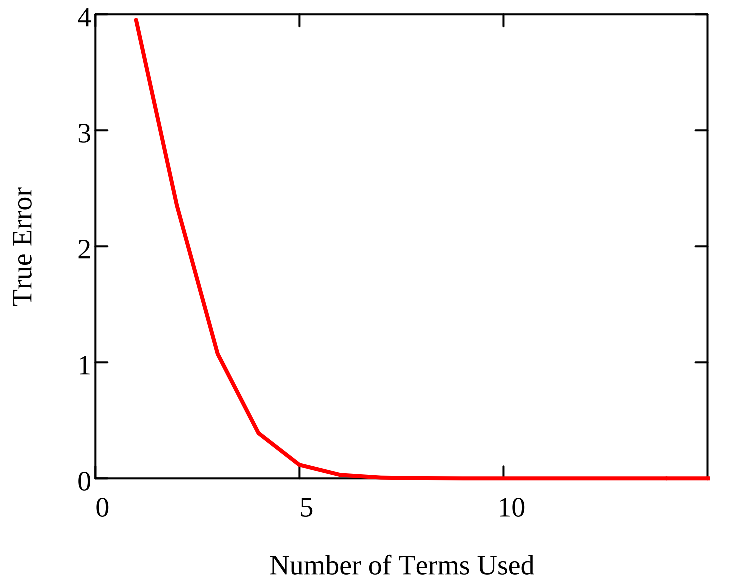
Section 5: Graphs

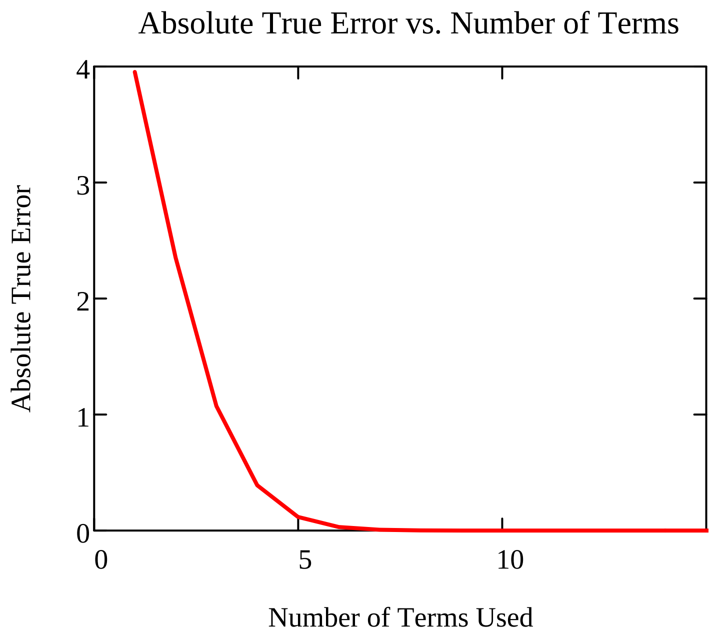
The following graphs show the calculated value of f(*x*) using Maclaurin series as a function, true error, absolute true error, absolute relative true error, relative true error, and least number of significant digits as a function of step size. Each graph displays the error results of each of the methods of approximation.

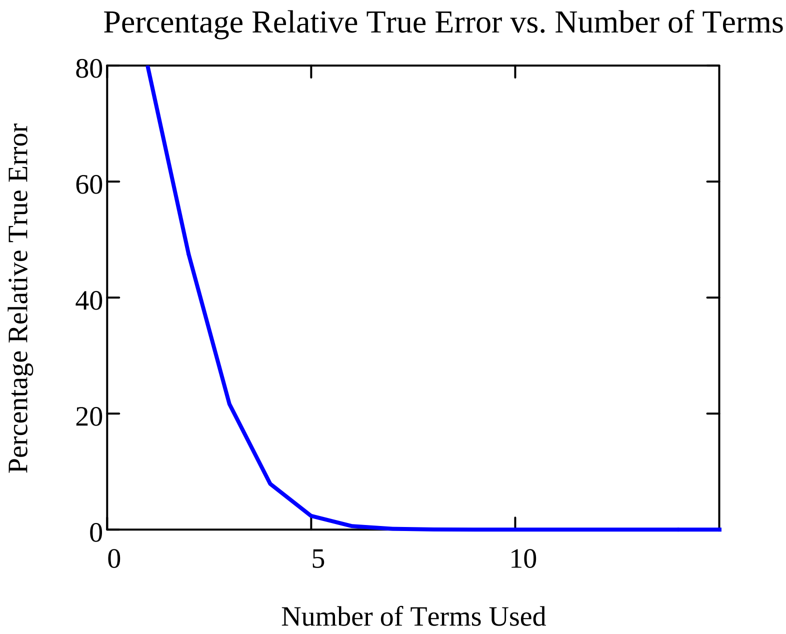
Calculated Value of f(x) Using Maclaurin Series vs. Number of Terms



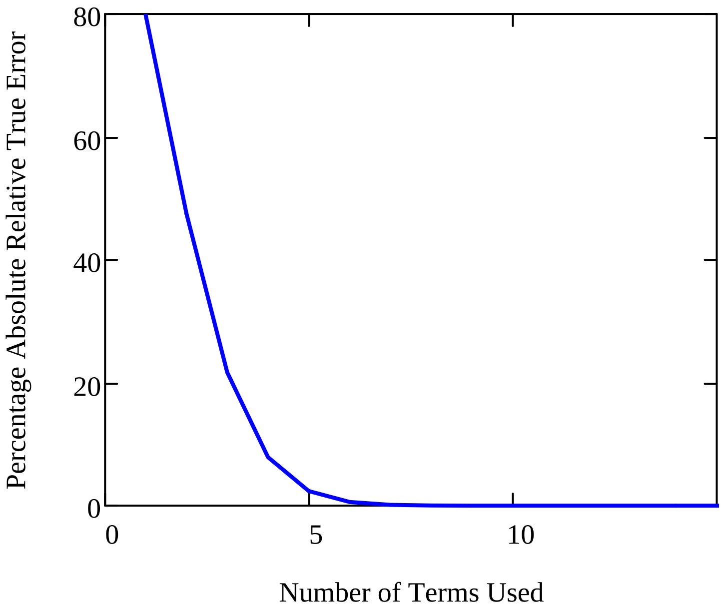
True Error vs. Number of Terms




Absolute True Error vs. Number of Terms
Absolute True Error
4
3
2
1
0
0
5
10
Number of Terms Used


Percentage Relative True Error vs. Number of Terms
Percentage Relative True Error
80
60
40
20
0
0
5
10
Number of Terms Used

Percentage Absolute Relative True Error vs. Number of Terms



Conclusion

This worksheet shows how the number of terms taken in a Maclaurin series affects the accuracy of the calculated answer through the analysis of error. Note that though true error shows the magnitude of the error, it does not indicate how bad the error is. Hence, relative true error is used here to give a more complete picture of the state of error.

References

Measuring Errors.
See:
http://numericalmethods.eng.usf.edu/mcd/gen/01aae/mcd_gen_aae_txt_measuringerror.pdf

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