Topic : Newton Raphson Method - Roots of Equations
Simulation : Convergence of Method
Language : Mathcad 2001
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Abstract : This simulation illustrates the convergence of the Newton-Raphson method of finding roots of the equation f(x)=0.

INPUTS: Enter the following

Function in f(x)=0 $f(x) := x^3 - 0.03 \cdot x^2 + 2.4 \cdot 10^{-6}$

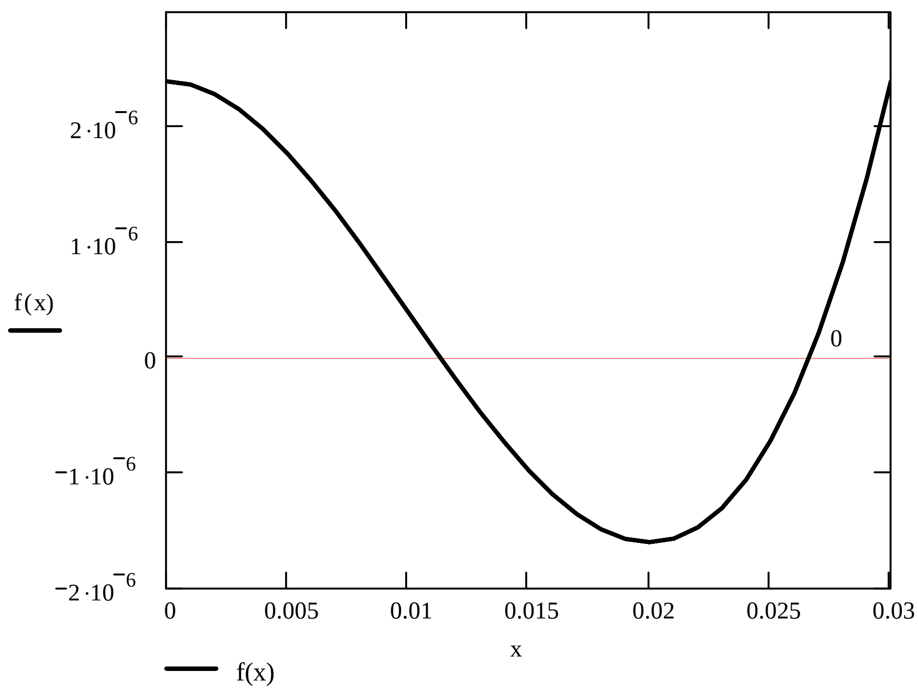
Range of x you want to see the function $x := 0, .001 .. .03$

Initial guess $x_{initial} := 0.005$

Maximum number of iterations $n_{max} := 5$

SOLUTION

Entered function at given interval



$g(x) := \frac{d}{dx} f(x)$

Exact Solution:

This is the true solution found by Mathcad.

$$TOL := 1\cdot 10^{-10}$$

$$x := x_{initial}$$

$$x_{true} := root(f(x), x)$$

$$x_{true} = 0.01134$$

Value of Root as a Function of Iterations:

The Newton Raphson method algorithm is applied to generate the values y, the roots, true error, absolute relative true error, approximate error, relative approximate error, and the number of significant digits at least correct in the estimated root as a funcion of number of iterations.

$$x_r(n) := \left| \begin{array}{l} i \leftarrow 1 \\ x \leftarrow x_{initial} \\ \text{while } i \le n \\ \quad \left| \begin{array}{l} x_{next} \leftarrow x - \dfrac{f(x)}{g(x)} \\ x \leftarrow x_{next} \\ i \leftarrow i + 1 \end{array} \right. \\ x_{next} \end{array} \right.$$

$$n := 1 .. n_{max}$$

Absolute true error:

$$E_t(n) := \left| x_{true} - x_r(n) \right|$$

Absolute relative true error:

$$\varepsilon_t(n) := \left| \frac{E_t(n)}{x_{true}} \right| \cdot 100$$

Absolute approximate error:

$$E_a(n) := \begin{cases} \left(x_r(1) - x_{initial}\right) & \text{if } n = 1 \\ \left(x_r(n) - x_r(n-1)\right) & \text{otherwise} \end{cases}$$

Absolute relative approximate error:

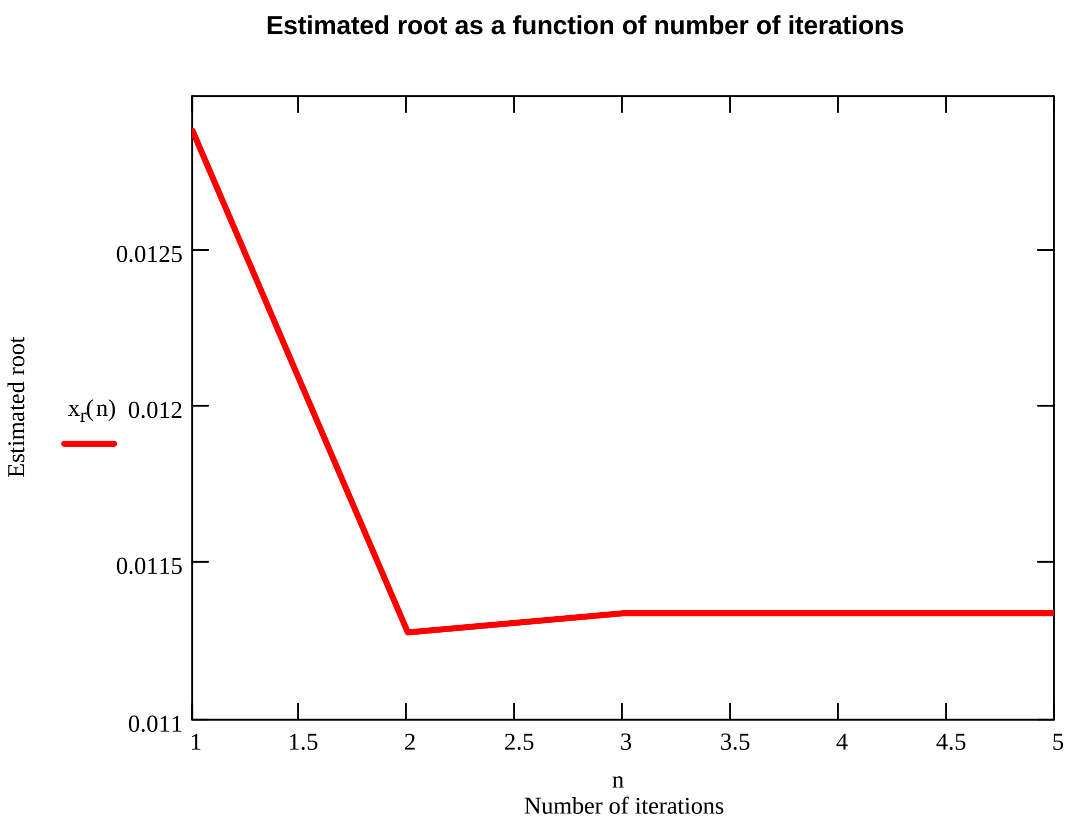
$$\varepsilon_a(n) := \left|\frac{E_a(n)}{x_r(n)}\right| \cdot 100$$

Significant digits at least correct:

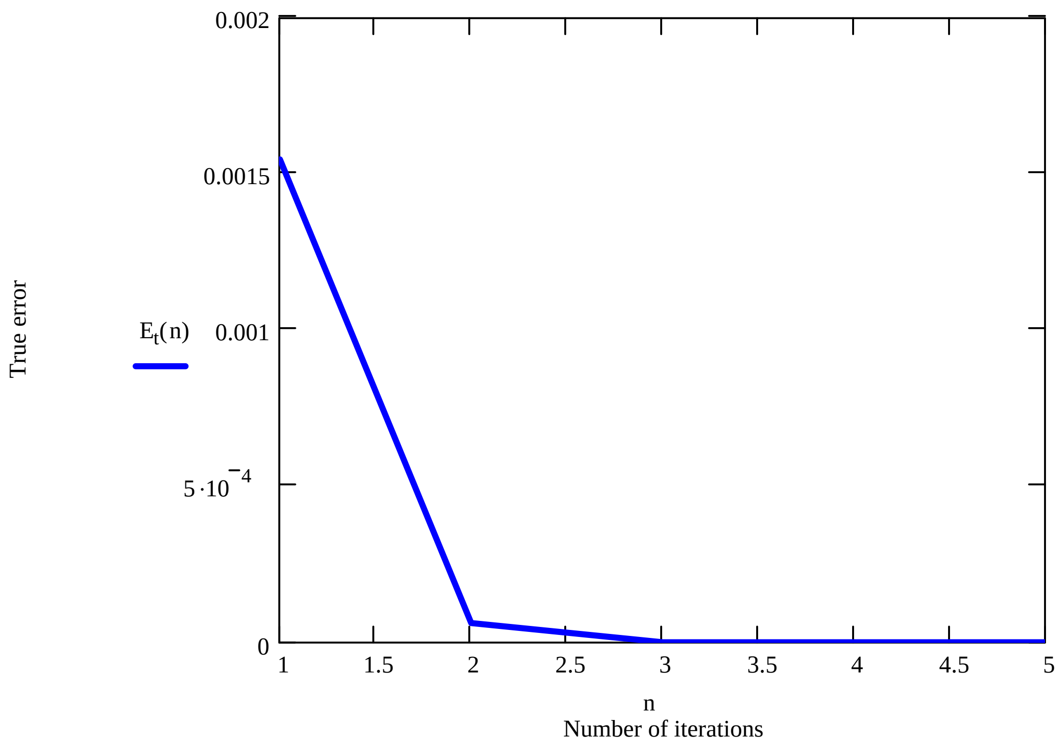
$$\text{sigdigits}(n) := \begin{cases} 0 & \text{if } \left|\varepsilon_a(n)\right| \le 0 \\ \left(2 - \log\left(\left|\frac{\left|\varepsilon_a(n)\right|}{0.5}\right|\right)\right) & \text{otherwise} \end{cases}$$

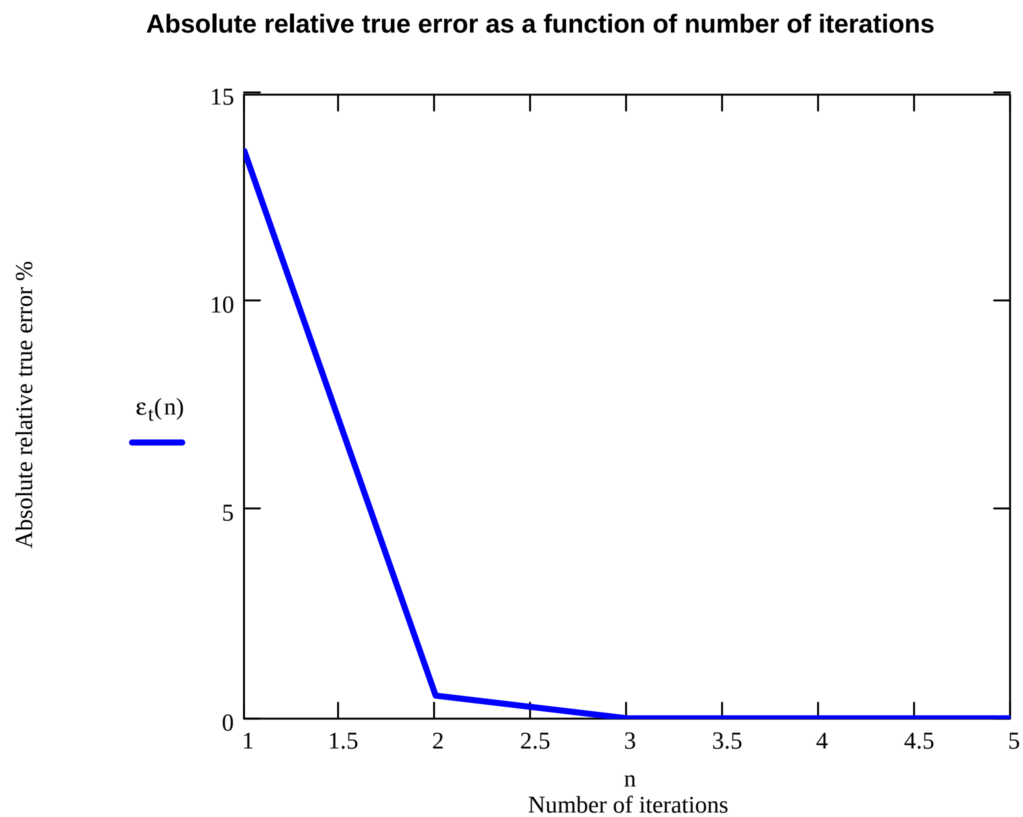
Table of Values:

$n =$	$x_r(n) =$	$E_t(n) =$	$\varepsilon_t(n) =$	$E_a(n) =$	$\varepsilon_a(n) =$	trunc(sigdigits(n)) =
1	0.01289	$1.54751\cdot10^{-3}$	13.6448	$7.88889\cdot10^{-3}$	61.2069	0
2	0.01128	$6.20045\cdot10^{-5}$	0.5467	$-1.60951\cdot10^{-3}$	14.26954	0
3	0.01134	$5.1106\cdot10^{-8}$	$4.50615\cdot10$	$6.19534\cdot10^{-5}$	0.54626	1
4	0.01134	$2.93808\cdot10^{-10}$	$2.59059\cdot10$	$5.08121\cdot10^{-8}$	$4.48024\cdot10^{-4}$	5
5	0.01134	$2.93773\cdot10^{-10}$	$2.59027\cdot10$	$3.52652\cdot10^{-14}$	$3.10943\cdot10^{-10}$	11

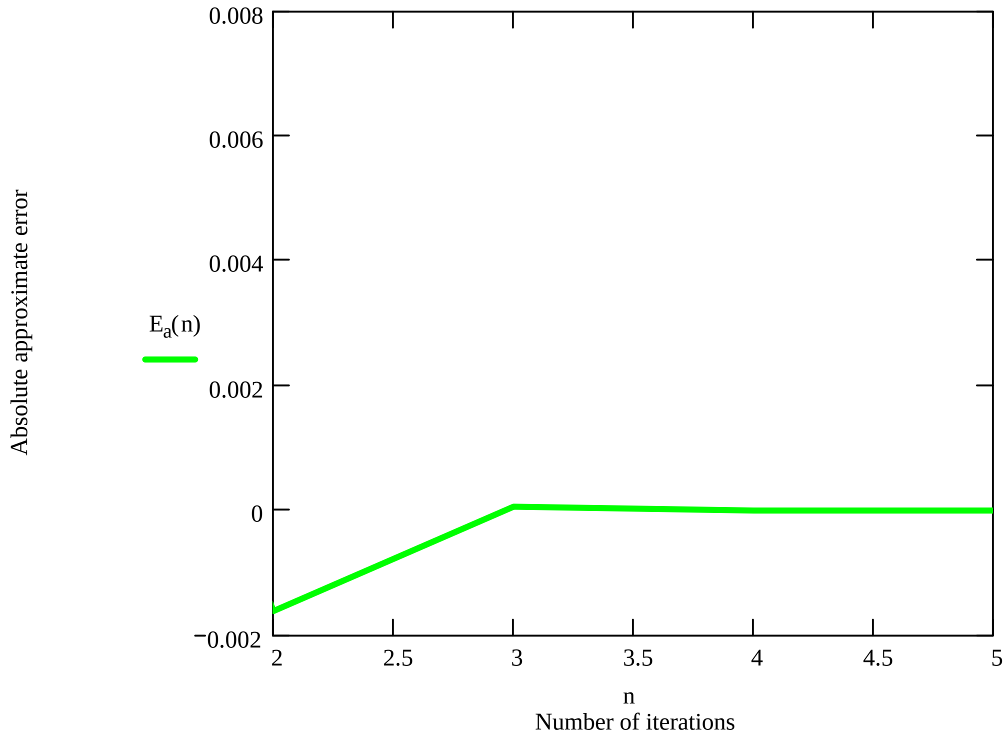

Estimated root as a function of number of iterations
Estimated root
$x_r(n)$
0.0125
0.012
0.0115
0.011
1
1.5
2
2.5
3
3.5
4
4.5
5
n
Number of iterations

Absolute true error as a function of number of iterations

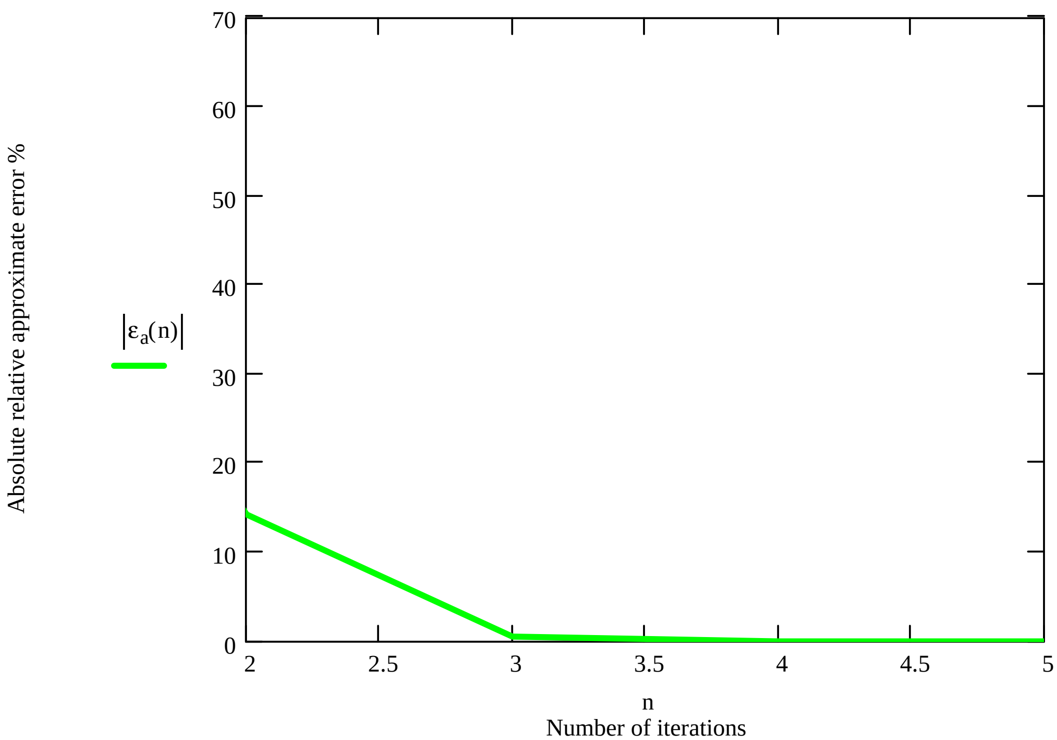



Absolute relative true error as a function of number of iterations
15
10
5
0
1
1.5
2
2.5
3
3.5
4
4.5
5
n
Number of iterations
Absolute relative true error %
$\varepsilon_t(n)$

Absolute approximate error as a function of number of iterations



Absolute relative approximate error as a function of number of iterations



Least number of significant digits at least correct as a function of number of iterations