Topic : Newton Raphson Method - Roots of Equations
Simulation : Pitfall - Slow convergence around inflection points
Language : Mathcad 2001
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Abstract : The following example illustrates slow convergence of Newton-Raphson method due to an inflection point occuring in the vicinity of the root.

INPUTS: Enter the following

Function in f(x)=0 $f(x) := (x-1)^3$

Range of x you want to see the function $x := -3, -2.99 .. 3$

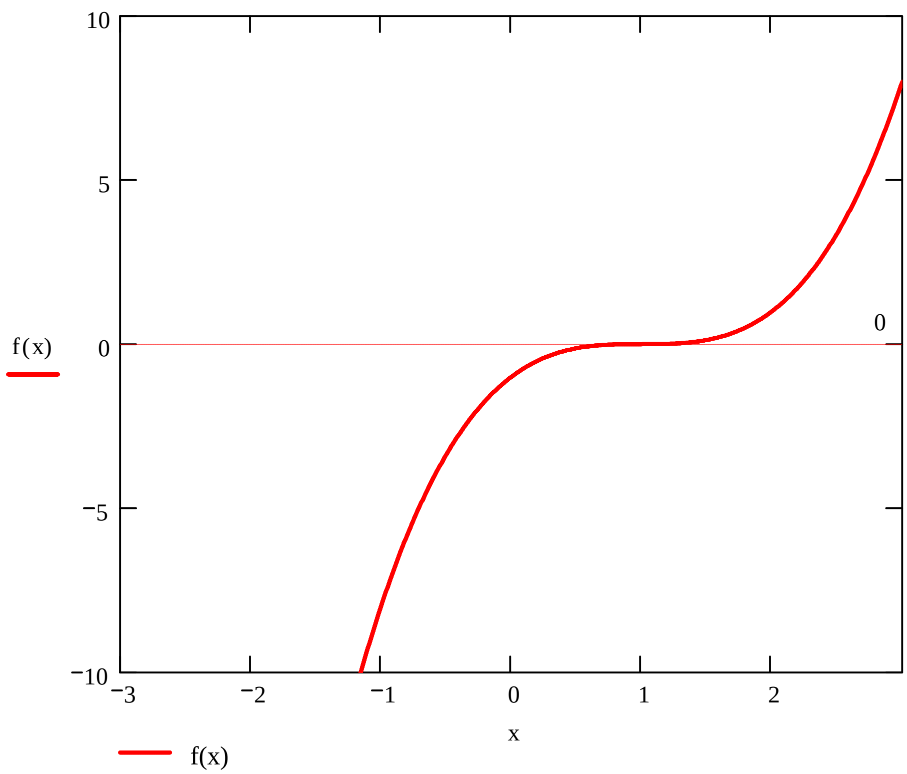
Initial guess $x_{initial} := -1$

Maximum number of iterations $n_{max} := 10$

SOLUTION:

$$g(x) := \frac{d}{dx}f(x)$$

Entered function at given interval



Iteration 1

$x_0 := x_{initial}$

$$x_1 := x_0 - \frac{f(x_0)}{g(x_0)}$$

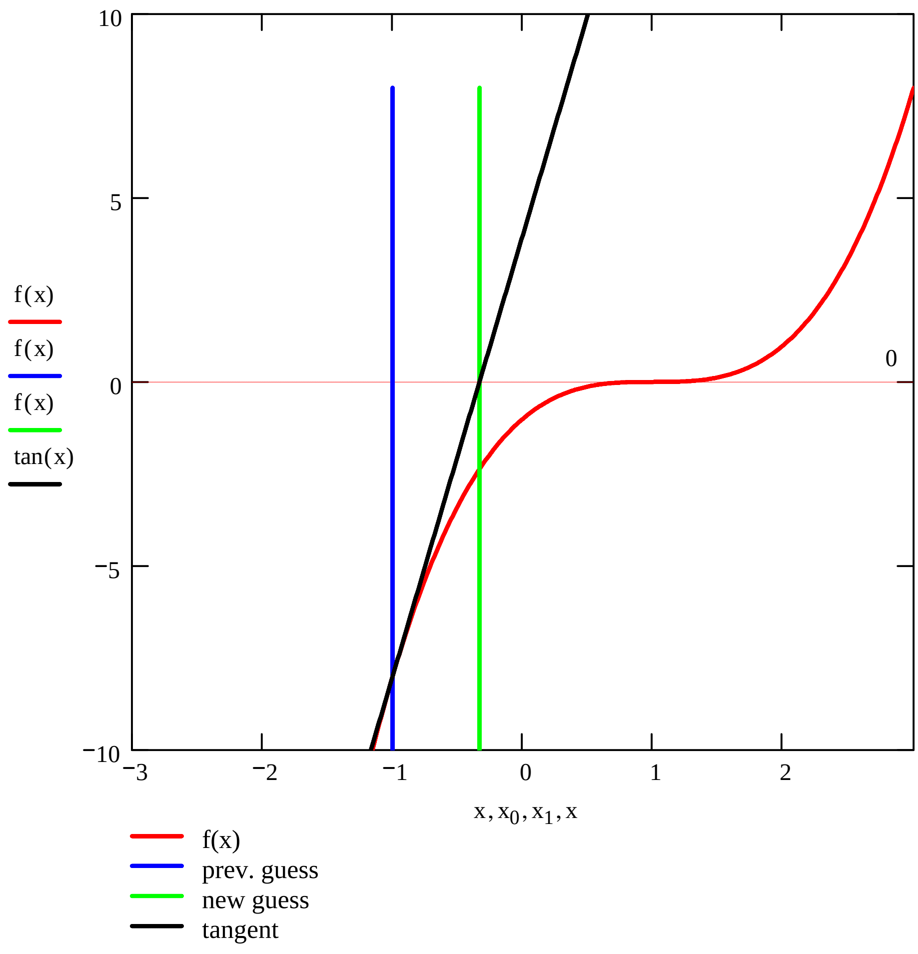
$x_1 = -0.333$

$$\tan(x) := f(x_0) + \frac{(0 - f(x_0))}{x_1 - x_0}(x - x_0)$$

$$\varepsilon_a := \left|\frac{x_1 - x_0}{x_1}\right| \cdot 100$$

$\varepsilon_a = 200$

Entered function along given interval with current and next root and the tangent line of the curve at the current root



Iteration 2

$$x_2 := x_1 - \frac{f(x_1)}{g(x_1)}$$

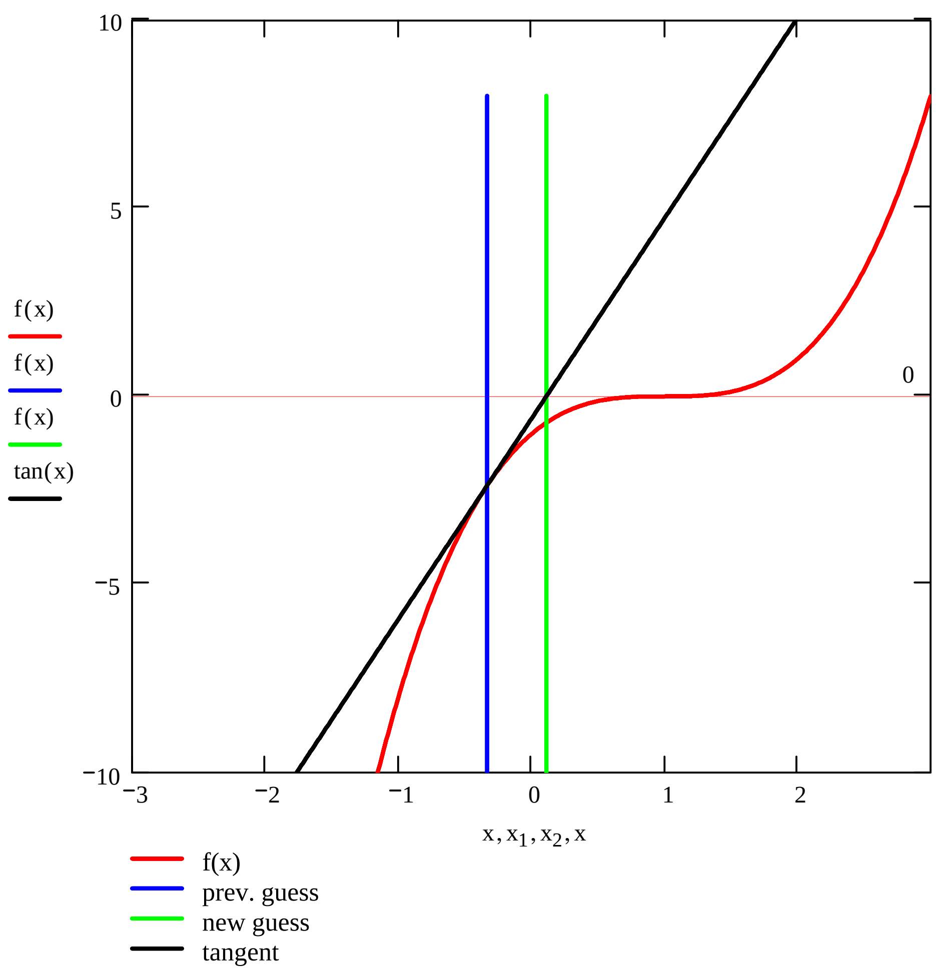
$$x_2 = 0.111$$

$$\tan(x) := f(x_1) + \frac{(0 - f(x_1))}{x_2 - x_1}(x - x_1)$$

$$\varepsilon_a := \left|\frac{x_2 - x_1}{x_2}\right| \cdot 100$$

$$\varepsilon_a = 400$$

Entered function along given interval with current and next root and the tangent line of the curve at the current root



Iteration 3

$$x_3 := x_2 - \frac{f(x_2)}{g(x_2)}$$

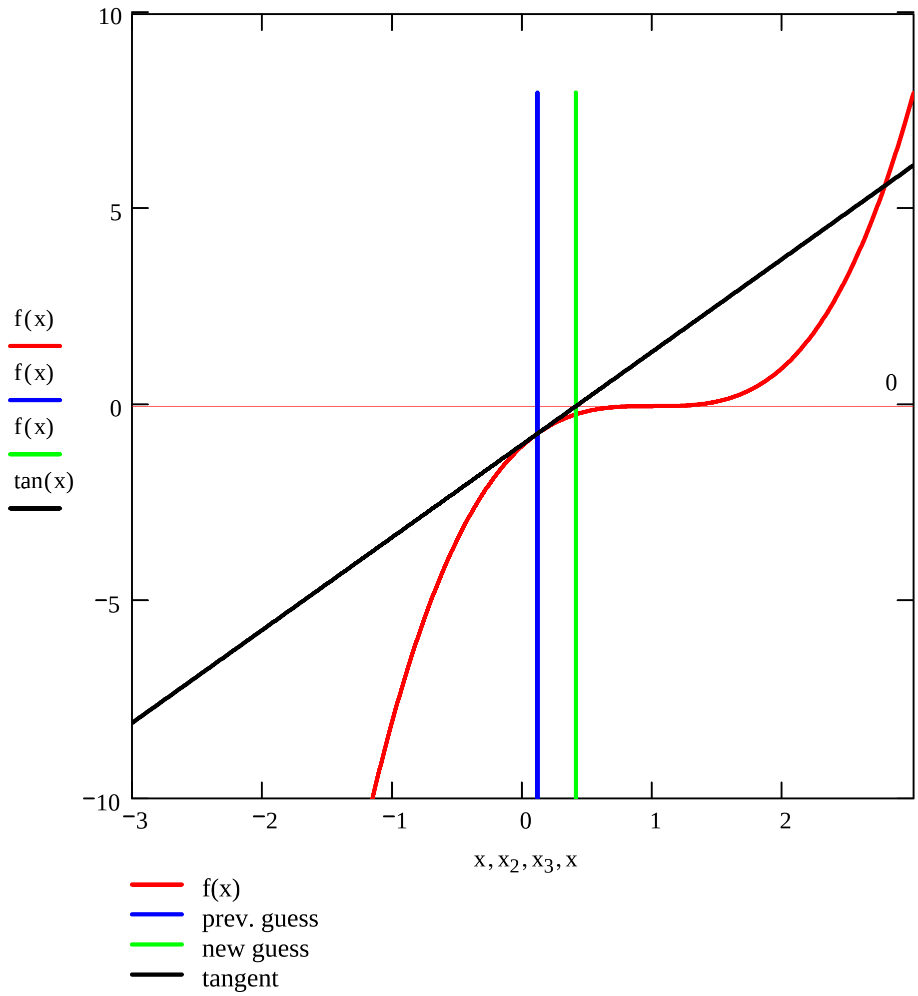
$x_3 = 0.407$

$$\tan(x) := f(x_2) + \frac{(0 - f(x_2))}{x_3 - x_2}(x - x_2)$$

$$\varepsilon_a := \left|\frac{x_3 - x_2}{x_3}\right| \cdot 100$$

$\varepsilon_a = 72.727$

Entered function along given interval with current and next root and the tangent line of the curve at the current root



Iteration 4

$$x_4 := x_3 - \frac{f(x_3)}{g(x_3)}$$

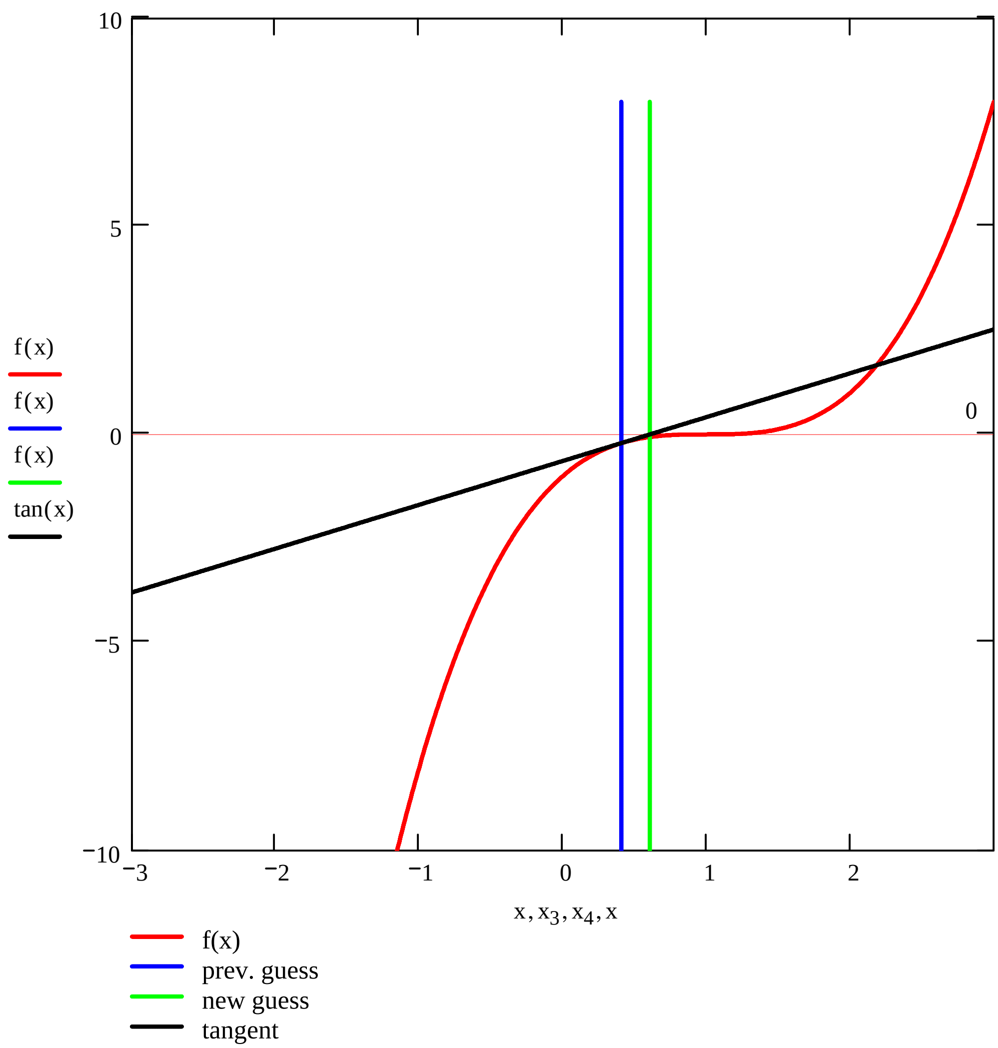
$x_4 = 0.605$

$$\tan(x) := f(x_3) + \frac{(0 - f(x_3))}{x_4 - x_3}(x - x_3)$$

$$\varepsilon_a := \left|\frac{x_4 - x_3}{x_4}\right| \cdot 100$$

$\varepsilon_a = 32.653$

Entered function along given interval with current and next root and the tangent line of the curve at the current root



Root Calculation:

$$x_r(n) := \left| \begin{array}{l} i \leftarrow 1 \\ x \leftarrow x_{initial} \\ \text{while } i \le n \\ \quad \left| \begin{array}{l} x_{next} \leftarrow x - \dfrac{f(x)}{g(x)} \\ x \leftarrow x_{next} \\ i \leftarrow i + 1 \end{array} \right. \\ x_{next} \end{array} \right.$$

$$n := 1, 2 .. n_{max}$$

Absolute approximate error:

$$E_a(n) := \left| \begin{array}{ll} (x_r(1) - x_{initial}) & \text{if } n = 1 \\ (x_r(n) - x_r(n-1)) & \text{otherwise} \end{array} \right.$$

Absolute relative approximate error:

$$\varepsilon_a(n) := \left| \frac{E_a(n)}{x_r(n)} \right| \cdot 100$$

Absolute relative approximate error versus number of iterations