Abstract: During the technology boom of the late 1990’s, the NASDAQ Composite Index that tracks mainly technology stocks had a phenomenal increase in its value. For example, at the end of year 1998, the NASDAQ index was at an all time high of 1570.35. It increase to 2192.69 by the end of 1999 and further to 4069.31 by the end of 2000. Many people who had never ventured into stocks started investing in the technology stocks and mutual funds. Dreams of doubling their money every year were not considered unrealistic. So given below is the data of the NASDAQ index from 1994 to 1999. If you were extrapolating the data, what would you estimate the NASDAQ index to be at the end of 2000 and 2001? How different did it turn out to be?

**INPUTS: Enter the following**

**Array of x data:**

```
End_of_year :=
```

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>751.96</td>
</tr>
<tr>
<td>2</td>
<td>1052.13</td>
</tr>
<tr>
<td>3</td>
<td>1291.03</td>
</tr>
<tr>
<td>4</td>
<td>1570.35</td>
</tr>
<tr>
<td>5</td>
<td>2192.69</td>
</tr>
<tr>
<td>6</td>
<td>4069.31</td>
</tr>
</tbody>
</table>

Range of years actually between 1994 (Year 1) and 1999 (Year 6). Numbers start from 1 to avoid round-off errors and near singularity in matrix calculations.

**Array of y data**

```
NASDAQ :=
```

<table>
<thead>
<tr>
<th>Year</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>751.96</td>
</tr>
<tr>
<td>1995</td>
<td>1052.13</td>
</tr>
<tr>
<td>1996</td>
<td>1291.03</td>
</tr>
<tr>
<td>1997</td>
<td>1570.35</td>
</tr>
<tr>
<td>1998</td>
<td>2192.69</td>
</tr>
<tr>
<td>1999</td>
<td>4069.31</td>
</tr>
</tbody>
</table>


**Extrapolated values**

```
e_1 := 7
e_2 := 8
```

In the above scheme of representing years, year 7 would correspond to 2000 and year 8 would correspond to 2001.
SOLUTION

Extrapolating using polynomial Interpolation:

Here we are passing a 5th order polynomial through the given 6 data points.

\[ n := \text{rows(End\_of\_year)} \]
\[ z := \text{regress(End\_of\_year,NASDAQ,n - 1)} \]

The "regress" function of MATHCAD is used as a way to conduct interpolation because a \((n-1)\)th order polynomial is represented through \(n\) data points, which is polynomial interpolation.

\[ f_1(a) := \text{interp}(z,\text{End\_of\_year},\text{NASDAQ},a) \]
\[ \text{End\_of\_years} := \text{End\_of\_year} \]
\[ \text{NASDAQs} := \text{NASDAQ} \]
\[ \text{End\_of\_years}_6 := e_1 \]
\[ \text{End\_of\_years}_7 := e_2 \]
\[ \text{NASDAQs}_6 := 2470.52 \]
\[ \text{NASDAQs}_7 := 1950.4 \]
Data from 1994 to 1999 extrapolated to yield results for 2000 and 2001

```
results := for i ∈ 0..1
           d_{i,0} ← i + 7
           d_{i,1} ← NASDAQs_{i+6}
           d_{i,2} ← f_1(e_{i+1})
           d_{i,3} ← 100 \cdot \frac{|d_{i,1} - d_{i,2}|}{d_{i,1}}
       d

<table>
<thead>
<tr>
<th>End of Year</th>
<th>Actual</th>
<th>Polynomial Interpolation</th>
<th>Percentage Relative True Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2.471 \times 10^3</td>
<td>9.128 \times 10^3</td>
<td>269.472</td>
</tr>
<tr>
<td>8</td>
<td>1.95 \times 10^3</td>
<td>2.072 \times 10^4</td>
<td>962.358</td>
</tr>
</tbody>
</table>
```
Cubic Spline:

Here we are using cubic spline interpolation to extrapolate the values. Will we get any different results?

S := cspline(End_of_year, NASDAQ)

f₃(x) := interp(S, End_of_year, NASDAQ, x)
Data from 1994 to 1999 extrapolated to yield results for 2000 and 2001

\[ \text{NASDAQs} \]

\[ f_3(\text{range}) \]

\[ f_3(e_1) \]

\[ f_3(e_2) \]

\[ f_3(e_i) \]

\[ (\text{End of years range}, e_1, e_2) \]

results :=

\[
\begin{align*}
\text{for } i & \in 0..1 \\
d_{i,0} & \leftarrow i + 7 \\
d_{i,1} & \leftarrow \text{NASDAQs}_{i+6} \\
d_{i,2} & \leftarrow f_3(e_{i+1}) \\
d_{i,3} & \leftarrow 100 \left| \frac{d_{i,1} - d_{i,2}}{d_{i,1}} \right|
\end{align*}
\]

<table>
<thead>
<tr>
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<th>Actual</th>
<th>Polynomial Interpolation</th>
<th>Percentage Relative True Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2.471\cdot10^3</td>
<td>8.26\cdot10^3</td>
<td>234.358</td>
</tr>
<tr>
<td>8</td>
<td>1.95\cdot10^3</td>
<td>1.583\cdot10^4</td>
<td>711.428</td>
</tr>
</tbody>
</table>
results := for i ∈ 0..1

\[ d_{i,0} \leftarrow i + 7 \]
\[ d_{i,1} \leftarrow \text{NASDAQs}_i+6 \]
\[ d_{i,2} \leftarrow f_1(e_{i+1}) \]
\[ d_{i,3} \leftarrow 100 \frac{d_{i,1} - d_{i,2}}{d_{i,1}} \]
\[ d_{i,4} \leftarrow f_3(e_{i+1}) \]
\[ d_{i,5} \leftarrow 100 \frac{d_{i,1} - d_{i,4}}{d_{i,1}} \]

**Summary of Extrapolated Data**

<table>
<thead>
<tr>
<th>End of Year</th>
<th>Actual</th>
<th>Polynomial Interpolation</th>
<th>Percentage Relative True Error</th>
<th>Cubic Spline Interpolation</th>
<th>Percentage Relative True Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>results</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>2471</td>
<td>9128</td>
<td>269</td>
<td>8260</td>
<td>234</td>
</tr>
<tr>
<td>8</td>
<td>1950</td>
<td>20720</td>
<td>962</td>
<td>15826</td>
<td>711</td>
</tr>
</tbody>
</table>