Topic : Bisection Method - Roots of Equations
Simulation : Pitfall Slow Convergence
Language : Mathcad 2001
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Abstract : This simulation illustrates the slow convergence of the bisection method of finding roots of an equation f(x)=0.

INPUTS: Enter the following

Function in f(x)=0 $f(x) := x^2 - 1$

Range of 'x' you want to see the function $x := -2, -1.99 .. 2$

Lower initial guess $x_l := -1.25$

Upper initial guess $x_u := -0.5$

Maximum number of iterations $n_{max} := 30$

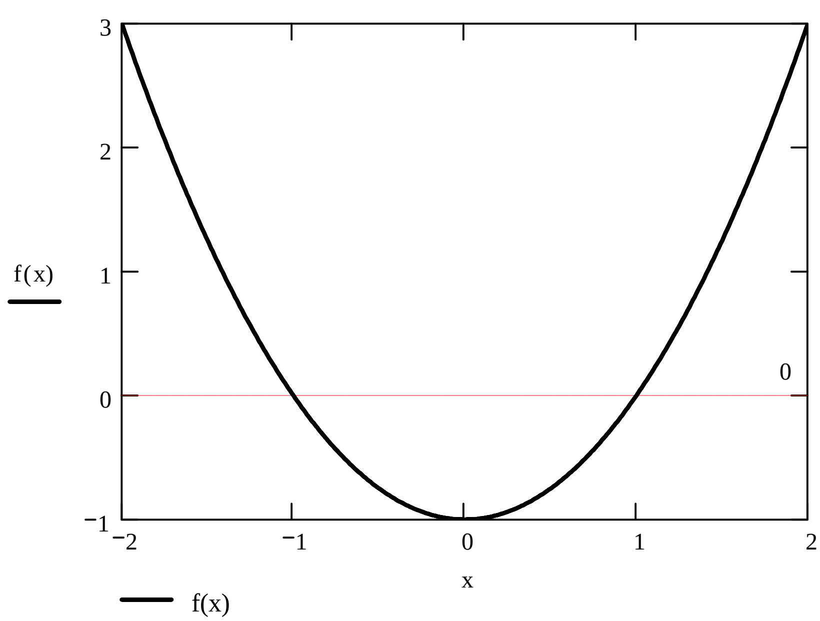
Initial guess for Mathcad solution $x_{guess} := -1.25$

SOLUTION

Check if the lower and upper guess bracket the root of the equation

$$f(x_l) \cdot f(x_u) = -0.42188$$

Entered function at given interval



Exact Solution:

This is the true solution found by Mathcad.

$x := x_{guess}$

$x_{true} := \mathrm{root}(f(x), x)$

$x_{true} = -1.00003$

Value of Root as a Function of Iterations:

Here the bisection method algorithm is applied to generate the values of the roots, true error, absolute relative true error, absolute approximate error, absolute relative approximate error, and the number of significant digits at least correct in the estimated root as a function of number of iterations.

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xr(n) := | i ← 1
         | xl ← x_l
         | xu ← x_u
         | while i ≤ n
         |   | xroot ← (xu + xl) / 2
         |   | xl ← | xroot  if f(xu)·f(xroot) ≤ 0
         |   |      | xl  otherwise
         |   | xu ← | xroot  if f(xu)·f(xroot) > 0
         |   |      | xu  otherwise
         |   | i ← i + 1
         | xroot
```

$n := 1 .. n_{max}$

Absolute true error:

$E_t(n) := \left| x_{true} - x_r(n) \right|$

Absolute relative true error:

$$\varepsilon_t(n) := \left| \frac{E_t(n)}{x_{true}} \right| \cdot 100$$

Absolute approximate error:

$$E_a(n) := \left| x_r(n) - x_r(n-1) \right|$$

Absolute relative approximate error:

$$\varepsilon_a(n) := \begin{cases} 0 & \text{if } n \le 1 \\ \left(\left| \dfrac{E_a(n)}{x_r(n)} \right| \cdot 100 \right) & \text{otherwise} \end{cases}$$

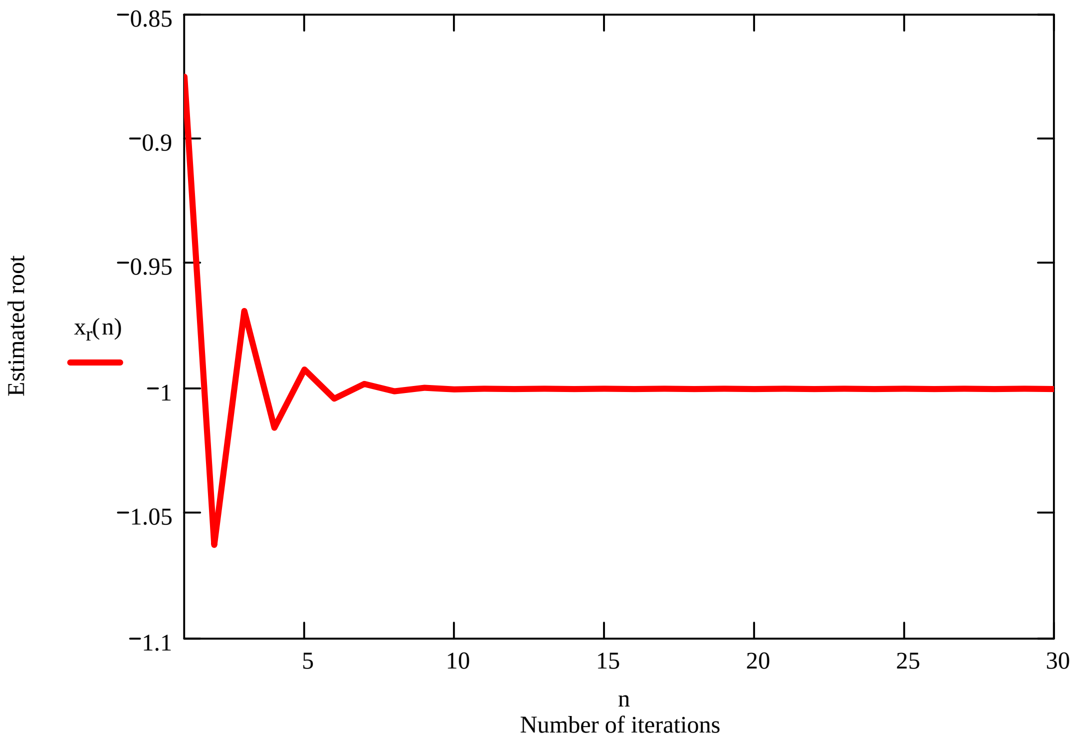
Significant digits at least correct:

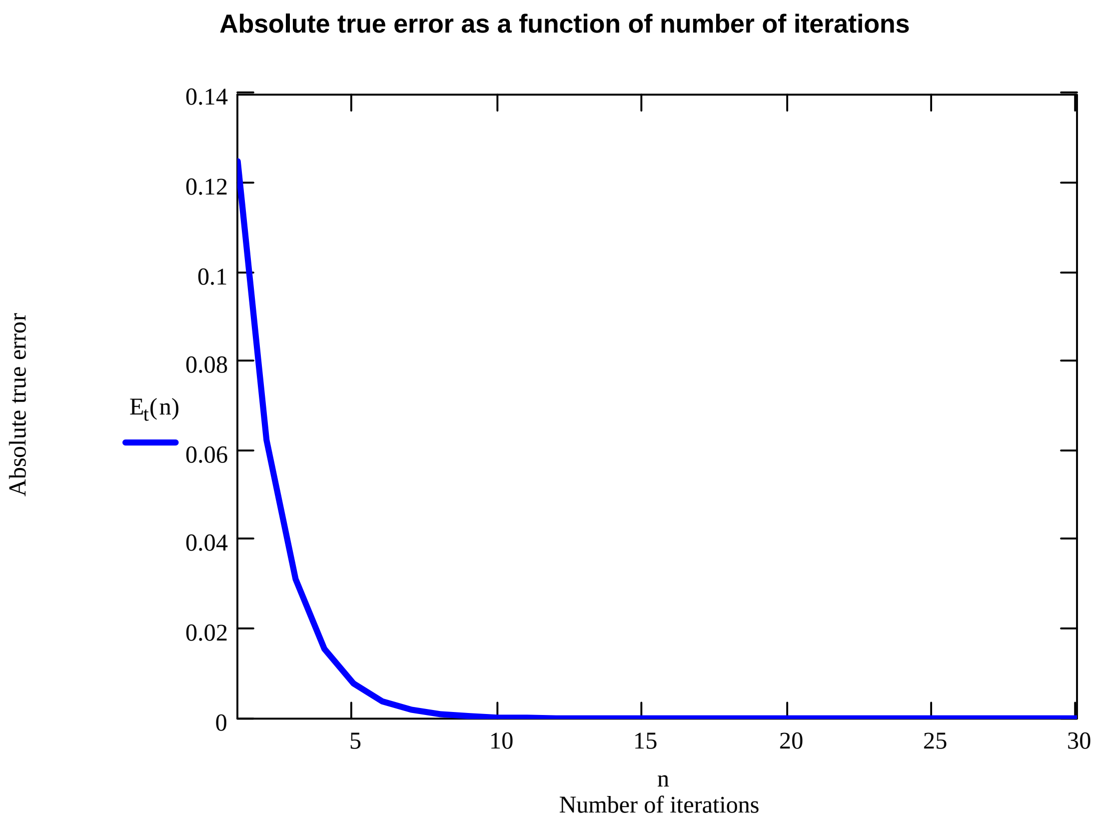
$$\text{sigdigits}(n) := \begin{cases} 0 & \text{if } \left|\varepsilon_a(n)\right| \le 0 \\ \left(2 - \log\left(\left| \dfrac{\left|\varepsilon_a(n)\right|}{0.5} \right| \right) \right) & \text{otherwise} \end{cases}$$

Table of Values:

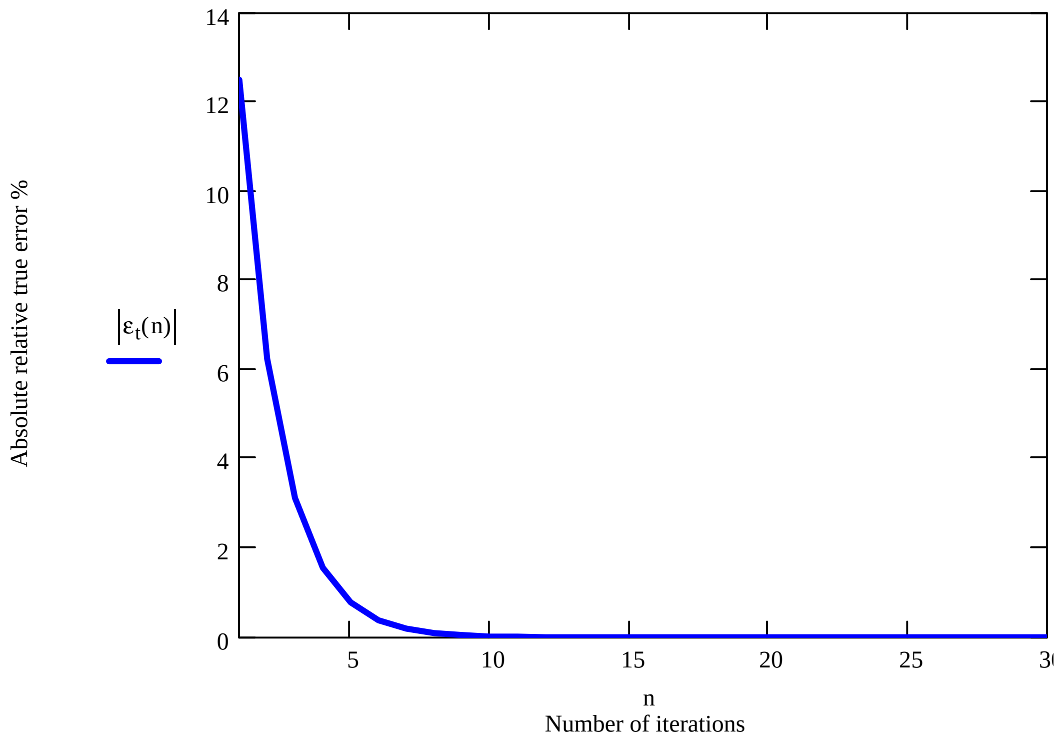
n =	$x_r(n)$ =	$E_t(n)$ =	$\varepsilon_t(n)$ =	$E_a(n)$ =	$\varepsilon_a(n)$ =	trunc(sigdigits(n)) =
1	-0.875	0.12503	12.50299	0.875	0	0
2	-1.0625	0.06247	6.24637	0.1875	17.64706	0
3	-0.96875	0.03128	3.12831	0.09375	9.67742	0
4	-1.01563	0.01559	1.55903	0.04688	4.61538	1
5	-0.99219	$7.84667 \cdot 10^{-3}$	0.78464	0.02344	2.3622	1
6	-1.00391	$3.87208 \cdot 10^{-3}$	0.38719	0.01172	1.16732	1
7	-0.99805	$1.9873 \cdot 10^{-3}$	0.19872	$5.85938 \cdot 10^{-3}$	0.58708	1
8	-1.00098	$9.4239 \cdot 10^{-4}$	0.09424	$2.92969 \cdot 10^{-3}$	0.29268	2
9	-0.99951	$5.22454 \cdot 10^{-4}$	0.05224	$1.46484 \cdot 10^{-3}$	0.14656	2
10	-1.00024	$2.09968 \cdot 10^{-4}$	0.021	$7.32422 \cdot 10^{-4}$	0.07322	2
11	-0.99988	$1.56243 \cdot 10^{-4}$	0.01562	$3.66211 \cdot 10^{-4}$	0.03663	3
12	-1.00006	$2.68628 \cdot 10^{-5}$	$2.68619 \cdot 10^{-3}$	$1.83105 \cdot 10^{-4}$	0.01831	3
13	-0.99997	$6.46899 \cdot 10^{-5}$	$6.46877 \cdot 10^{-3}$	$9.15527 \cdot 10^{-5}$	$9.15555 \cdot 10^{-3}$	3
14	-1.00002	$1.89135 \cdot 10^{-5}$	$1.89129 \cdot 10^{-3}$	$4.57764 \cdot 10^{-5}$	$4.57757 \cdot 10^{-3}$	4
15	-0.99999	$4.18017 \cdot 10^{-5}$	$4.18003 \cdot 10^{-3}$	$2.28882 \cdot 10^{-5}$	$2.28884 \cdot 10^{-3}$	4
16	-1	$3.03576 \cdot 10^{-5}$	$3.03566 \cdot 10^{-3}$	$1.14441 \cdot 10^{-5}$	$1.1444 \cdot 10^{-3}$	4

Estimated root as a function of number of iterations

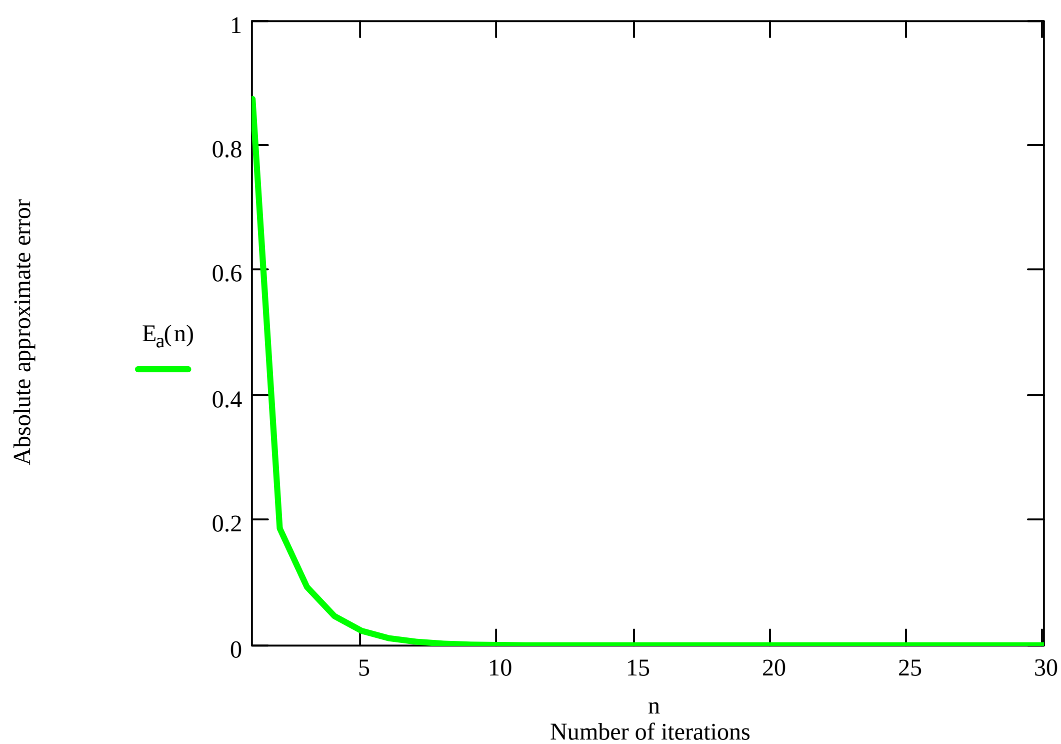



Absolute true error as a function of number of iterations
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
5
10
15
20
25
30
Absolute true error
E_t(n)
n
Number of iterations

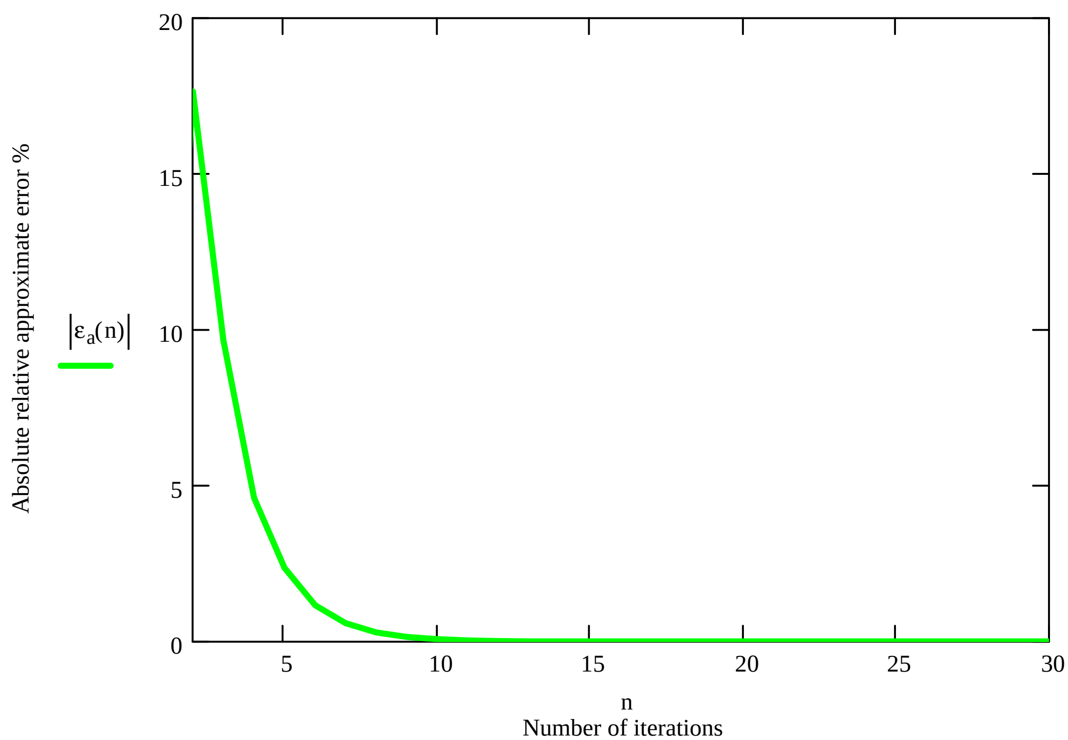
Absolute relative true error as a function of number of iterations



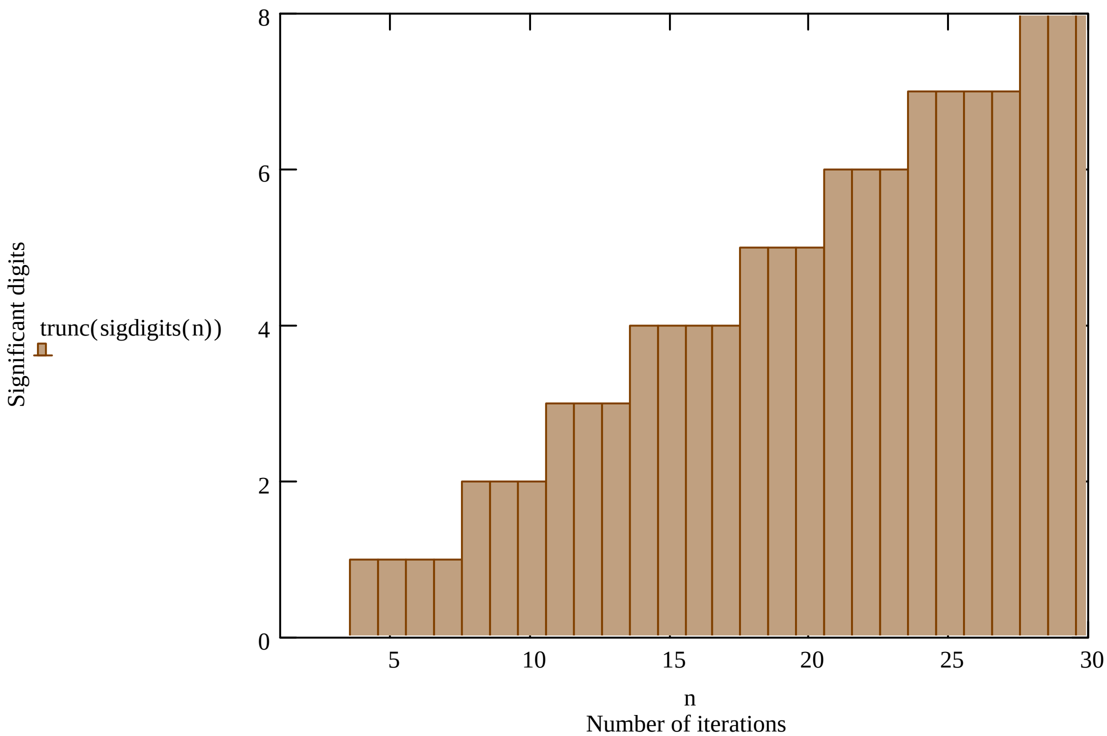
Absolute approximate error as a function of number of iterations



Absolute relative approximate error as a function of number of iterations



Least number of significant digits at least correct as a function of number of iterations



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