SOLUTION OF REGRESSION MODELS

NonLinear Regression Models without Data Linearization

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Introduction

This worksheet illustrates finding the constants of nonlinear regression models without d linearization. Three common nonlinear models are illustrated -

1) **Exponential:** $y = ae^{bx}$
2) **Power:** $y = ax^b$
3) **Saturation:** $y = \frac{(ax)}{(b+x)}$

where *a* and *b* are constants of the model.

Given *n* data points (x_1,y_1), (x_2, y_2), (x_3,y_3),....,(x_n,y_n), you can best fit one of the nonlinea models to the data. In this worksheet, the constants *a* and *b* are calculated in the followin steps:

1) Finding the sum of the squares of the residuals, *Sr*
2) Minimizing *Sr* by differentiating with respect to *a* and *b* and setting the resulti equations to zero
3) Solving for the two nonlinear equations simultaneously

Mathcad will then return the real solutions of *a* and *b*. To learn more about nonlinear regression models without data linearization see the **Nonlinear Regression Model** workshe

Section 1: Input data

Below are the input parameters to begin the simulation. This is the only section that requires user input. The user can choose the nonlinear model that is desired with the *model_type* variable. For reasonable initial guess values, use numbers that are less than or more than the solution from the **Nonlinear Regression with data linearization** worksheet.
Note: the origin has been set to one to redefine the starting index of all arrays. The user SHOULD NOT change this value. ORIGIN := 1

Input Parameters:

- Number of data points, *n*

  n := 3

- Array of *x* values, "X"

$$X := \begin{pmatrix} 0.5 \\ 1 \\ 3 \end{pmatrix}$$

- Array of *y* values, "Y"

$$Y := \begin{pmatrix} 3.29744 \\ 5.43656 \\ 40.1711 \end{pmatrix}$$

- For ***exponential*** model call model type to be **"exponential"**
  For ***power*** model assign the model type variable as **"power"**
  For ***saturation growth*** model, assign the model type variable to be **"growth"**

  model_type := "power"

- Insert your initial guess values for *a* and *b* here. Reasonable initial guesses for *a* and
  can be obtained from data linearization models.

  Ainit := 7.3

  Binit := 1.39

In section 3, *Solve Block* is used to find the solution to the constants of the regression mo without data linearization. You may need to use a different method to solve if you are una to get a solution. Change the algorithm by right clicking on the *Find* command and check different nonlinear method in the popup window.

Section 2: Finding the constants of the Model

Assigning the proper regression model

$$f(x,a,b) := \begin{cases} a\cdot \exp(b\cdot x) & \text{if } model\_type = \text{"exponential"} \\ a\cdot x^{b} & \text{if } model\_type = \text{"power"} \\ \dfrac{a\cdot x}{b+x} & \text{if } model\_type = \text{"growth"} \end{cases}$$

Calculating the sum of the square of the residuals, *Sr*:

$$Sr(x,a,b,y,n) := \begin{array}{|l} Sr \leftarrow 0 \\ \text{for } i \in 1..n \\ \quad Sr \leftarrow Sr + \left(y_i - f\left(x_i,a,b\right)\right)^2 \\ Sr \end{array}$$

Differentiating *Sr* with respect to the constants of the model *a* and *b* to setup two equation and two unknowns.

$$g(a,b) := \frac{d}{da}Sr(X,a,b,Y,n)$$

$$h(a,b) := \frac{d}{db}Sr(X,a,b,Y,n)$$

Setting *g(a,b)* and *h(a,b)* equal to zero, then solving the two nonlinear equations for *a* and

Guess

$a := Ainit$

$b := Binit$

Given

$g(a,b) = 0$

$h(a,b) = 0$

$constants\_of\_model := Find(a,b)$

The constants of the desired nonlinear model are:

$a := constants\_of\_model_1$ $b := constants\_of\_model_2$

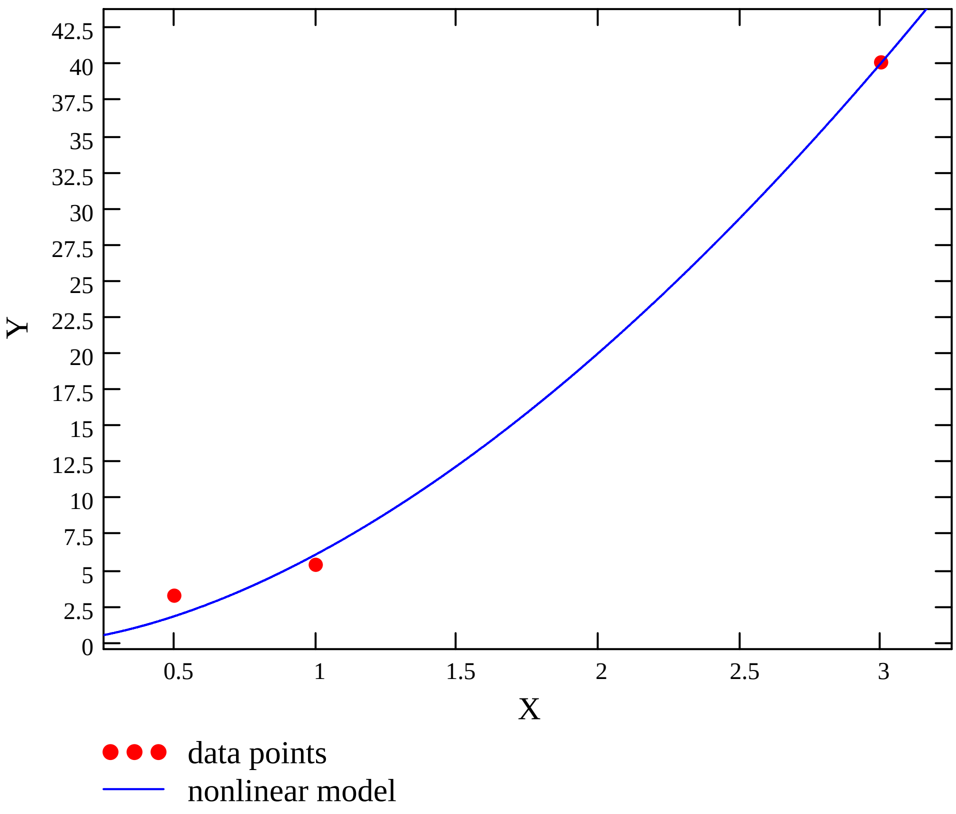
$a = 6.144$ $b = 1.708$

Below, the nonlinear model is plotted versus the data set. Recall that the model type is assigned as

model_type = "power"

$$F(x) := \begin{cases} a \cdot \exp(b \cdot x) & \text{if model\_type} = \text{"exponential"} \\ a \cdot x^{b} & \text{if model\_type} = \text{"power"} \\ \dfrac{a \cdot x}{b + x} & \text{if model\_type} = \text{"growth"} \end{cases}$$

Figure 1: Nonlinear Regression Model, y vs. x



References

[1] Autar Kaw, *Holistic Numerical Methods Institute*, *http://numericalmethods.eng.usf.edu/mcd*, See
How does Nonlinear Regression work?

Conclusion

Mathcad helped us apply our knowledge of least squares regression method to regress a given data set to a nonlinear model.

Question 1: Verify each of the models by using data that exactly follows the regression model.

Question 2: What is the difference in the solution between nonlinear models obtained via data that is linearized and data that is not linearized.