Topic : Additional Interpolation Topics
Simulation : Danger of Extrapolation
Language : Mathcad 2001
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Abstract : During the technology boom of the late 1990's, the NASDAQ Composite Index that tracks mainly technology stocks had a phenomenal increase in its value. For example, at the end of year 1998, the NASDAQ index was at an all time high of 1570.35. It increase to 2192.69 by the end of 1999 and further to 4069.31 by the end of 2000. Many people who had never ventured into stocks started investing in the technology stocks and mutual funds. Dreams of doubling their money every year were not considered unrealistic. So given below is the data of the NASDAQ index from 1994 to 1999. If you were **extrapolating** the data, what would you estimate the NASDAQ index to be at the end of 2000 and 2001? How different did it turn out to be?

INPUTS: Enter the following

Array of x data:

$$End\_of\_year := \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{pmatrix}$$

Range of years actually between 1994 (Year 1) and 1999 (Year 6). Numbers start from 1 to avoid round-off errors and near singularity in matrix calculations.

Array of y data

$$NASDAQ := \begin{pmatrix} 751.96 \\ 1052.13 \\ 1291.03 \\ 1570.35 \\ 2192.69 \\ 4069.31 \end{pmatrix}$$

End of Year NASDAQ Composite Data taken from www.bigcharts.com

Extrapolated values

$e_1 := 7$ $\qquad e_2 := 8$

In the above scheme of representing years, year 7 would correspond to 2000 and year 8 would correspond to 2001

SOLUTION

Extrapolating using polynomial Interpolation:

Here we are passing a 5th order polynomail through the given 6 data points.

$n := \text{rows}(\text{End\_of\_year})$

$z := \text{regress}(\text{End\_of\_year}, \text{NASDAQ}, n - 1)$

The "regress" function of MATHCAD is used as a way to conduct interpolation because a (n-1)th order polynomial is represented through n data points, which is polynomial interpolation.

$f_1(a) := \text{interp}(z, \text{End\_of\_year}, \text{NASDAQ}, a)$

$\text{End\_of\_years} := \text{End\_of\_year}$

$\text{NASDAQs} := \text{NASDAQ}$

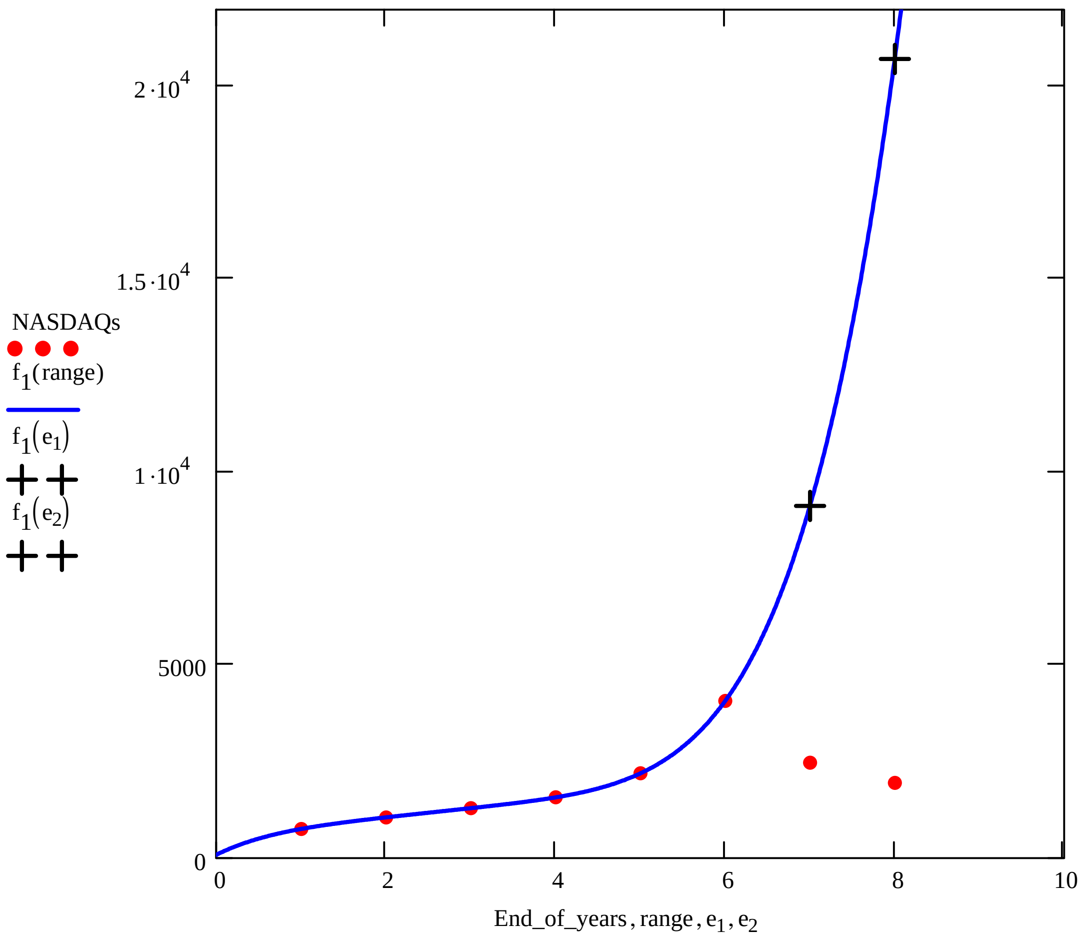
$\text{End\_of\_years}_6 := e_1$

$\text{End\_of\_years}_7 := e_2$

$\text{NASDAQs}_6 := 2470.52$

$\text{NASDAQs}_7 := 1950.4$

Data from 1994 to 1999 extrapolated to yield results for 2000 and 2001



$$
\text{results} := \left|
\begin{array}{l}
\text{for } i \in 0..1 \\
\quad \left|
\begin{array}{l}
d_{i,0} \leftarrow i + 7 \\
d_{i,1} \leftarrow \text{NASDAQs}_{i+6} \\
d_{i,2} \leftarrow f_1\left(e_{i+1}\right) \\
d_{i,3} \leftarrow 100 \left| \dfrac{d_{i,1} - d_{i,2}}{d_{i,1}} \right|
\end{array}
\right. \\
d
\end{array}
\right.
$$

results =

End of Year	Actual	Polynomial Iterpolation	Percentage Relative True Error
7	$2.471 \cdot 10^3$	$9.128 \cdot 10^3$	269.472
8	$1.95 \cdot 10^3$	$2.072 \cdot 10^4$	962.358

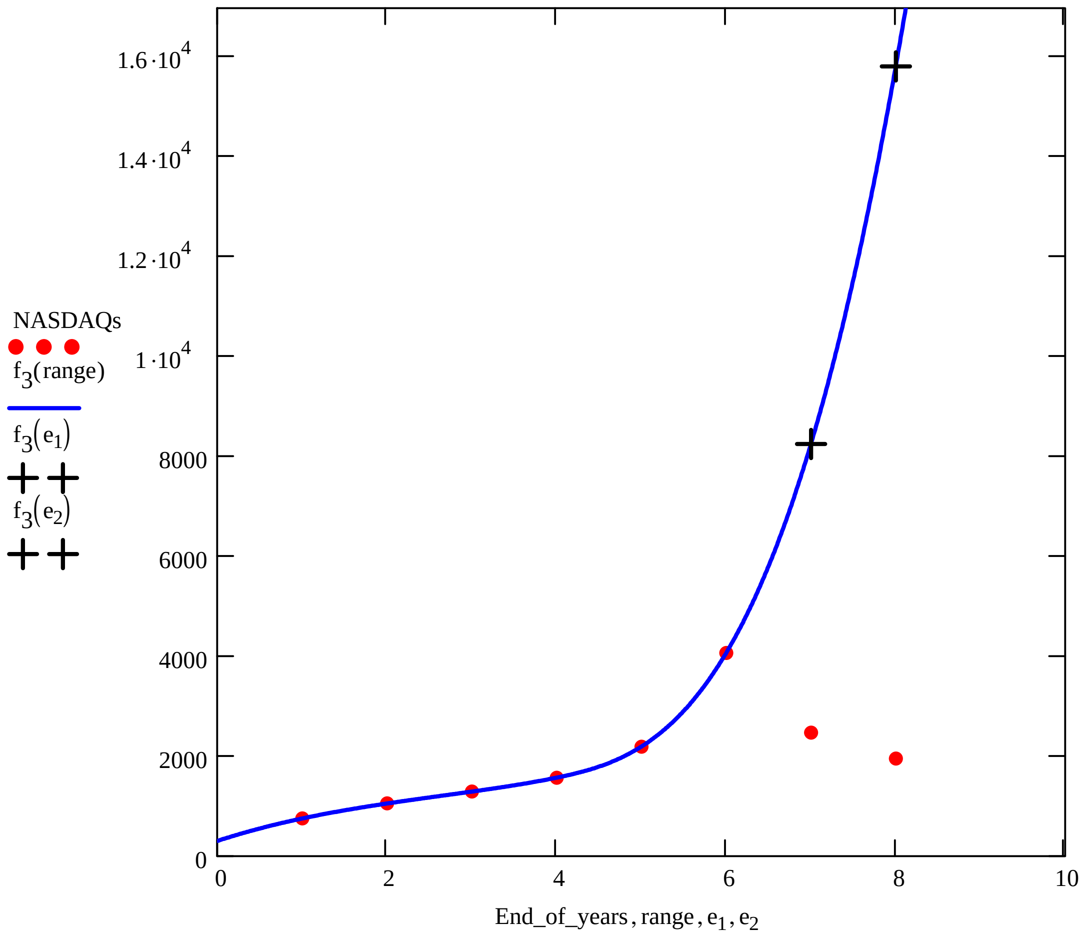
Cubic Spline:

Here we are using cubic spline interpolation to extrapolate the values. Will we get any different results?

$$S := \mathrm{cspline}(End\_of\_year, NASDAQ)$$

$$f_3(x) := \mathrm{interp}(S, End\_of\_year, NASDAQ, x)$$

Data from 1994 to 1999 extrapolated to yield results for 2000 and 2001



$$
\text{results} := \left|\begin{array}{l}
\text{for } i \in 0..1 \\
\quad \left|\begin{array}{l}
d_{i,0} \leftarrow i + 7 \\
d_{i,1} \leftarrow \text{NASDAQs}_{i+6} \\
d_{i,2} \leftarrow f_3\left(e_{i+1}\right) \\
d_{i,3} \leftarrow 100 \left| \dfrac{d_{i,1} - d_{i,2}}{d_{i,1}} \right|
\end{array}\right. \\
d
\end{array}\right.
$$

	End of Year	Actual	Polynomial Iterpolation	Percentage Relative True Error
results =	7	$2.471 \cdot 10^3$	$8.26 \cdot 10^3$	234.358
	8	$1.95 \cdot 10^3$	$1.583 \cdot 10^4$	711.428

$$
\text{results} := \left|\begin{array}{l}
\text{for } i \in 0..1 \\
\quad \left|\begin{array}{l}
d_{i,0} \leftarrow i + 7 \\
d_{i,1} \leftarrow NASDAQs_{i+6} \\
d_{i,2} \leftarrow f_1\left(e_{i+1}\right) \\
d_{i,3} \leftarrow 100 \left|\dfrac{d_{i,1} - d_{i,2}}{d_{i,1}}\right| \\
d_{i,4} \leftarrow f_3\left(e_{i+1}\right) \\
d_{i,5} \leftarrow 100 \left|\dfrac{d_{i,1} - d_{i,4}}{d_{i,1}}\right|
\end{array}\right. \\
d
\end{array}\right.
$$

Summary of Extrapolated Data

results =

End of Year	Actual	Polynomial Interpolation	Percentage Relative True Error	Cubic Spline Interpoaltion	Percentage Relative True Error
7	2471	9128	269	8260	234
8	1950	20720	962	15826	711