Topic : Additional Interpolation Topics
Simulation : Higher Order Interpolation is a Bad Idea
Language : Mathcad 2001
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Abstract : In 1901, Carl Runge published his work on dangers of higher order polynomial interpolation. He took a simple looking function $f(x)=1/(1+25x^2)$ on the interval of [-1,1]. He took points equidistantly spaced in [-1,1] and interpolated the points with polynomials. He found that as he took more points, the polynomials and the original curve differed considerably in their value.

INPUTS: Enter the following

You can make 3 seperate choices for the number of equidistant points in [-1,1]. So if you choose n_1:= 5, you are using a 4^{th} order polynomial to approximate $1/(1+25x^2)$ in [-1,1].

$n_1 := 5$

$n_2 := 9$

$n_3 := 17$

SOLUTION

$$f(x) := \frac{1}{1 + 25 \cdot x^2} \qquad x := -1, -.99 .. 1$$

When n is given, this function returns a matrix containing sequential x values

$$x_{points}(n) := \left| \begin{array}{l} \text{for } i \in 0..n-1 \\ \quad c_i \leftarrow \frac{2}{n-1} \cdot i - 1 \\ c \end{array} \right.$$

$$Y(n) := f\left(x_{points}(n)\right)$$

$$X(n) := x_{points}(n)$$

When x and y data and order is given, this function constructs the matrix whose inverse is needed to find coefficients of the polynomial which approximates the data.

$$M(n, X, Y) := \left| \begin{array}{l} \text{for } i \in 0..n-1 \\ \quad \text{for } j \in 0..n-1 \\ \quad\quad a_{i,j} \leftarrow \left(X_i\right)^j \\ a \end{array} \right.$$

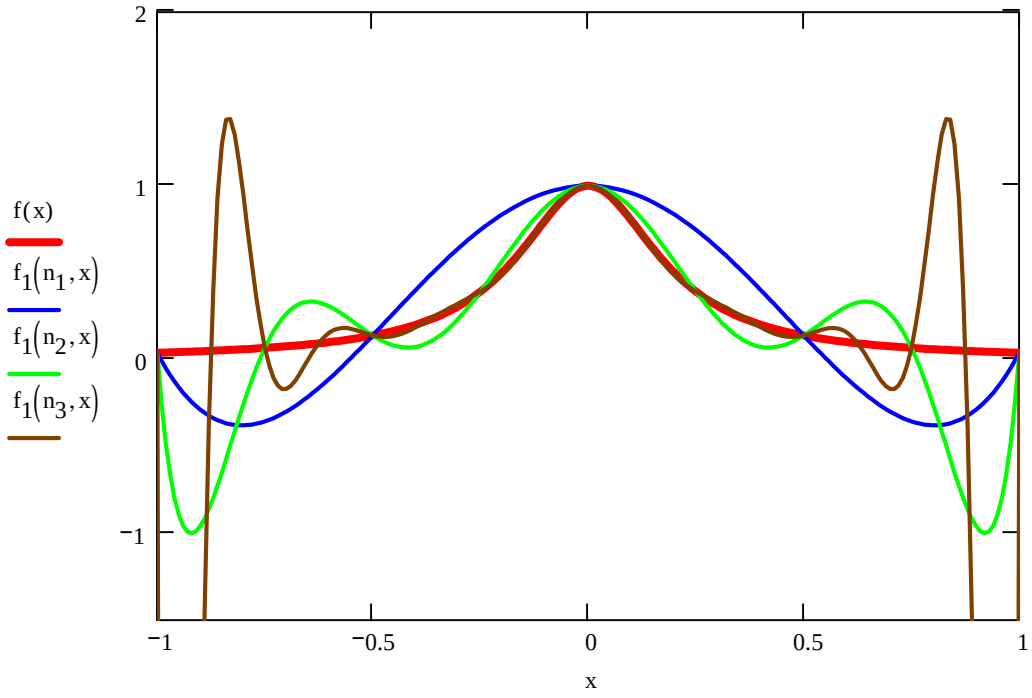
This function generates the coefficients for the polynomial that approximates the x and y data.

$$a(n) := M(n, X(n), Y(n))^{-1} \cdot Y(n)$$

When given the polynomial order and specific x, this function uses the above calculated coefficients to calculate the approximated f(x)

$$f_1(n, x) := \left| \begin{array}{l} c \leftarrow 0 \\ \text{for } i \in 0..n-1 \\ \quad \left| \begin{array}{l} d \leftarrow a(n)_i \cdot x^i + c \\ c \leftarrow d \end{array} \right. \\ d \end{array} \right.$$

Runge's function and three interpolents using equidistantly spaced points



This function is similar to the first function in that it generates 2n-1 equidistanly spaced points. These points are used below to numerically illustrate the exponentially growing error.

$$\text{int}(n) := \left| \begin{array}{l} \text{for } i \in 0..2(n-1) \\ \quad c_i \leftarrow \dfrac{2}{2\cdot(n-1)} \cdot i - 1 \\ c \end{array} \right.$$

The results function calculates the actual values, the interpolated values, and the percent true error, and places them in the tabular form seen below.

$$
\begin{aligned}
&\text{results}(n) := \left|
\begin{array}{l}
i \leftarrow 0, 1 .. 2 \cdot n - 2 \\
j \leftarrow 0 \\
\text{for } q \in 0 .. 2 \cdot n - 2 \\
\quad M_{q,j} \leftarrow \text{int}(n)_q \\
j \leftarrow 1 \\
\text{for } q \in 0 .. 2 \cdot n - 2 \\
\quad M_{q,j} \leftarrow f(\text{int}(n))_q \\
j \leftarrow 2 \\
\text{for } q \in 0 .. 2 \cdot n - 2 \\
\quad M_{q,j} \leftarrow f_1(n, \text{int}(n))_q \\
j \leftarrow 3 \\
\text{for } q \in 0 .. 2 \cdot n - 2 \\
\quad M_{q,j} \leftarrow \left| \dfrac{f(\text{int}(n))_q - f_1(n, \text{int}(n))_q}{f(\text{int}(n))_q} \right| \cdot 100 \\
M
\end{array}
\right.
\end{aligned}
$$

For $n_1 = 5$

$\text{results}(n_1) =$

X	Actual	Interpolated	Percent True Error
-1	0.03846	0.03846	0
-0.75	0.06639	-0.35683	637.4693
-0.5	0.13793	0.13793	0
-0.25	0.39024	0.74563	91.06704
0	1	1	0
0.25	0.39024	0.74563	91.06704
0.5	0.13793	0.13793	0
0.75	0.06639	-0.35683	637.4693
1	0.03846	0.03846	0

For $n_2 = 9$

$\text{results}(n_2) =$

X	Actual	Interpolated	Percent True Error
-1	0.03846	0.03846	0
-0.875	0.04965	-0.83102	1773.71986
-0.75	0.06639	0.06639	0
-0.625	0.09289	0.32826	253.38855
-0.5	0.13793	0.13793	0
-0.375	0.22145	0.09196	58.47428
-0.25	0.39024	0.39024	0
-0.125	0.7191	0.8083	12.40364
0	1	1	0
0.125	0.7191	0.8083	12.40364
0.25	0.39024	0.39024	0
0.375	0.22145	0.09196	58.47428
0.5	0.13793	0.13793	0
0.625	0.09289	0.32826	253.38855
0.75	0.06639	0.06639	0
0.875	0.04965	-0.83102	1773.71986
1	0.03846	0.03846	0

For $n_3 = 17$

$\text{results}(n_3) =$

X	Actual	Interpolated	Percent True Error
-1	0.03846	0.03846	0
-0.9375	0.04353	-10.17387	23472.07535
-0.875	0.04965	0.04965	0
-0.8125	0.05713	1.18181	1968.62567
-0.75	0.06639	0.06639	0
-0.6875	0.07802	-0.14606	287.20003
-0.625	0.09289	0.09289	0
-0.5625	0.11223	0.1806	60.92122
-0.5	0.13793	0.13793	0
-0.4375	0.17286	0.14337	17.05794
-0.375	0.22145	0.22145	0
-0.3125	0.29058	0.30751	5.82725
-0.25	0.39024	0.39024	0
-0.1875	0.53222	0.52071	2.16441
-0.125	0.7191	0.7191	0
-0.0625	0.91103	0.91622	0.56958
0	1	1	0
0.0625	0.91103	0.91622	0.56958
0.125	0.7191	0.7191	0
0.1875	0.53222	0.52071	2.16441
0.25	0.39024	0.39024	0
0.3125	0.29058	0.30751	5.82725
0.375	0.22145	0.22145	0
0.4375	0.17286	0.14337	17.05794
0.5	0.13793	0.13793	0
0.5625	0.11223	0.1806	60.92122
0.625	0.09289	0.09289	0
0.6875	0.07802	-0.14606	287.20003
0.75	0.06639	0.06639	0
0.8125	0.05713	1.18181	1968.62567
0.875	0.04965	0.04965	0
0.9375	0.04353	-10.17387	23472.07535
1	0.03846	0.03846	0