Topic: Additional Interpolation TopicsSimulation: The Need for Spline InterpolationLanguage: Mathcad 2001Authors: Nathan Collier, Autar Kaw, Ginger FisherDate: 25 June 2002Abstract: This simulation shows the need for spline interpolationas opposed to using polynomial interpolation. The function chosenwas first used by Runge in 1901 when he wanted to show that higherorder interpolation is a bad idea. He took a simple looking functionf(x)=1/(1+25x2) and chose equidistantly spaced points to interpolate

INPUTS: Enter the following

Enter the number of equidistantly spaced points in [-1,1]

n := 9

SOLUTION

$$f(x) := \frac{1}{1 + 25 \cdot x^2}$$
 $x := -1, -.99..1$

When n is given, this function returns a matrix containing sequential x values

$$\begin{array}{l} x_{\text{points}}(n) \coloneqq & \text{for } i \in 0..n - 1 \\ c_i \leftarrow \frac{2}{n-1} \cdot i - 1 \\ c \end{array}$$
$$Y(n) \coloneqq f(x_{\text{points}}(n))$$
$$X(n) \coloneqq x_{\text{points}}(n)$$

When x and y data and order is given, this function constructs the matrix whose inverse is needed to find coefficients of the polynomial which approximates the data.

$$M(n, X, Y) := \begin{cases} \text{for } i \in 0 .. n - 1 \\ \text{for } j \in 0 .. n - 1 \\ a_{i, j} \leftarrow (X_i)^j \\ a \end{cases}$$

This function generates the coefficients for the polynomial that approximates the x and y data.

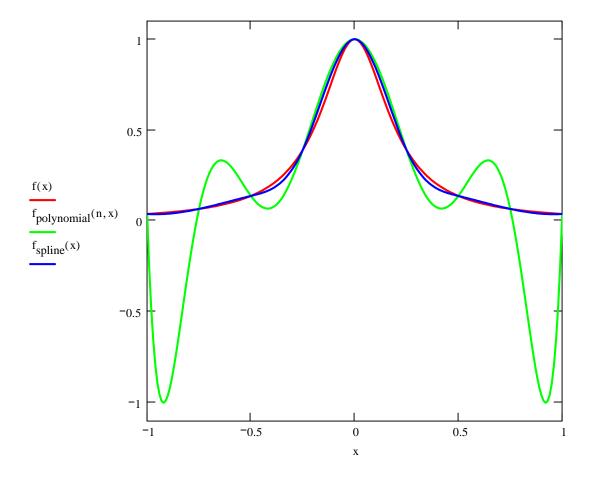
 $a(n) \coloneqq M(n, X(n), Y(n))^{-1} \cdot Y(n)$

When given the polynomial order and specific x, this function uses the above calculated coefficients to calculate the approximated f(x)

$$f_{\text{polynomial}}(n, x) := \begin{vmatrix} c \leftarrow 0 \\ \text{for } i \in 0.. n - 1 \\ d \leftarrow a(n)_i \cdot x^i + c \\ c \leftarrow d \\ d \end{vmatrix}$$

S := cspline(X(n), Y(n))

 $f_{spline}(x) := interp(S, X(n), Y(n), x)$



Runge's function interpolated using polynomial interpolation and cubic spline interpolation