Special Topics - Integration

Integrating a Discrete Function

Article Information

Subject: This worksheet demonstrates the use of Mathcad to illustrate the integration of a discrete function using different methods.
Revised: 4 October 2004
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Version: Mathcad 2001

Introduction

The following worksheet will illustrate how to integrate a discrete function. This is usually either by calculating the area of the trapezoids defined by the data points or by integrating interpolant. So here we show the answer by trapezoidal rule as well as with 3 types of interpolation. For more information on interpolation see the **topic**. The following outlines sheet:

1. Trapezoidal Rule
2. Polynomial Interpolation
3. Linear Spline Interpolation
4. Cubic Spline Interpolation

See the following links for notes and presentations. [click **here** for textbook notes] [click **h** for power point presentation].

Inputs

The user may enter any set of data X, and Y, and the lower and upper limit for the functioı be integrated. By entering these data, the program will calculate the average value of the iı using the trapezoidal rule, the polynomial interpolation, and the spline interpolation.

Set of Data, *X* $X := \begin{pmatrix} 0 \\ 10 \\ 15 \\ 20 \\ 22.5 \\ 30 \end{pmatrix}$ Set of Data, Y $Y := \begin{pmatrix} 320 \\ 120 \\ 620 \\ 7720 \\ 320 \\ 620 \end{pmatrix}$

NOTE: Make sure you enter the same number of data in both X and Y.

Lower limit of the integral *a* $a := 0.1$

Upper limit of the integral *b* $b := 30$

$$range := X_0, X_0 + 0.01 .. X_{rows(X)-1}$$

Data Checks

The data set X and Y and limits a and b must have certain characteristics. The following checks that certain criteria are true.

Are the limits a and b in the data range of X? If $check_0$ returns YES, then you may procee
If it reports NO then you need to redefine either or both a and b so that they are within the data range of X.

$$check_0 := \begin{array}{|l} \text{"YES"} \quad \text{if } a \geq \min(X) \wedge b \leq \max(X) \\ \text{"NO"} \quad \text{otherwise} \end{array}$$

$$check_0 = \text{"YES"}$$

Are the values of X in increasing order? If $check_1$ returns YES, then you may proceed. If i
reports NO, then you need to resort X and Y so that X is in ascending order.

$$check_1 := \begin{array}{|l} XX \leftarrow \text{sort}(X) \\ \text{"YES"} \quad \text{if } X = XX \\ \text{"NO"} \quad \text{otherwise} \end{array}$$

$$check_1 = \text{"YES"}$$

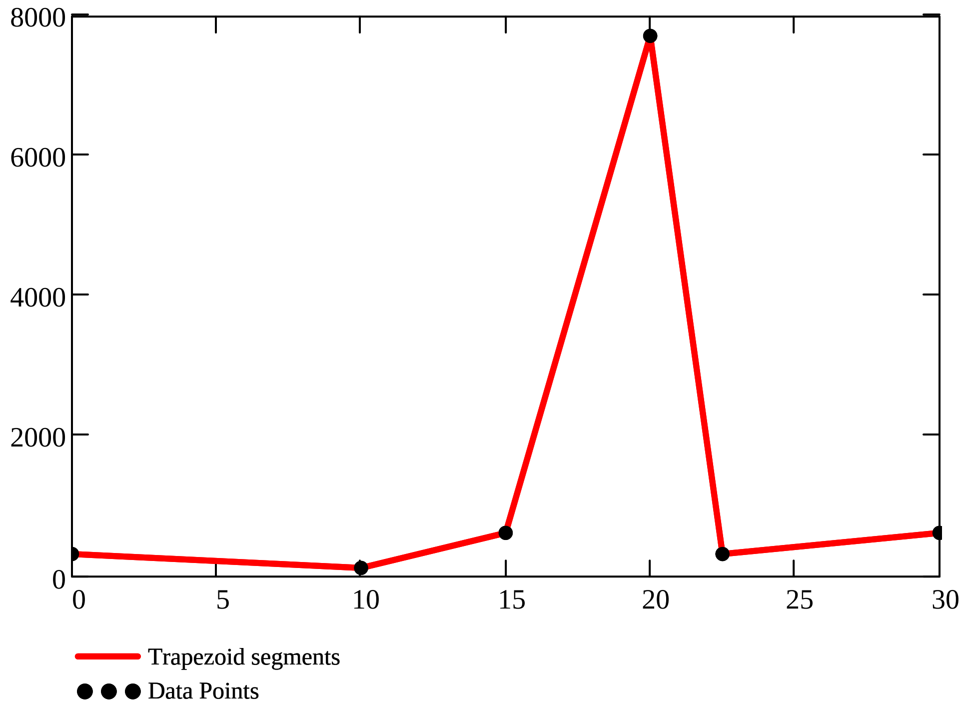
Trapezoidal Rule

$$
\text{trap}_d(X,Y,a,b) := \left|
\begin{array}{l}
n \leftarrow \text{rows}(X) \\
I \leftarrow 0 \\
\text{for } i \in 0..n-2 \\
\quad \left| \begin{array}{l} ain \leftarrow i \ \text{ if } a > X_i \\ bin \leftarrow i \ \text{ if } b > X_i \end{array} \right. \\
ya \leftarrow \dfrac{a - X_{ain}}{X_{ain+1} - X_{ain}} \left(Y_{ain+1} - Y_{ain}\right) + Y_{ain} \\
yb \leftarrow \dfrac{b - X_{bin}}{X_{bin+1} - X_{bin}} \left(Y_{bin+1} - Y_{bin}\right) + Y_{bin} \\
I \leftarrow \left(X_{ain+1} - a\right)\left(ya + \dfrac{Y_{ain+1} - ya}{2}\right) \\
I \leftarrow I + \left(b - X_{bin}\right)\left(Y_{bin} + \dfrac{yb - Y_{bin}}{2}\right) \\
i \leftarrow ain + 1 \\
\text{while } i \le bin - 1 \\
\quad \left| \begin{array}{l} I \leftarrow I + \left(X_{i+1} - X_i\right)\left(Y_i + \dfrac{Y_{i+1} - Y_i}{2}\right) \\ i \leftarrow i + 1 \end{array} \right. \\
I
\end{array}
\right.
$$

$AV_0 := \text{trap}_d(X, Y, a, b)$

$AV_0 = 38443.1$

Figure 1: Trapezoidal Rule



Polynomial Interpolation

NOTE: Regression of a data set to the order of one less than the number of data sets is polynomial interpolation.

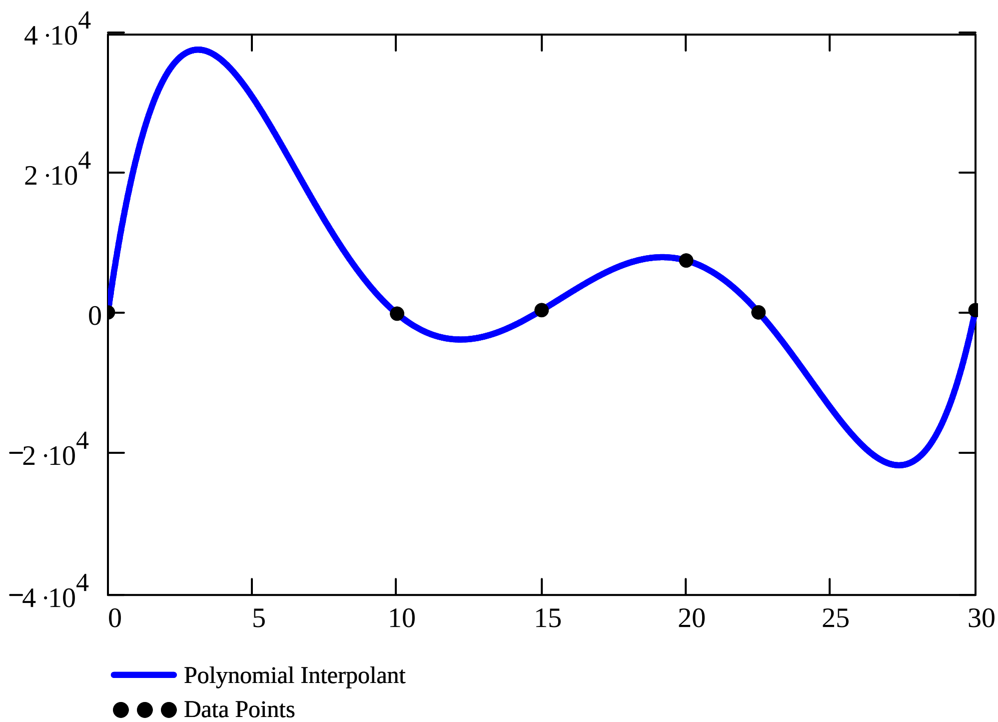
$$z := \mathrm{regress}(X, Y, \mathrm{rows}(X) - 1)$$

$$f_{polynomial}(x) := \mathrm{interp}(z, X, Y, x)$$

$$AV_1 := \int_a^b f_{polynomial}(x)\,dx$$

$$AV_1 = 151251.755$$

Figure 2: Polynomial Interpolation



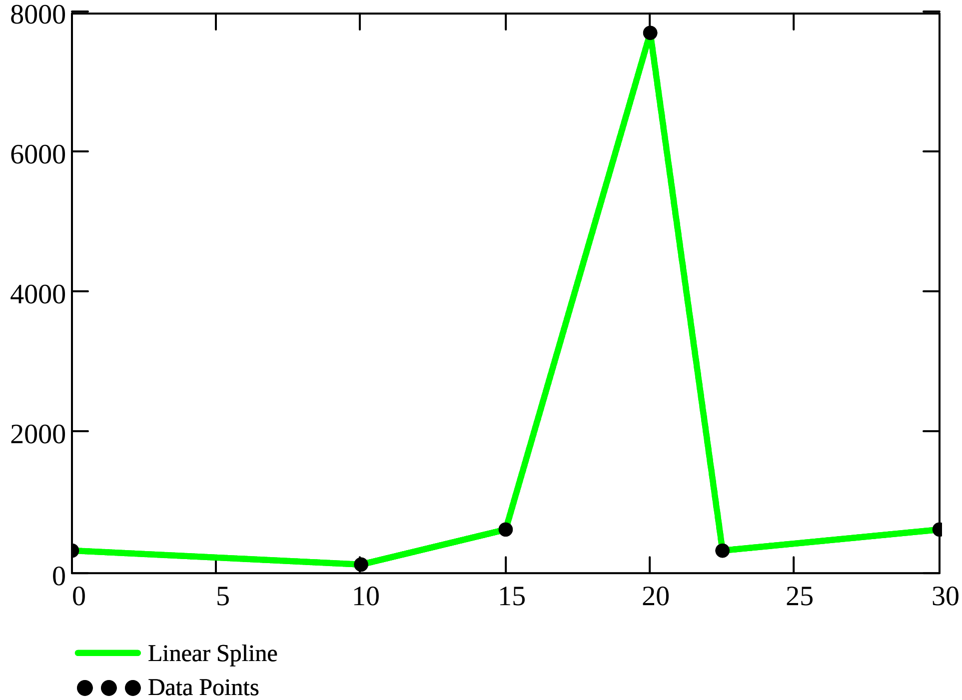
Linear Spline Interpolation

$$f_{linear}(x) := \mathrm{linterp}(X, Y, x)$$

$$AV_2 := \int_a^b f_{linear}(x)\,dx$$

$$AV_2 = 38444.302$$

Figure 3: Linear Spline Interpolation



Cubic Spline Interpolation

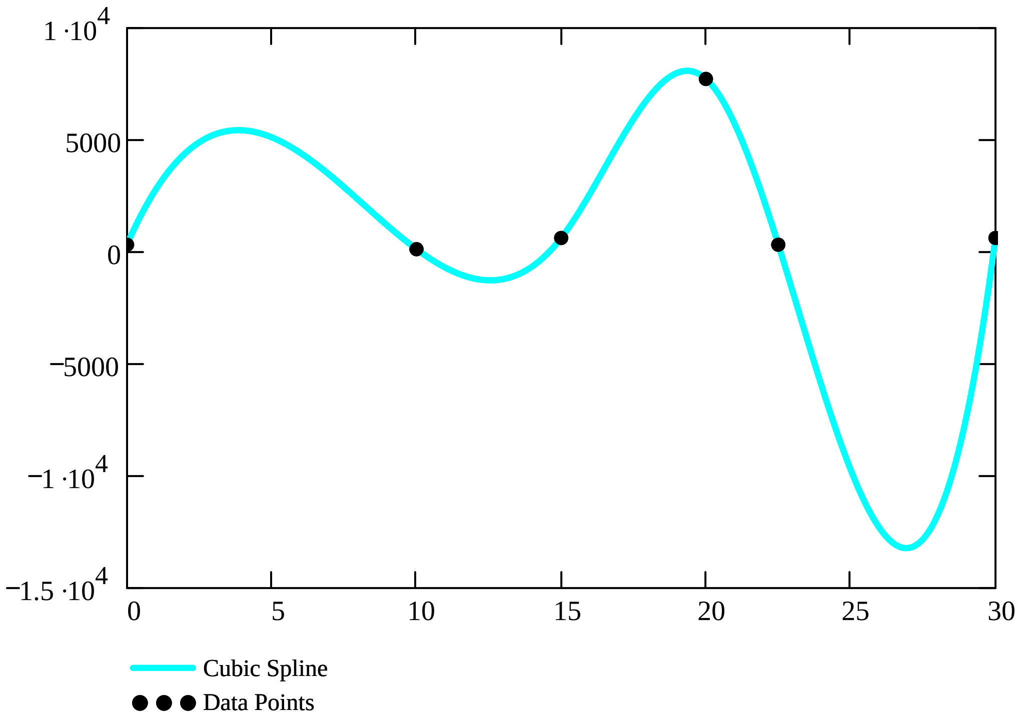
$$v := \text{cspline}(X, Y)$$

$$f_{cubic}(x) := \text{interp}(v, X, Y, x)$$

$$AV_3 := \int_a^b f_{cubic}(x)\, dx$$

$$AV_3 = 6928.386$$

Figure 4: Cubic Spline Interpolation



Conclusions

Below is a table showing all the results for comparison purposes.

AV =

	0	
0	38443.1	rapezoidal Rule
1	151251.755	Polynomial Interpolation
2	38444.302	Linear Splines
3	6928.386	Cubic Splines