Topic : Secant Method - Roots of Equations
Simulation : Convergence of Method
Language : Mathcad 2001
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Abstract : The simulation the convergence of secant method of finding the root of an equation f(x)=0.

INPUTS: Enter the following

Function in f(x)=0 $f(x) := x^3 - 0.03 \cdot x^2 + 2.4 \cdot 10^{-6}$

Range of x you want to see the function $x := 0, .001 .. .03$

Upper guess $x_{guess1} := 0.02$

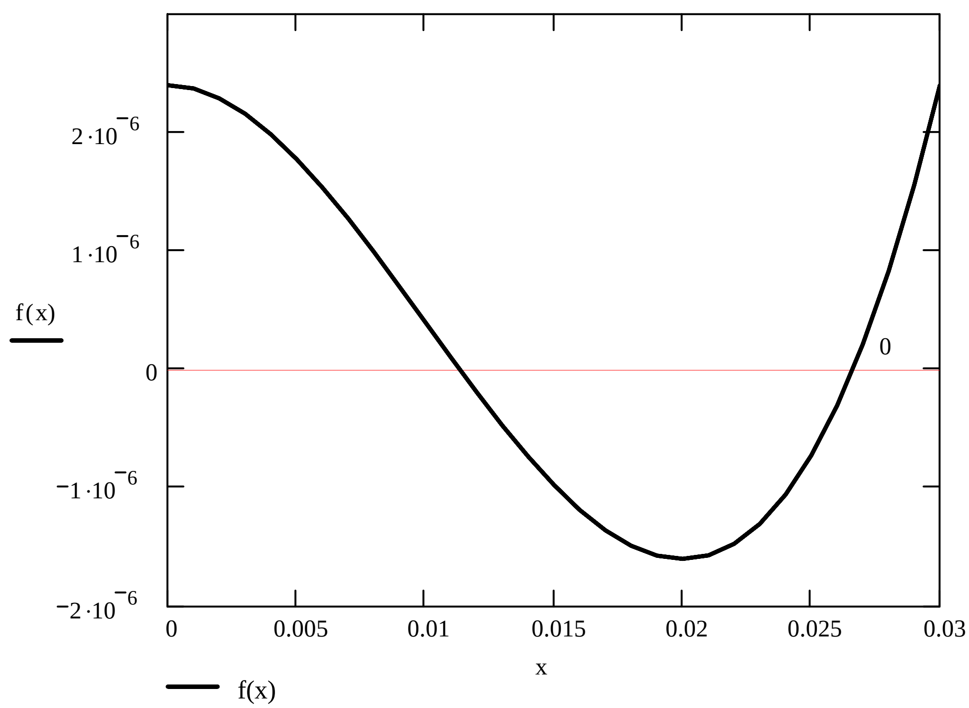
Lower guess $x_{guess2} := 0.01$

Maximum number of iterations $n_{max} := 7$

Initial guess for Mathcad solution $x_{guess} := 0.01$

SOLUTION

Entered function at given interval



Exact Solution:

This is the true solution found by Mathcad.

$$x := x_{guess}$$

$$x_{true} := \mathrm{root}(f(x), x)$$

$$x_{true} = 0.01133$$

Here the secant method algorithm is applied to generate the estimated root, true error, absolute relative true error, absolute approximate error, absolute relative approximate error, and the number of significant digits at least correct in the estimated root as a funcion of number of iterations.

$$x_r(n) := \left| \begin{array}{l} i \leftarrow 1 \\ x1 \leftarrow x_{guess1} \\ x2 \leftarrow x_{guess2} \\ \text{while } i \le n \\ \quad \left| \begin{array}{l} x_{root} \leftarrow x2 - \dfrac{[f(x2)(x1 - x2)]}{f(x1) - f(x2)} \\ x1 \leftarrow x2 \\ x2 \leftarrow x_{root} \\ i \leftarrow i + 1 \end{array} \right. \\ x_{root} \end{array} \right.$$

$$n := 1 .. n_{max}$$

True error:

$$E_t(n) := x_{true} - x_r(n)$$

Absolute relative true error:

$$\epsilon_t(n) := \left| \frac{E_t(n)}{x_{true}} \right| \cdot 100$$

Absolute approximate error:

$$E_a(n) := x_r(n) - x_r(n-1)$$

Absolute relative approximate error:

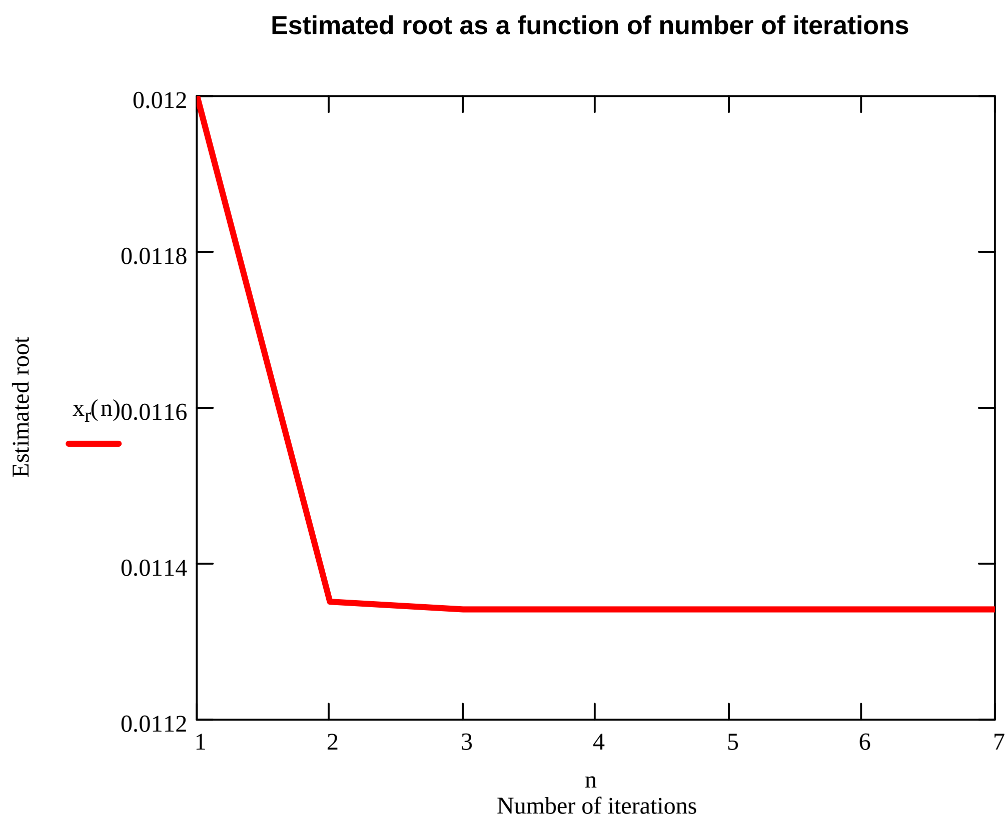
$$\varepsilon_a(n) := \begin{cases} 0 & \text{if } n \le 1 \\ \left(\left| \dfrac{E_a(n)}{x_r(n)} \right| \cdot 100 \right) & \text{otherwise} \end{cases}$$

Significant digits at least correct:

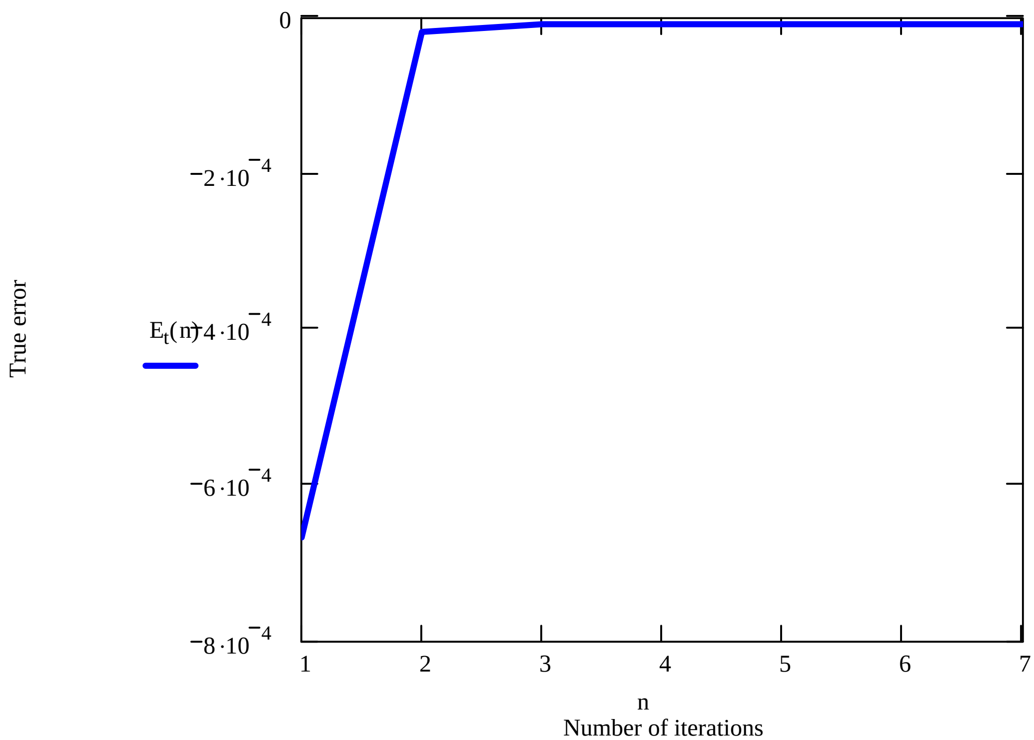
$$\text{sigdigits}(n) := \begin{cases} 0 & \text{if } \left|\varepsilon_a(n)\right| \le 0 \\ \left(2 - \log\left(\left| \dfrac{\left|\varepsilon_a(n)\right|}{0.5} \right| \right) \right) & \text{otherwise} \end{cases}$$

Table of Values:

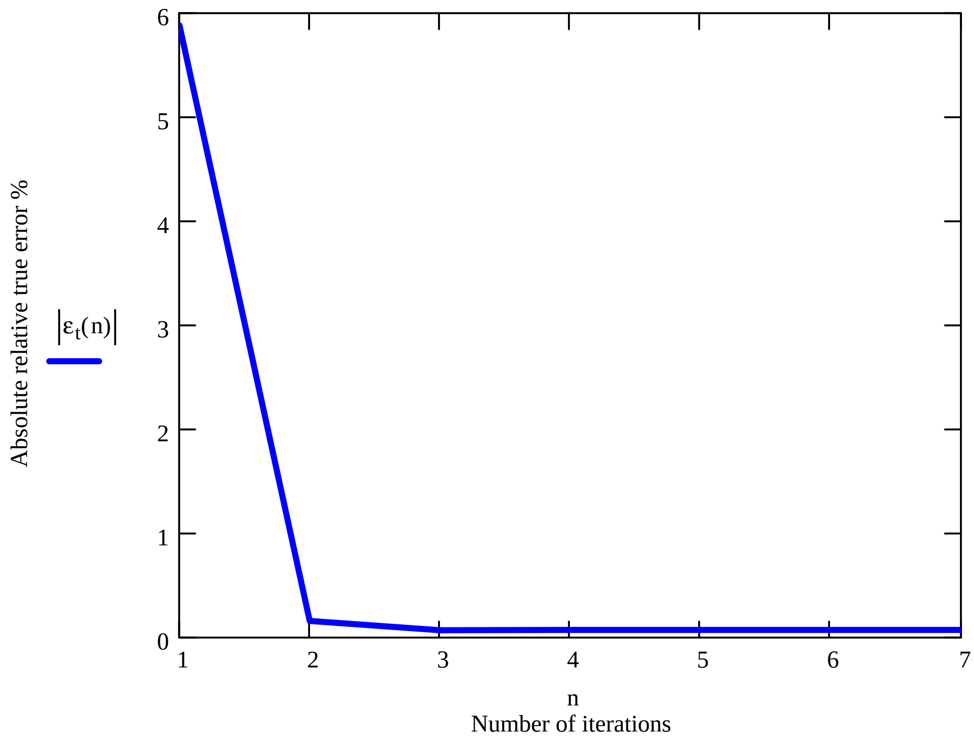
$n =$	$x_r(n) =$	$E_t(n) =$	$\varepsilon_t(n) =$	$E_a(n) =$	$\left\|\varepsilon_a(n)\right\| =$	trunc(sigdigits(n)) =
1	0.012	$-6.66666 \cdot 10^{-4}$	5.88235	0.012	0	0
2	0.01135	$-1.80176 \cdot 10^{-5}$	0.15898	$-6.48649 \cdot 10^{-4}$	5.71429	0
3	0.01134	$-7.93893 \cdot 10^{-6}$	0.07005	$-1.00786 \cdot 10^{-5}$	0.08887	2
4	0.01134	$-8.04469 \cdot 10^{-6}$	0.07098	$1.05766 \cdot 10^{-7}$	$9.32568 \cdot 10^{-4}$	4
5	0.01134	$-8.04468 \cdot 10^{-6}$	0.07098	$-1.44433 \cdot 10^{-11}$	$1.27351 \cdot 10^{-7}$	8
6	0.01134	$-8.04468 \cdot 10^{-6}$	0.07098	0	$1.68251 \cdot 10^{-13}$	14
7	0.01134	$-8.04468 \cdot 10^{-6}$	0.07098	0	$3.05911 \cdot 10^{-14}$	15

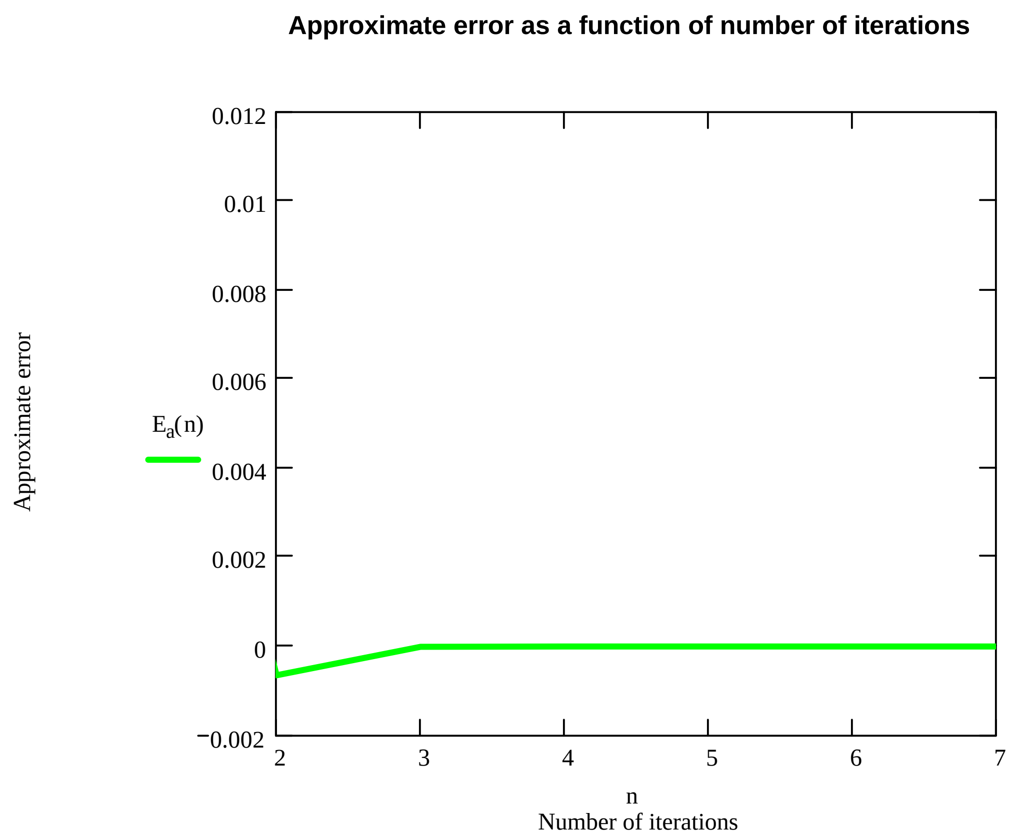

Estimated root as a function of number of iterations
0.012
0.0118
0.0116
0.0114
0.0112
Estimated root
x_r(n)
1
2
3
4
5
6
7
n
Number of iterations

True error as a function of number of iterations

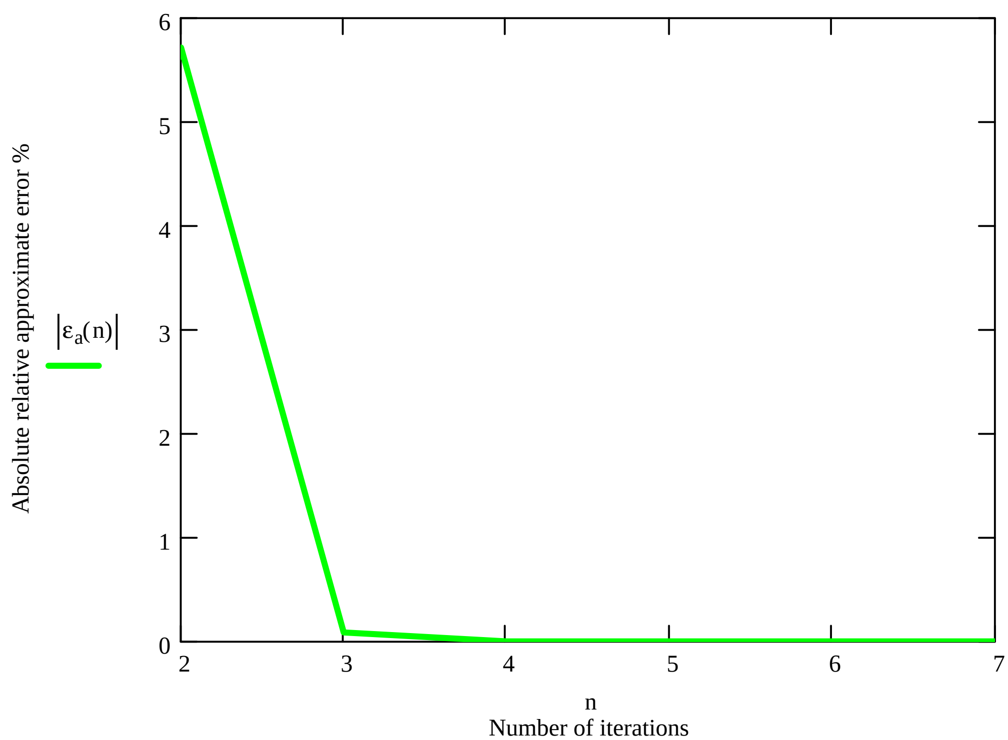


Absolute relative true error as a function of number of iterations




Approximate error as a function of number of iterations
Approximate error
$E_a(n)$
0.012
0.01
0.008
0.006
0.004
0.002
0
−0.002
2
3
4
5
6
7
n
Number of iterations

Absolute relative approximate error as a function of number of iterations



Least number of significant digits at least correct as a function of number of iterations