Topic : Bisection Method - Roots of Equations
Simulation : Graphical Simulation of the Method
Language : Mathcad 2001
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Abstract : This simulation shows how the bisection method for finding roots of an equation f(x)=0 works.

INPUTS: Enter the following

Function in f(x)=0 $f(x) := x^3 - 0.03 \cdot x^2 + 2.4 \cdot 10^{-6}$

Range of 'x' you want to see the function $x := 0, .001 .. .03$

Lower initial guess $x_l := 0.01$

Upper initial guess $x_u := 0.02$

SOLUTION

Check first if the lower and upper guesses bracket the root of the equation f(x)=0

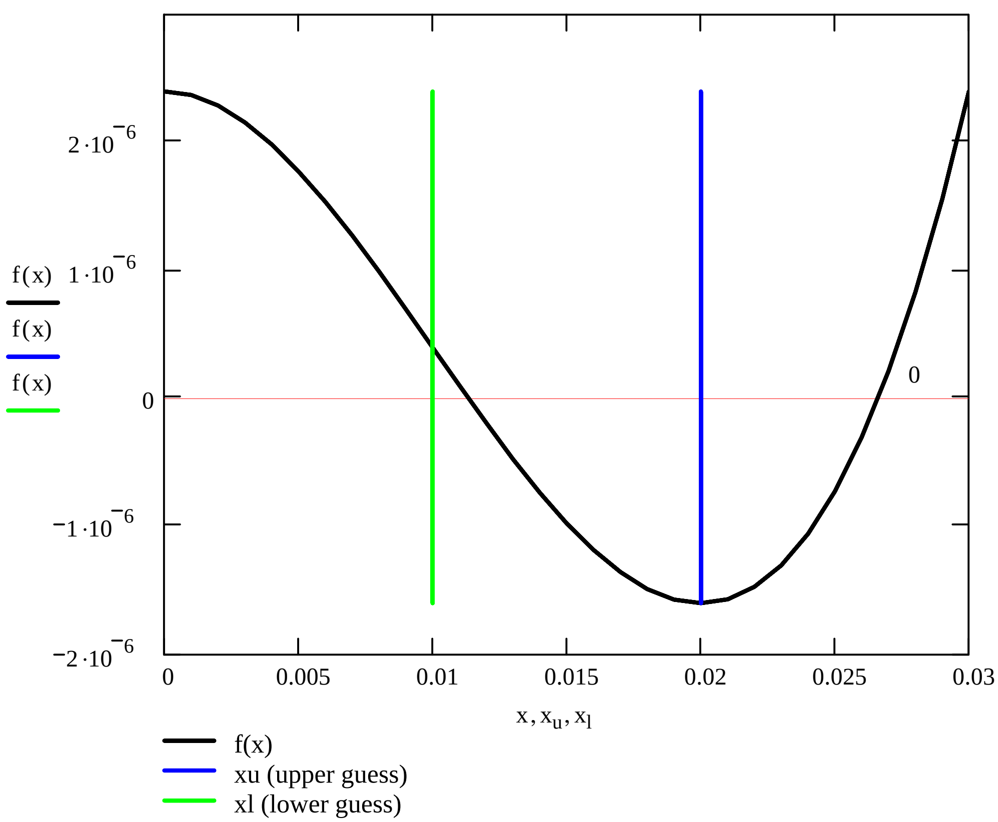
$$f(x_l) = 4 \times 10^{-7}$$

$$f(x_u) = -1.6 \times 10^{-6}$$

$$f(x_l) \cdot f(x_u) = -6.4 \times 10^{-13}$$

The blue line represents the upper initial guess, while the green line represents the lower initial guess.

Entered function on given interval with initial upper and lower guesses



Iteration 1:

New estimate of root

$$x_r := \frac{x_u + x_l}{2}$$

$x_r = 0.015$

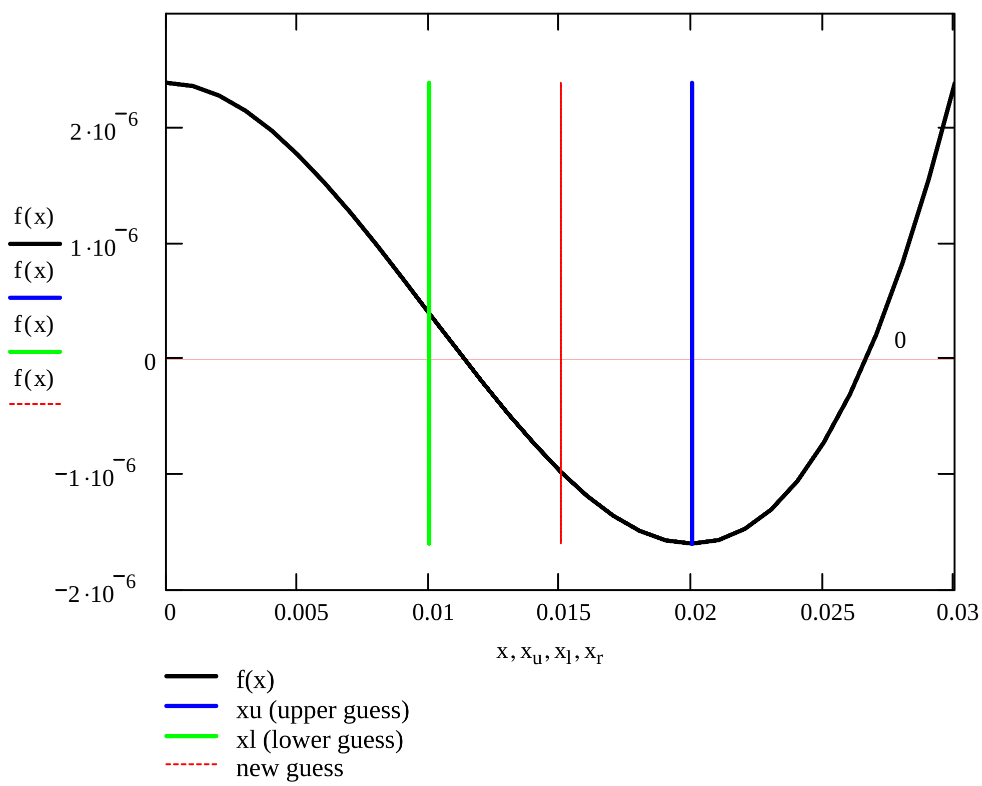
Finding value of function at the lower and upper guesses and the estimated root

$f(x_l) = 4 \times 10^{-7}$

$f(x_u) = -1.6 \times 10^{-6}$

$f(x_r) = -9.75 \times 10^{-7}$

Entered function on given interval with upper and lower guesses and estimated root



Check the interval between which the root lies. Does it lie in (x_l, x_r) or (x_r, x_u)?

$$x_l := \begin{cases} x_r & \text{if } f(x_u)\cdot f(x_r) \le 0 \\ x_l & \text{otherwise} \end{cases}$$

$x_l = 0.01$

$$x_u := \begin{cases} x_r & \text{if } f(x_u)\cdot f(x_r) > 0 \\ x_u & \text{otherwise} \end{cases}$$

$x_u = 0.015$

Store the value of the previous guess in x_p to calculate absolute relative approximate error.

$x_p := x_r$

Iteration 2:

$$x_r := \frac{x_u + x_l}{2}$$

$x_r = 0.0125$

Finding value of function at the lower and upper guesses and the estimated root

$f(x_u) = -9.75 \times 10^{-7}$

$f(x_l) = 4 \times 10^{-7}$

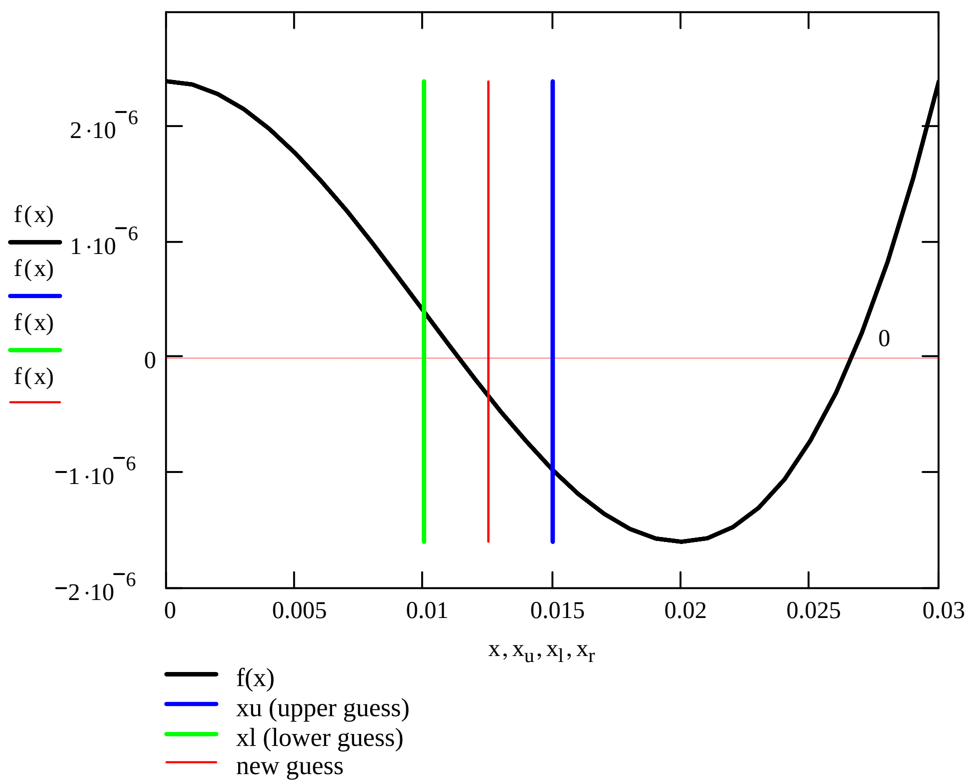
$f(x_r) = -3.34375 \times 10^{-7}$

Absolute relative approximate error, $|\epsilon_a|$.

$$\varepsilon_a := \left| \frac{x_r - x_p}{x_r} \right| \cdot 100$$

$\varepsilon_a = 20$

Entered function on given interval with upper and lower guesses and estimated root



Check the interval between which the root lies. Does it lie in (x_l, x_r) or (x_r, x_u)?

$$x_l := \begin{cases} x_r & \text{if } f(x_u)\cdot f(x_r) \le 0 \\ x_l & \text{otherwise} \end{cases}$$

$x_l = 0.01$

$$x_u := \begin{cases} x_r & \text{if } f(x_u)\cdot f(x_r) > 0 \\ x_u & \text{otherwise} \end{cases}$$

$x_u = 0.0125$

Store the value of the previous guess in x_p to calculate absolute relative approximate error.

$x_p := x_r$

Iteration 3:

$$x_r := \frac{x_u + x_l}{2}$$

$$x_r = 0.01125$$

Finding value of function at the lower and upper guesses and the estimated root

$$f(x_u) = -3.34375 \times 10^{-7}$$

$$f(x_l) = 4 \times 10^{-7}$$

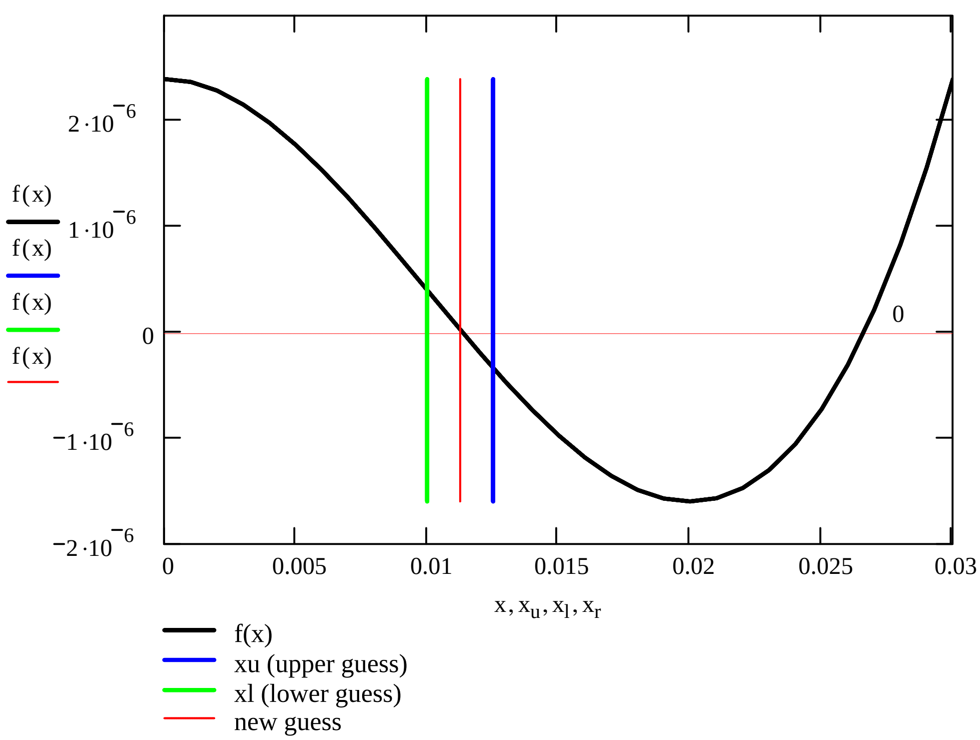
$$f(x_r) = 2.69531 \times 10^{-8}$$

Absolute relative approximate error, $|\epsilon_a|$.

$$\varepsilon_a := \left| \frac{x_r - x_p}{x_r} \right| \cdot 100$$

$$\varepsilon_a = 11.11111$$

Entered function on given interval with upper and lower guesses and estimated root



Check the interval between which the root lies. Does it lie in (x_l, x_r) or (x_r, x_u)?

$$x_l := \begin{cases} x_r & \text{if } f(x_u)\cdot f(x_r) \le 0 \\ x_l & \text{otherwise} \end{cases}$$

$x_l = 0.01125$

$$x_u := \begin{cases} x_r & \text{if } f(x_u)\cdot f(x_r) > 0 \\ x_u & \text{otherwise} \end{cases}$$

$x_u = 0.0125$

Store the value of the previous guess in x_p to calculate absolute relative approximate error.

$x_p := x_r$

Iteration 4:

$$x_r := \frac{x_u + x_l}{2}$$

$x_r = 0.01188$

Finding value of function at the lower and upper guesses and the estimated root

$f(x_u) = -3.34375 \times 10^{-7}$

$f(x_l) = 2.69531 \times 10^{-8}$

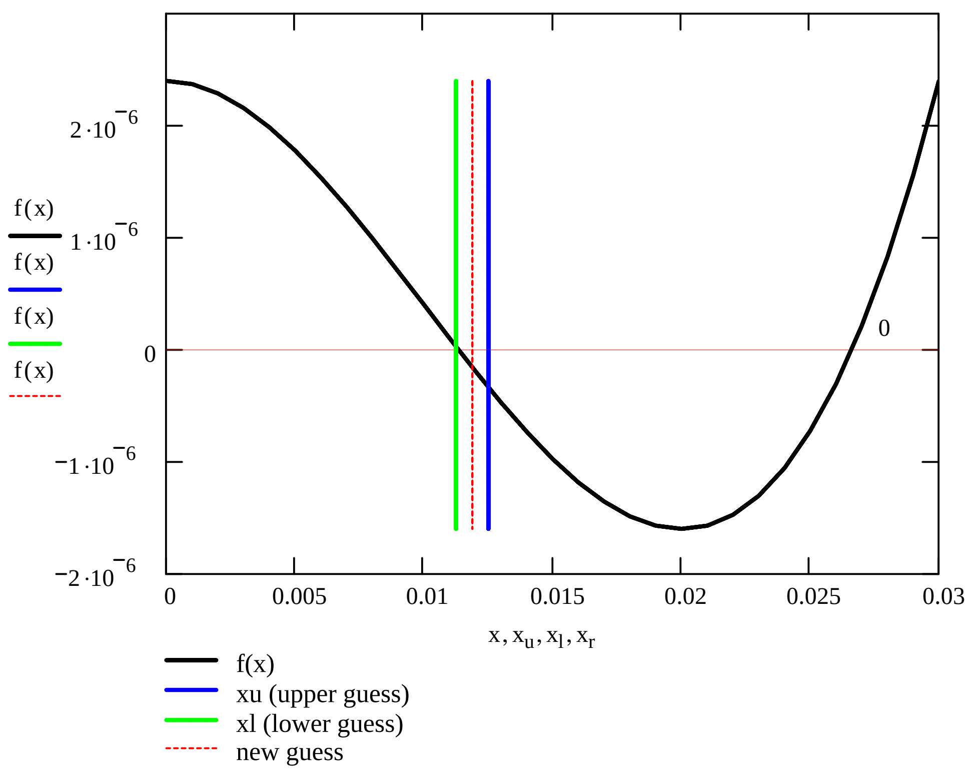
$f(x_r) = -1.55908 \times 10^{-7}$

Absolute relative approximate error, $|\epsilon_a|$.

$$\varepsilon_a := \left| \frac{x_r - x_p}{x_r} \right| \cdot 100$$

$\varepsilon_a = 5.26316$

Entered function on given interval with upper and lower guesses and estimated root



Check the interval between which the root lies. Does it lie in (x_l, x_r) or (x_r, x_u)?

$$x_l := \begin{cases} x_r & \text{if } f(x_u)\cdot f(x_r) \le 0 \\ x_l & \text{otherwise} \end{cases}$$

$x_l = 0.01125$

$$x_u := \begin{cases} x_r & \text{if } f(x_u)\cdot f(x_r) > 0 \\ x_u & \text{otherwise} \end{cases}$$

$x_u = 0.01188$

Store the value of the previous guess in x_p to calculate absolute relative approximate error.

$x_p := x_r$

Iteration 5:

$$x_r := \frac{x_u + x_l}{2}$$

$x_r = 0.01156$

Finding value of function at the lower and upper guesses and the estimated root

$f(x_u) = -1.55908 \times 10^{-7}$

$f(x_l) = 2.69531 \times 10^{-8}$

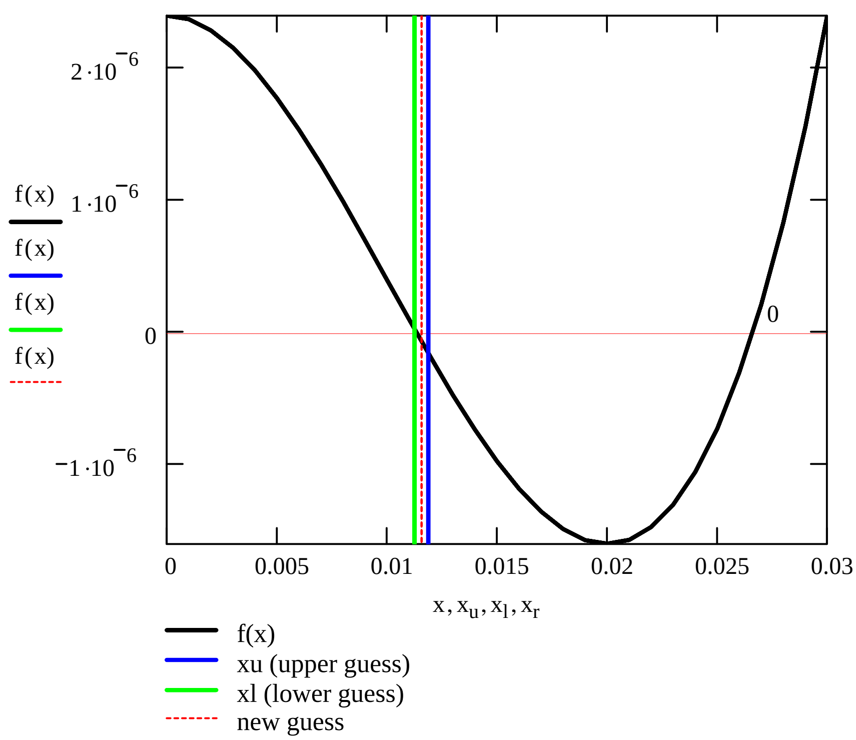
$f(x_r) = -6.49353 \times 10^{-8}$

Calculate absolute relative approximate error, $|\epsilon_a|$.

$$\epsilon_a := \left|\frac{x_r - x_p}{x_r}\right| \cdot 100$$

$\epsilon_a = 2.7027$

Entered function on given interval with upper and lower guesses and estimated root



Check the interval between which the root lies. Does it lie in (x_l, x_r) or (x_r, x_u)?

$$x_l := \begin{vmatrix} x_r & \text{if } f(x_u)\cdot f(x_r) \le 0 \\ x_l & \text{otherwise} \end{vmatrix}$$

$x_l = 0.01125$

$$x_u := \begin{vmatrix} x_r & \text{if } f(x_u)\cdot f(x_r) > 0 \\ x_u & \text{otherwise} \end{vmatrix}$$

$x_u = 0.01156$

Store the value of the previous guess in x_p to calculate absolute relative approximate error.

$x_p := x_r$