Topic : Bisection Method - Roots of Equations
Simulation : Convergence of Method
Language : Mathcad 2001
Authors : Nathan Collier, Autar Kaw, Ginger Fisher
Date : 28 June 2002
Abstract : This simulation illustrates the convergence of the bisection method of finding roots of an equation f(x)=0.

INPUTS: Enter the following

Function in f(x)=0 $f(x) := x^3 - 0.03 \cdot x^2 + 2.4 \cdot 10^{-6}$

Range of 'x' you want to see the function $x := 0, .001 .. .03$

Lower initial guess $x_l := 0.01$

Upper initial guess $x_u := 0.02$

Maximum number of iterations $n_{max} := 30$

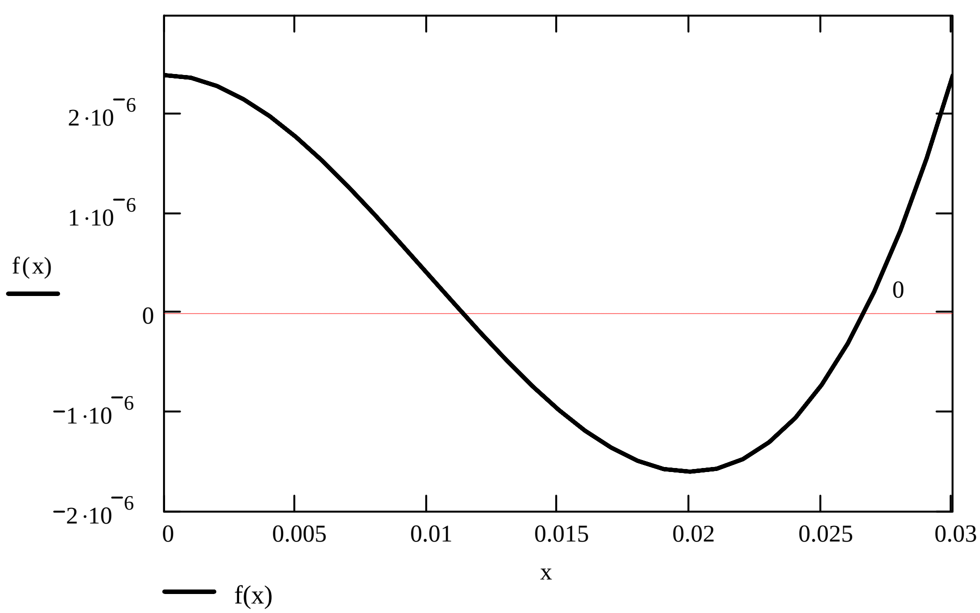
Initial guess for Mathcad numerical solution $x_{guess} := 0.01$

SOLUTION

Check if the lower and upper guess bracket the root of the equation

$$f(x_l) \cdot f(x_u) = -6.4 \times 10^{-13}$$

Entered function at given interval



True Solution:

This is the solution found by Mathcad.

$x := x_{guess}$

$x_{true} := \text{root}(f(x), x)$

$x_{true} = 0.01133$

Value of Root as a Function of Iterations:

Here the bisection method algorithm is applied to generate the values of the roots, true error, absolute relative true error, approximate error, absolute relative approximate error, and the number of significant digits at least correct in the estimated root as a function of number of iterations.

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x_r(n) := | i ← 1
          | xl ← x_l
          | xu ← x_u
          | while i ≤ n
          |   | x_root ← (xu + xl)/2
          |   | xl ← | x_root  if f(xu)·f(x_root) ≤ 0
          |   |      | xl  otherwise
          |   | xu ← | x_root  if f(xu)·f(x_root) > 0
          |   |      | xu  otherwise
          |   | i ← i + 1
          | x_root
```

$n := 1 .. n_{max}$

Absolute true error:

$E_t(n) := \left| x_{true} - x_r(n) \right|$

Absolute relative true error:

$\varepsilon_t(n) := \left| \frac{E_t(n)}{x_{true}} \right| \cdot 100$

Absolute approximate error:

$$E_a(n) := \left| x_r(n) - x_r(n-1) \right|$$

Absolute relative approximate error:

$$\varepsilon_a(n) := \begin{cases} 0 & \text{if } n \le 1 \\ \left(\left| \dfrac{E_a(n)}{x_r(n)} \right| \cdot 100 \right) & \text{otherwise} \end{cases}$$

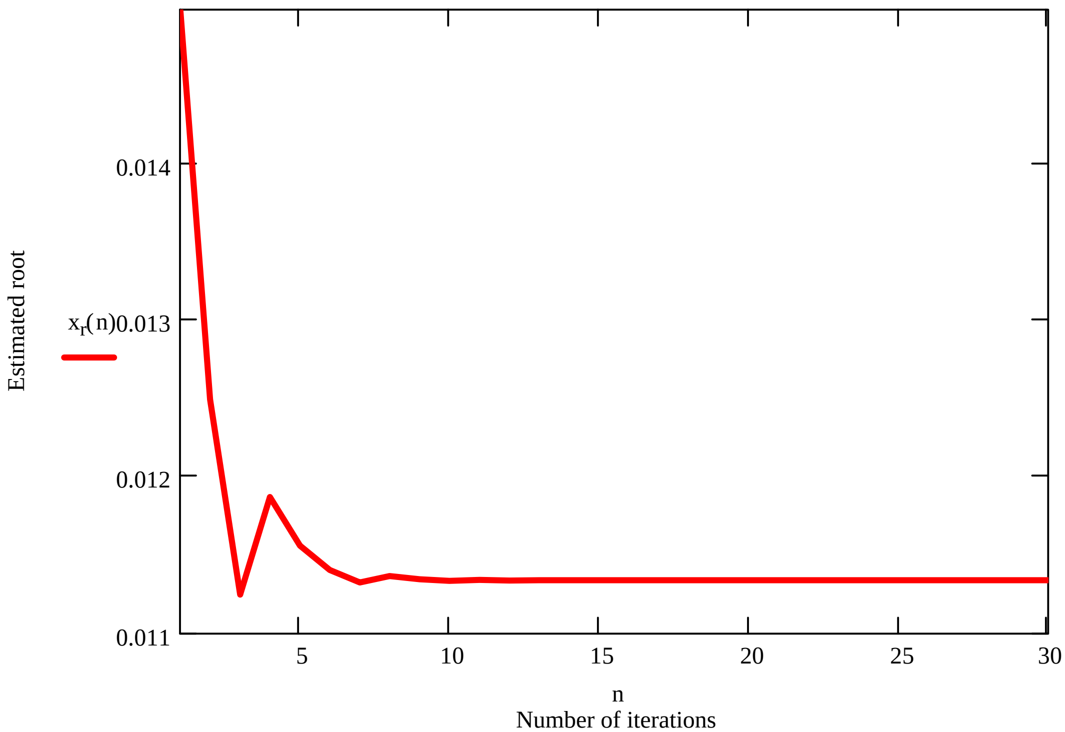
Significant digits at least correct:

$$\text{sigdigits}(n) := \begin{cases} 0 & \text{if } \left|\varepsilon_a(n)\right| \le 0 \\ \left(2 - \log\left(\left| \dfrac{\left|\varepsilon_a(n)\right|}{0.5} \right| \right) \right) & \text{otherwise} \end{cases}$$

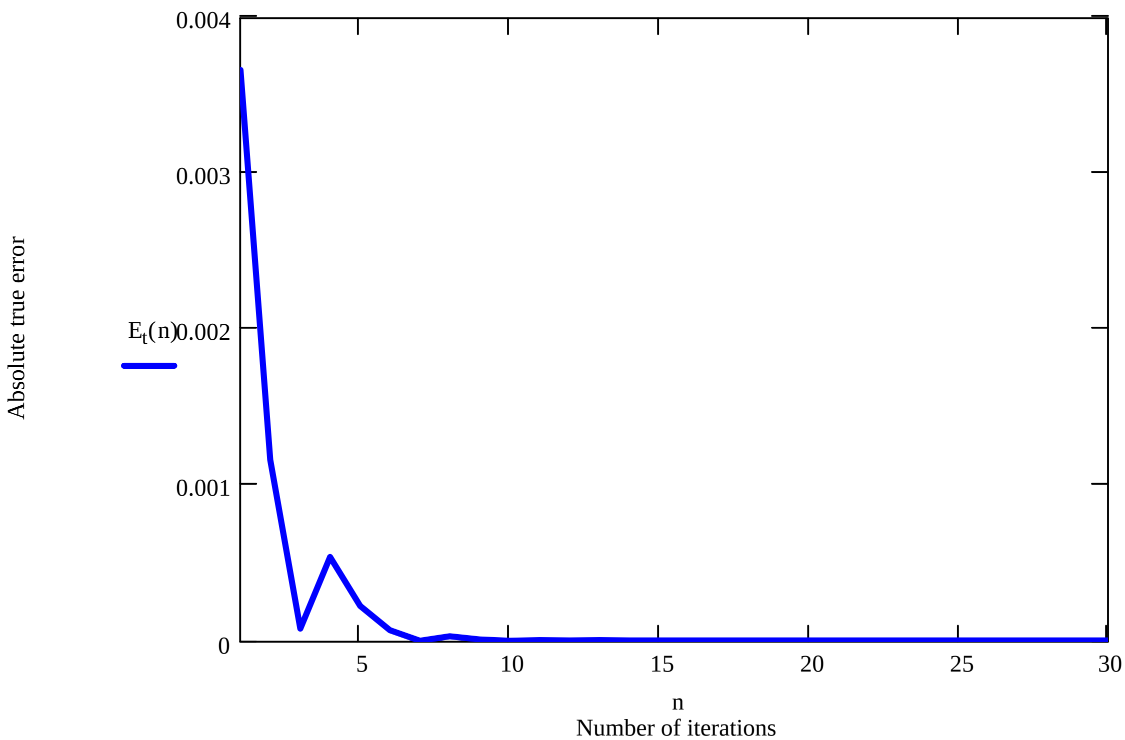
Table of Values:

$n =$	$x_r(n) =$	$E_t(n) =$	$\varepsilon_t(n) =$	$E_a(n) =$	$\varepsilon_a(n) =$	trunc(sigdigits(n)) =
1	0.015	$3.66667\cdot10^{-3}$	32.35294	0.015	0	0
2	0.0125	$1.16667\cdot10^{-3}$	10.29411	$2.5\cdot10^{-3}$	20	0
3	0.01125	$8.33338\cdot10^{-5}$	0.7353	$1.25\cdot10^{-3}$	11.11111	0
4	0.01188	$5.41666\cdot10^{-4}$	4.77941	$6.25\cdot10^{-4}$	5.26316	0
5	0.01156	$2.29166\cdot10^{-4}$	2.02205	$3.125\cdot10^{-4}$	2.7027	1
6	0.01141	$7.29162\cdot10^{-5}$	0.64338	$1.5625\cdot10^{-4}$	1.36986	1
7	0.01133	$5.20878\cdot10^{-6}$	0.04596	$7.8125\cdot10^{-5}$	0.68966	1
8	0.01137	$3.38537\cdot10^{-5}$	0.29871	$3.90625\cdot10^{-5}$	0.34364	2
9	0.01135	$1.43225\cdot10^{-5}$	0.12637	$1.95312\cdot10^{-5}$	0.17212	2
10	0.01134	$4.55685\cdot10^{-6}$	0.04021	$9.76563\cdot10^{-6}$	0.08613	2
11	0.01134	$9.43966\cdot10^{-6}$	0.08329	$4.88281\cdot10^{-6}$	0.04305	3
12	0.01134	$6.99825\cdot10^{-6}$	0.06175	$2.44141\cdot10^{-6}$	0.02153	3
13	0.01134	$8.21896\cdot10^{-6}$	0.07252	$1.2207\cdot10^{-6}$	0.01076	3
14	0.01134	$7.60861\cdot10^{-6}$	0.06713	$6.10352\cdot10^{-7}$	$5.38184\cdot10^{-3}$	3
15	0.01134	$7.91378\cdot10^{-6}$	0.06983	$3.05176\cdot10^{-7}$	$2.69085\cdot10^{-3}$	4
16	0.01134	$8.06637\cdot10^{-6}$	0.07117	$1.52588\cdot10^{-7}$	$1.34541\cdot10^{-3}$	4

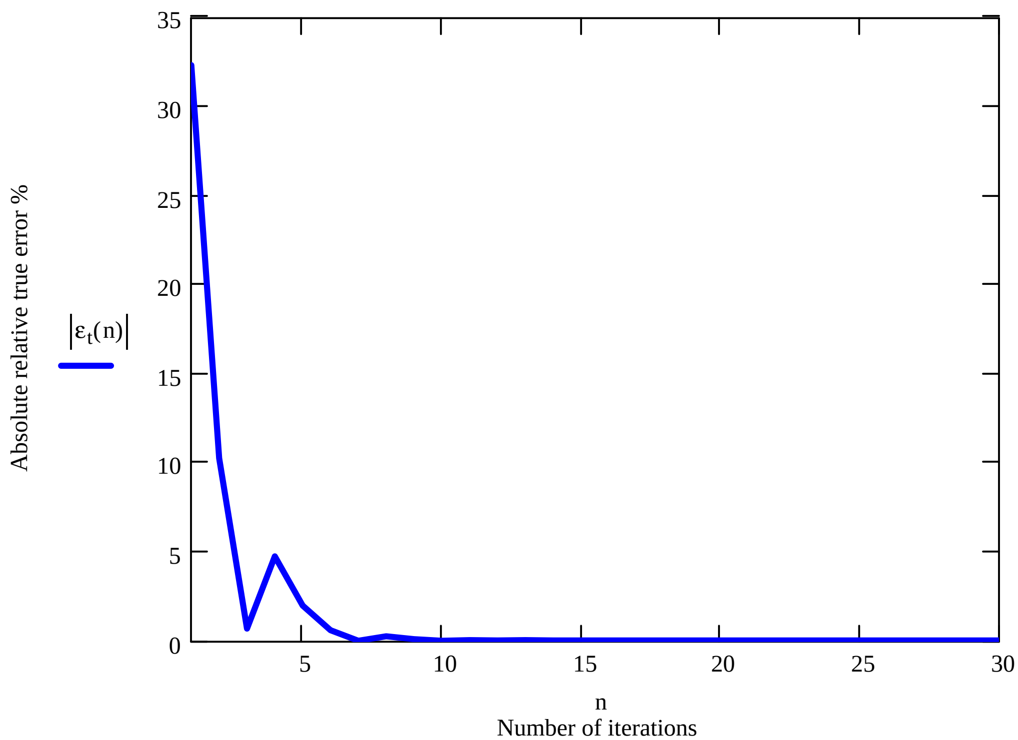
Estimated root as a function of number of iterations



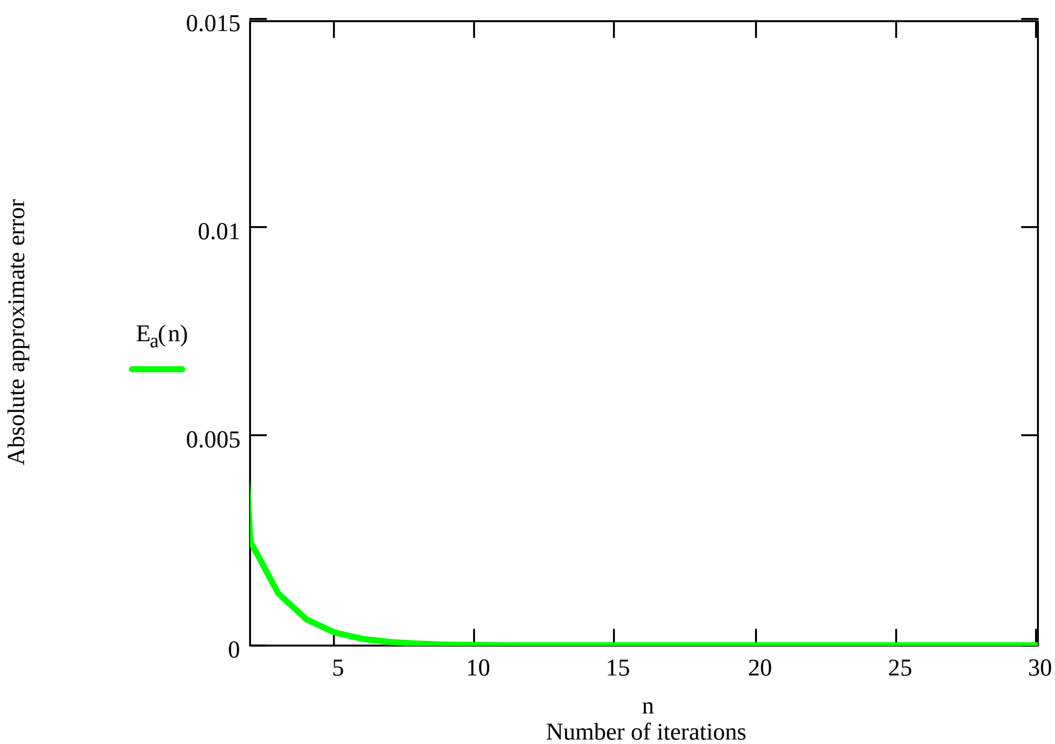
Absolute true error as a function of number of iterations



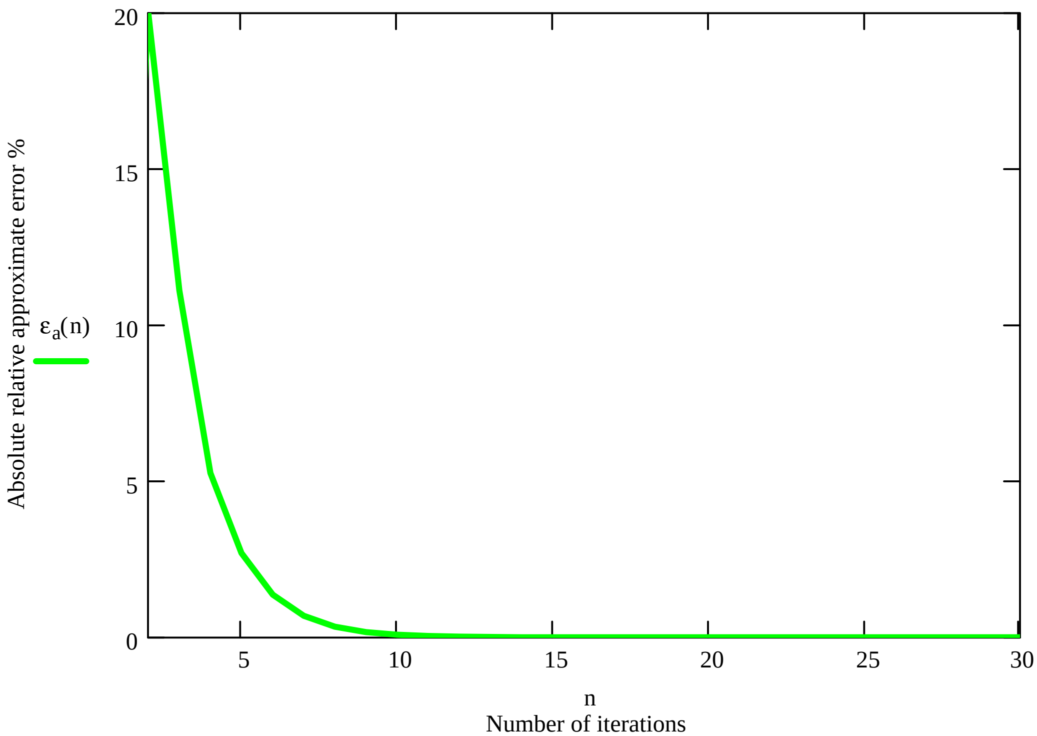
Absolute relative true error as a function of number of iterations



Absolute approximate error as a function of number of iterations



Absolute relative approximate error as a function of number of iterations



Number of significant digits at least correct as a function of number of iterations