Topic : Additional Interpolation Topics
Simulation : Comparing Polynomial Interpolation and Spline Interpolation
Language : Mathcad 2001
Authors : Nathan Collier, Autar Kaw, Ginger Fisher
Date : 25 June 2002
Abstract : A rapid robot arm with a laser is used to make seven holes on a rectangular plate 12" X 8" at six points as shown. The path of the robot going from one indentation point to another needs to be smooth so as to avoid sharp jerks in the arm that can otherwise create premature wear and tear of the robot arm. One suggestion has been to fit a fifth order polynomial through the six points. Another suggestion was to fit a quadratic spline through the holes. Which suggestion is better so that the robot path is shorter but also smooth?

INPUTS: The following is the data (x-y) coordinate data of the center of the six holes.

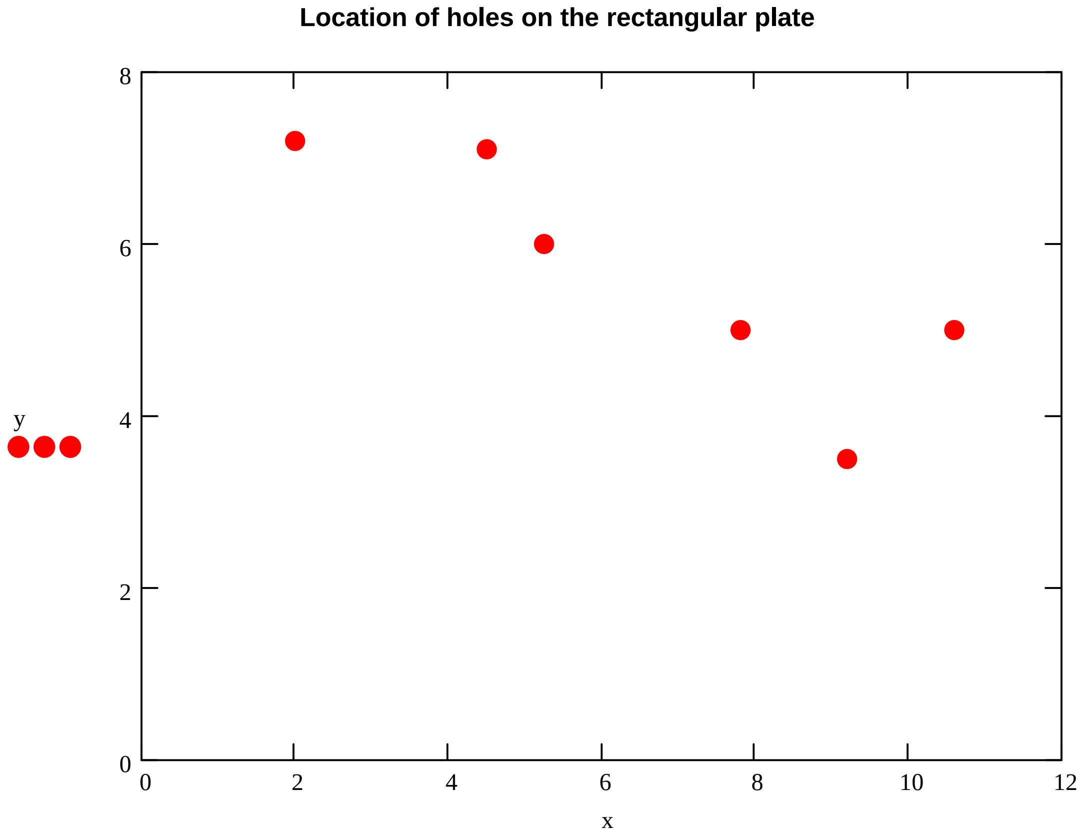
Array of x data:

$$x := \begin{pmatrix} 2 \\ 4.5 \\ 5.25 \\ 7.81 \\ 9.2 \\ 10.6 \end{pmatrix}$$

Array of y data

$$y := \begin{pmatrix} 7.2 \\ 7.1 \\ 6 \\ 5 \\ 3.5 \\ 5 \end{pmatrix}$$

SOLUTION



Polynomial Interpolation:

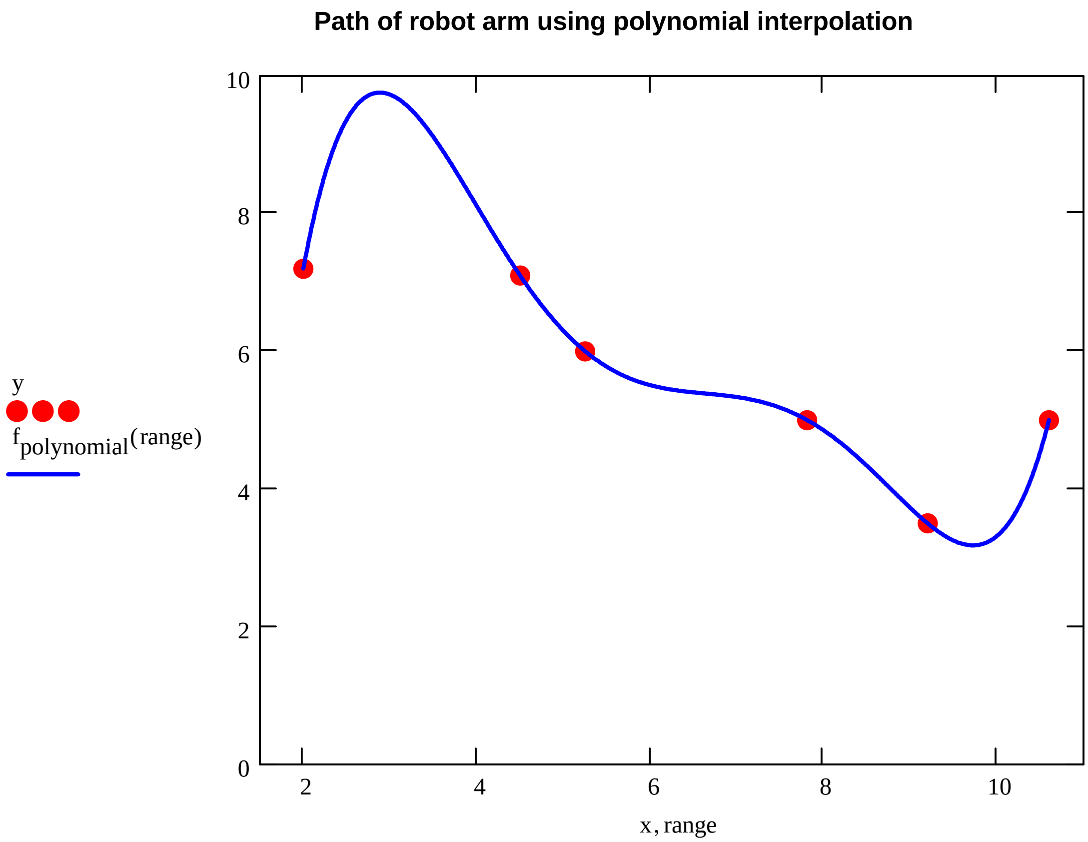
Using polynomial interpolation to find the path that goes through the six data points.

The "regress" function of MATHCAD is used to conduct polynomail interpolation. If one regresses 'n' data points to a 'n-1'th order polynomial, it is interpolation.

$z := \mathrm{regress}(x, y, \mathrm{rows}(x) - 1)$

$f_{polynomial}(a) := \mathrm{interp}(z, x, y, a)$

$\mathrm{range} := x_0, x_0 + 0.01 .. x_5$



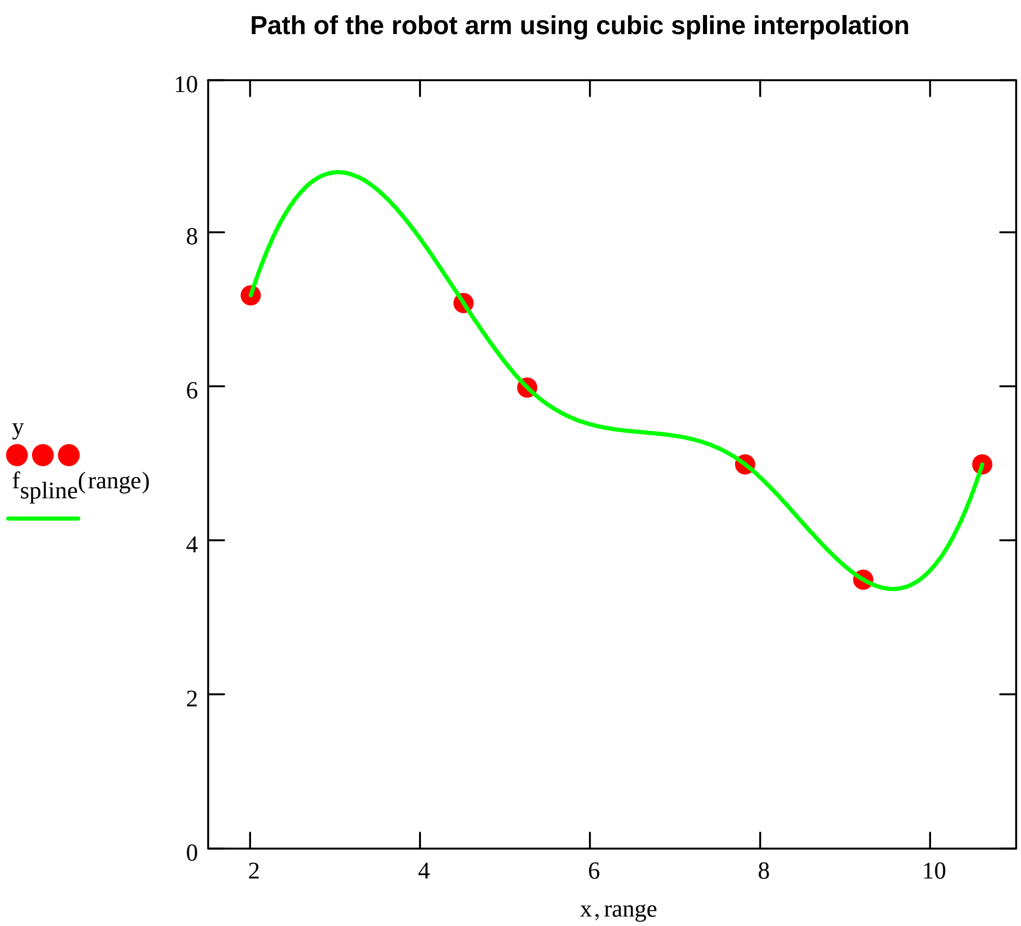
$$L_{polynomial} := \int_{x_0}^{x_5} \sqrt{1 + \left(\frac{d}{dx} f_{polynomial}(x)\right)^2}\, dx$$

$$L_{polynomial} = 14.919$$

Cubic Splines:

$S := \mathrm{cspline}(x, y)$

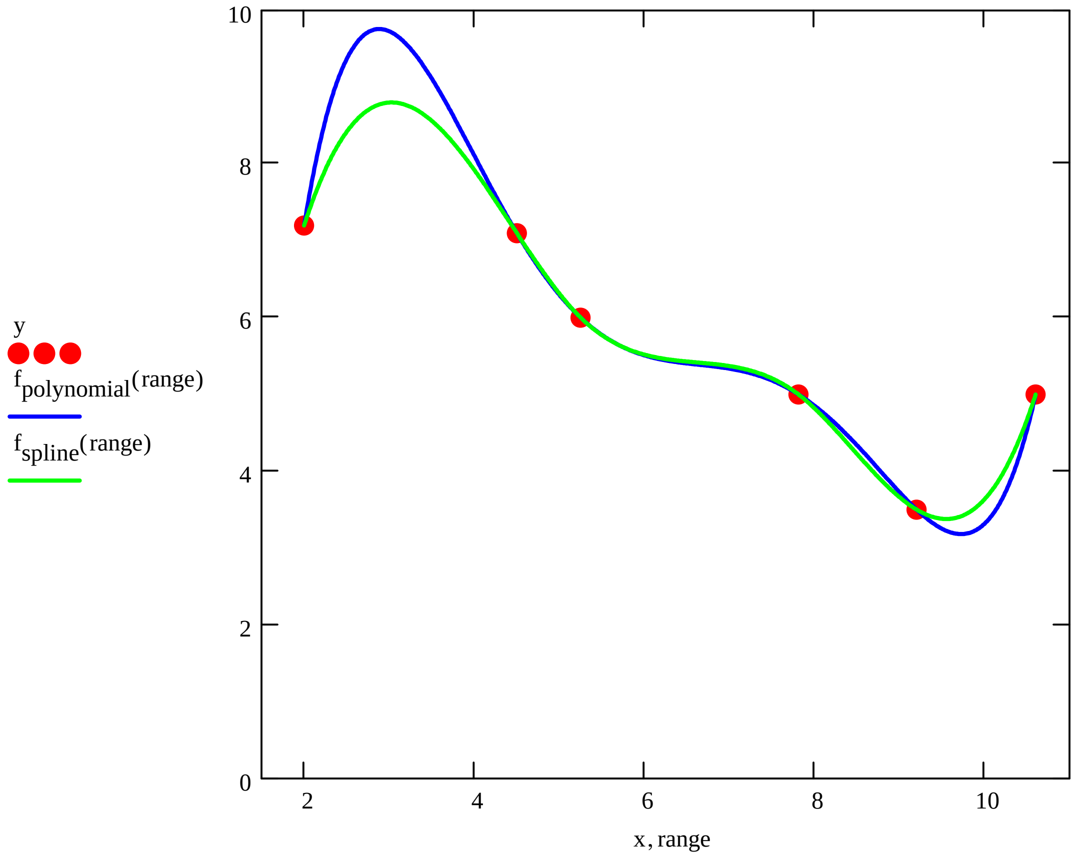
$f_{spline}(z) := \mathrm{interp}(S, x, y, z)$



$$L_{spline} := \int_{x_0}^{x_5} \sqrt{1 + \left(\frac{d}{dx} f_{spline}(x)\right)^2}\, dx$$

$L_{spline} = 12.932$

Path of the robot arm compared using polynomial interpolation and cubic spline interpolation



$$\text{results} := \left| \begin{array}{l} c_{0,0} \leftarrow \text{"Type of interpolation"} \\ c_{1,0} \leftarrow \text{"Length of curve"} \\ c_{0,1} \leftarrow \text{"Polynomial"} \\ c_{0,2} \leftarrow \text{"Cubic Spline"} \\ c_{1,2} \leftarrow L_{spline} \\ c_{1,1} \leftarrow L_{polynomial} \\ c \end{array} \right.$$

results =

"Type of interpolation"	"Polynomial"	"Cubic Spline"
"Length of curve"	14.919	12.932