HNMI α to ω

Simpson's 1/3 Rule - Integration

Graphical Simulation of the Method

Article Information

Subject: This worksheet demonstrates multiple segment Simpson's 1/3rd rule of integration.
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Introduction

Simpson's rule is based on the Newton-Cotes formula that if one approximates the integrar of the integral by an nth order polynomial, then the integral of the function is approximate the integral of that nth order polynomial. Integration of polynomials is simple and is based the calculus. Simpson's $1/3^{rd}$ rule is the area under the curve where the function is approximated by a second order polynomial. [click **here** for textbook notes] [click **here** fo power point presentation].

Inputs

The following simulation illustrates the Simpson's 1/3rd rule of integration. This section is only section where the user interacts with the program. The user enters any function *f(x),* tl lower and upper limit of the integration. By entering this data, the program will calculate t exact value of the integral, followed by the results using the Simpson's 1/3rd rule with *n = 4, 6, 8* segments.

Integrand *f(x)* $f(x) := \frac{300 \cdot x}{1 + e^{x}}$

Lower limit of the integral *a* $a := 0$

Upper limit of the integral *b* $b := 10$

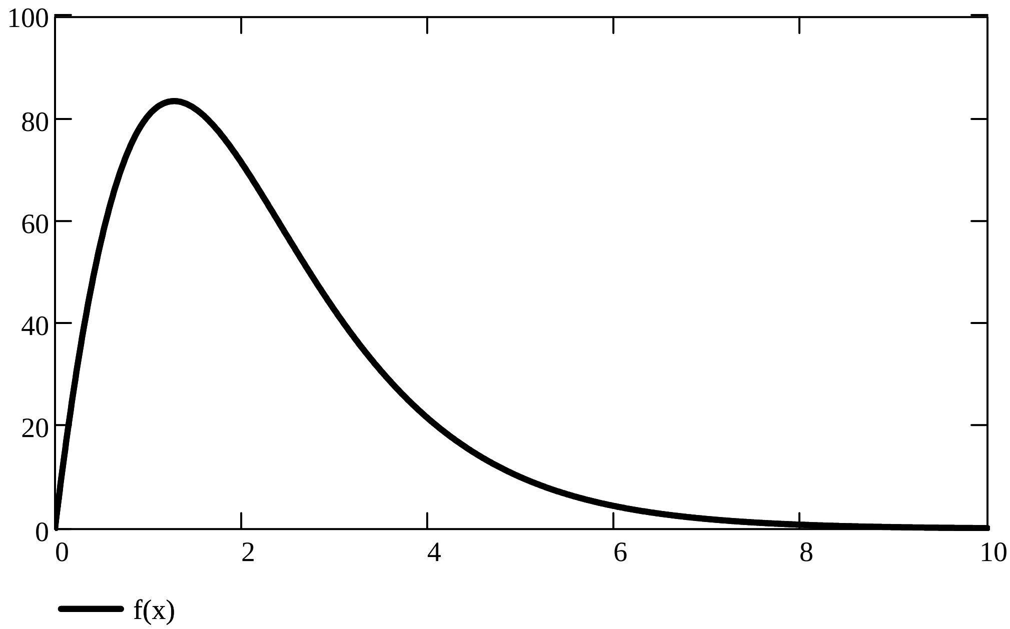
Exact Solution

In this section, the program will evaluate the exact value for the integral of the function *f(*
evaluated at the limits *a* and *b*.

$$s_{exact} := \int_a^b f(x)\, dx$$

$$s_{exact} = 246.59029$$

Figure 1: Entered function on given interval



2 Segment Simpon 1/3 Rule

$n := 2$

$h_2 := \frac{b - a}{n}$

$h_2 = 5$

The integral of the function *f(x)* from *a* to *b* using Simpson's rule with two segments will be equal to:

$s_2 := h_2 \cdot \frac{f(a) + 4 \cdot f\left(a + h_2\right) + f(b)}{3}$

$s_2 = 67.1555$

4 Segment Simpon 1/3 Rule

$$n := 4$$

$$h_4 := \frac{b - a}{n}$$

$$h_4 = 2.5$$

The integral of the function *f(x)* from *a* to *b* using Simpson's rule with four segments will be equal to:

$$s_4 := h_4 \cdot \frac{f(a) + 4 \cdot \left(f\left(a + h_4 \right) + f\left(a + 3 \cdot h_4 \right) \right) + 2 \cdot f\left(a + 2 \cdot h_4 \right) + f(b)}{3}$$

$$s_4 = 210.63691$$

The approximate error (E_a):

$$E_{a4} := s_4 - s_2$$

$$E_{a4} = 143.481$$

The absolute relative approximate percentage error (ε_a):

$$\varepsilon_{a4} := \left| \frac{E_{a4}}{s_4} \right| \cdot 100$$

$$\varepsilon_{a4} = 68.118$$

6 Segment Simpon 1/3 Rule

$n := 6$

$h_6 := \frac{b - a}{n}$

$h_6 = 1.667$

The integral of the function *f(x)* from *a* to *b* using Simpson's rule with four segments will be equal to:

$i := 1, 3 .. n - 1 \qquad j := 2, 4 .. n - 2$

$$s_6 := h_6 \cdot \frac{f(a) + 4 \cdot \sum_i f\left(a + i \cdot h_6\right) + 2 \cdot \sum_j f\left(a + j \cdot h_6\right) + f(b)}{3}$$

$s_6 = 241.33838$

The approximate error (E_a):

$E_{a6} := s_6 - s_4$

$E_{a6} = 30.701$

The absolute relative approximate percentage error (ε_a):

$$\varepsilon_{a6} := \left| \frac{E_{a6}}{s_6} \right| \cdot 100$$

$\varepsilon_{a6} = 12.721$

8 Segment Simpon 1/3 Rule

$n := 8$

$$h_8 := \frac{b - a}{n}$$

$h_8 = 1.25$

The integral of the function *f(x)* from *a* to *b* using Simpson's rule with four segments will be equal to:

$i := 1, 3 .. n - 1 \qquad j := 2, 4 .. n - 2$

$$s_8 := h_8 \cdot \frac{f(a) + 4 \cdot \sum_i f(a + i \cdot h_8) + 2 \cdot \sum_j f(a + j \cdot h_8) + f(b)}{3}$$

$s_8 = 245.85499$

The approximate error (E_a):

$E_{a8} := s_8 - s_6$

$E_{a8} = 4.517$

The absolute relative approximate percentage error (ε_a):

$$\varepsilon_{a8} := \left| \frac{E_{a8}}{s_8} \right| \cdot 100$$

$\varepsilon_{a8} = 1.837$