Topic : Additional Interpolation Topics
Simulation : The Effect of Choice of Points on Interpolation
Language : Mathcad 2001
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Abstract : In 1901, Carl Runge published his work on dangers of higher order interpolation. He took a simple looking function $f(x)=1/(1+25x^2)$ on the interval [-1,1]. He took points equidistantly spaced in [-1,1] and interpolated the points with polynomials. He found that as he took more points, the polynomials and the original curve differed considerably. However, if he took data points close to the ends of the interval [-1,1], the problem of large differences between interpolated and actual values was less pronounced. This simulation shows you this phenomena.

INPUTS: Enter the following

Enter the number of points chosen for interpolation in [-1,1].

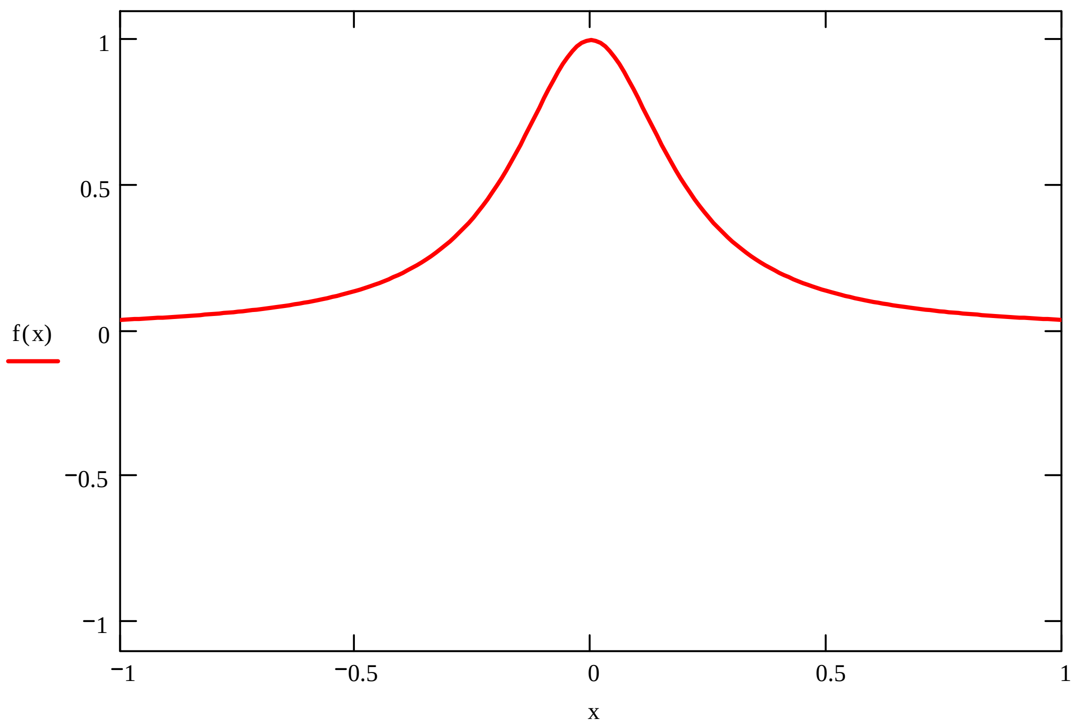
$n := 13$

For the number scheme to fun to number of points should be an odd number.

SOLUTION

$$f(x) := \frac{1}{1 + 25 \cdot x^2} \qquad x := -1, -.99 .. 1$$

Runge's Function



Interpolated polynomial with points equally spaced

When n is given, this function returns a matrix containing sequential x values

$$x_{points}(n) := \left| \begin{array}{l} \text{for } i \in 0..n-1 \\ \quad c_i \leftarrow \dfrac{2}{n-1} \cdot i - 1 \\ c \end{array} \right.$$

$$Y(n) := f\left(x_{points}(n)\right)$$

$$X(n) := x_{points}(n)$$

When x and y data and order is given, this function constructs the matrix whose inverse is needed to find coefficients of the polynomial which approximates the data.

$$M(n, X, Y) := \left| \begin{array}{l} \text{for } i \in 0..n-1 \\ \quad \text{for } j \in 0..n-1 \\ \quad\quad a_{i,j} \leftarrow \left(X_i\right)^j \\ a \end{array} \right.$$

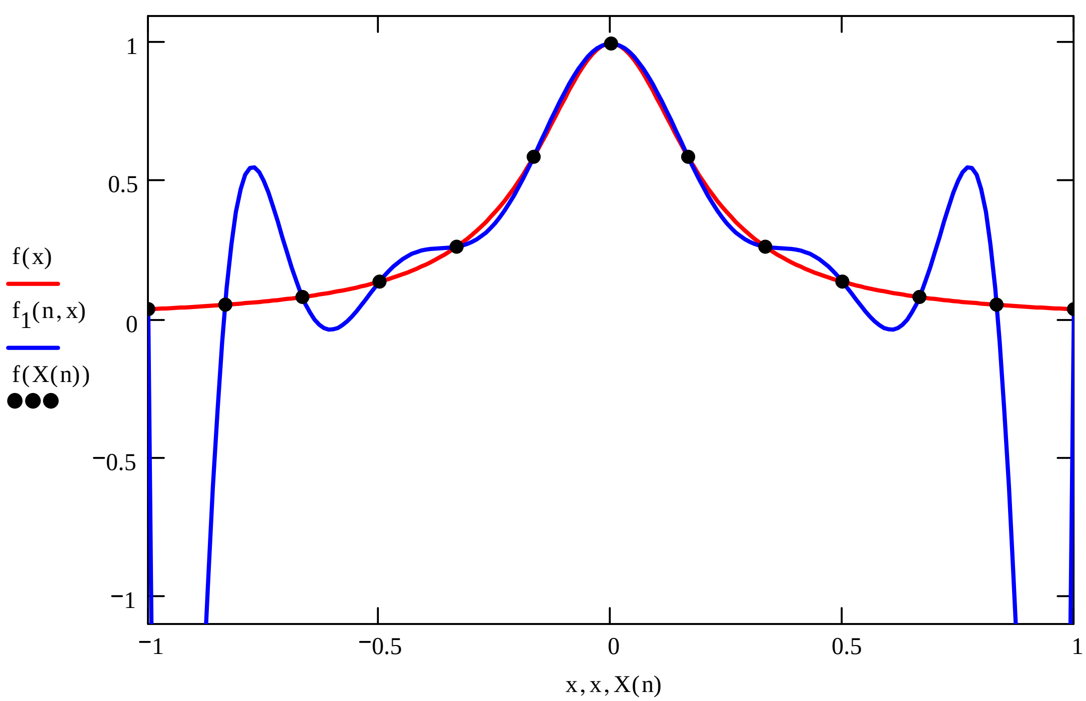
This function generates the coefficients for the polynomial that approximates the x and y data.

$$a(n) := M(n, X(n), Y(n))^{-1} \cdot Y(n)$$

When given the polynomial order and specific x, this function uses the above calculated coefficients to calculate the approximated f(x)

$$f_1(n, x) := \left|\begin{array}{l} c \leftarrow 0 \\ \text{for } i \in 0..n-1 \\ \quad \left|\begin{array}{l} d \leftarrow a(n)_i \cdot x^i + c \\ c \leftarrow d \end{array}\right. \\ d \end{array}\right.$$

Runge's function and interpolant with equidistantly spaced points



Fitted Polynomial with More Points at Ends

The idea of this function is to place more points at the ends of the interval than in the middle. If the distance between the first and second points is x, then the distance between the second and third is 2x. Each subsequent point is twice as far away from the next point as the previous. After passing the middle, the function reverses. The output is a matrix of x values which are biased to the end points.

$$
x_{biased}(n) := \left|
\begin{array}{l}
d \leftarrow 0 \\
\text{for } i \in 1 .. \dfrac{(n-1)}{2} \\
\quad d \leftarrow d + 2^{i-1} \\
l \leftarrow \dfrac{1}{d} \\
c_0 \leftarrow -1 \\
\text{for } i \in 1 .. \dfrac{(n-1)}{2} \\
\quad c_i \leftarrow c_{i-1} + 2^{i-1} \cdot l \\
\text{for } i \in \dfrac{(n-1)}{2} + 1 .. n-1 \\
\quad c_i \leftarrow c_{i-1} + 2^{n-i-1} \cdot l \\
c
\end{array}
\right.
$$

This assigns the biased values as X and the Y values as f(X)

$$X(n) := x_{biased}(n)$$

$$Y(n) := f\left(x_{biased}(n)\right)$$

When x and y data and order is given, this function constructs the matrix whose inverse is needed to find coefficients of the polynomial which approximates the data.

$$
M(n, X, Y) := \left|
\begin{array}{l}
\text{for } i \in 0 .. n-1 \\
\quad \text{for } j \in 0 .. n-1 \\
\quad\quad a_{i,j} \leftarrow \left(X_i\right)^j \\
a
\end{array}
\right.
$$

This function generates the coefficients for the polynomial that approximates the x and y data.

$$a(n) := M(n, X(n), Y(n))^{-1} \cdot Y(n)$$

When given the polynomial order and specific x, this function uses the above calculated coefficients to calculate the approximated f(x)

$$f_1(n,x) := \left| \begin{array}{l} c \leftarrow 0 \\ \text{for } i \in 0..n-1 \\ \quad \left| \begin{array}{l} d \leftarrow a(n)_i \cdot x^i + c \\ c \leftarrow d \end{array} \right. \\ d \end{array} \right.$$

Runge's function and interpolated polynomial with more points near the ends of the interval [-1,1]