HNMI α to ω

Trapezoidal Method - Integration

Graphical Simulation of the Method

Article Information

Subject: The following demonstrates the Trapezoidal method of estimating integrals of continuous functions.

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Version: Mathcad 2001

Introduction

Trapezoidal rule is based on the Newton-Cotes formula that if one approximates the integrand of the integral by an n^{th} order polynomial, then the integral of the function is approximated by the integral of that n^{th} order polynomial. Integration of polynomials is sir and is based on the calculus. Trapezoidal rule is the area under the curve for a first order polynomial (straight line) that approximates the integrand. [click **here** for textbook notes] [click **here** for power point presentation].

Inputs

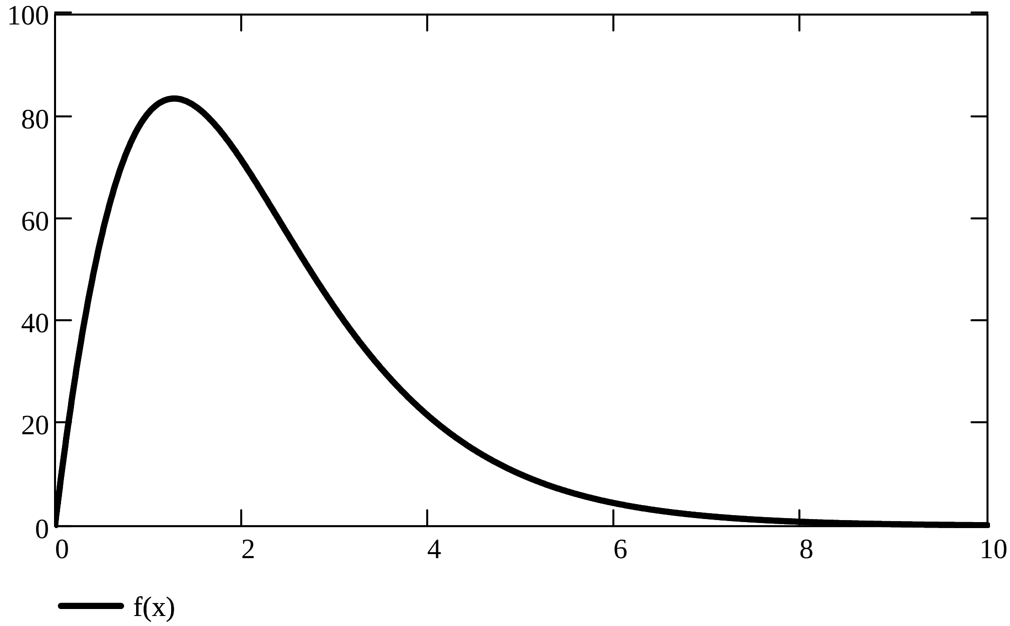
The following simulation illustrates the Trapezoidal rule of integration. This section is the section where the user interacts with the program. The user enters any function *f(x)*, the lo and upper limits of the integration, *a* and *b* respectively. By entering this data, the progran will calculate the results using Trapezoidal rule for *n* = 1, 2, 3, and 4 segments.

Integrand *f(x)* $f(x) := \frac{300 \cdot x}{1 + e^{x}}$

Lower limit of the integral *a* $a := 0$

Upper limit of the integral *b* $b := 10$

Figure 1: Entered function on given interval



Big, Ugly, Function

The following function draws the trapezoids in the graphs below:

$$
\text{drawtrap}(seg) := \left|
\begin{array}{l}
X_0 \leftarrow a \\
X_1 \leftarrow a \\
X_{1+seg} \leftarrow b \\
X_{2+seg} \leftarrow b \\
Y_{1+seg} \leftarrow f(b) \\
Y_{2+seg} \leftarrow 0 \\
Y_0 \leftarrow 0 \\
Y_1 \leftarrow f(a) \\
h \leftarrow \dfrac{b-a}{seg} \\
\text{for } i \in 1..seg-1 \\
\quad \left|
\begin{array}{l}
X_{i+1} \leftarrow a + i \cdot h \\
Y_{i+1} \leftarrow f\left(X_{i+1}\right) \\
\text{for } j \in 1..3 \\
\quad \left|
\begin{array}{l}
X_{2+seg+(i-1)\cdot 3+j} \leftarrow a + (seg-i)\cdot h \\
Y_{2+seg+(i-1)\cdot 3+j} \leftarrow f[a + (seg-i)\cdot h] \quad \text{if } j = 2 \\
0 \quad \text{otherwise}
\end{array}
\right.
\end{array}
\right. \\
X_{2+seg+3\cdot(seg-1)+1} \leftarrow a \\
Y_{2+seg+3\cdot(seg-1)+1} \leftarrow 0 \\
\text{augment}(X, Y)
\end{array}
\right.
$$

1 Segment Trapezoidal Rule

$n := 1$

$h_1 := \frac{b - a}{n}$

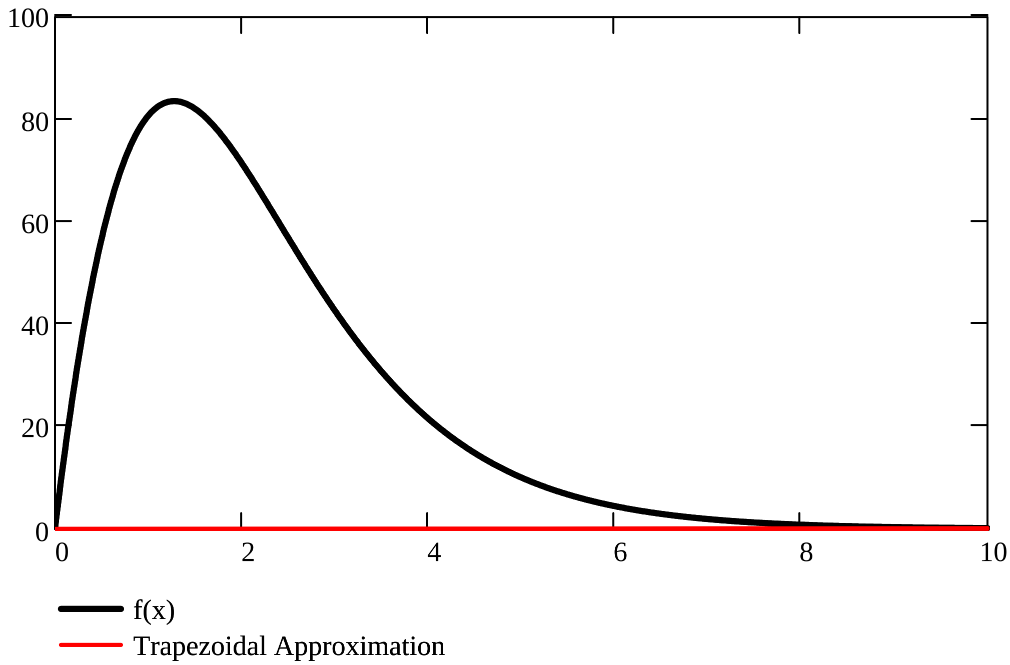
$h_1 = 10$

The integral of the function *f(x)* from *a* to *b* using the trapezoidal rule with one segment will be equal to

$s_1 := h_1 \cdot \frac{f(a) + f(b)}{2}$

$s_1 = 0.68097$

Figure 2: Integral approximated by 1 segment trapezoidal rule



The approximate error and absolute relative approximate error for the first iteration are undefined.

2 Segment Trapezoidal Rule

$$n := 2$$

$$h_2 := \frac{b - a}{n}$$

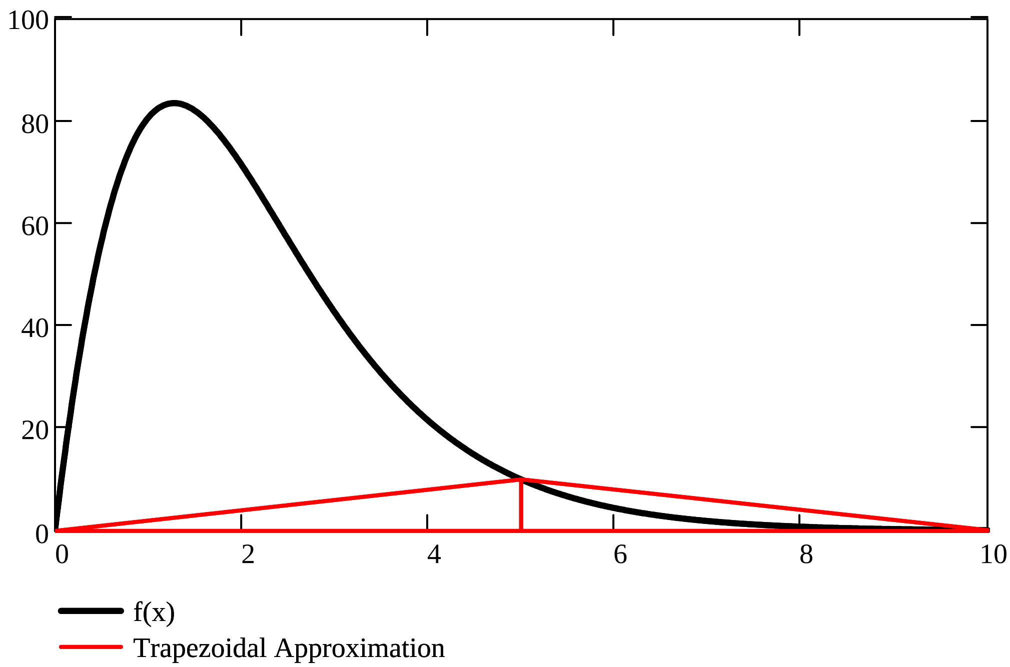
$$h_2 = 5$$

The integral of the function *f(x)* from *a* to *b* using the trapezoidal rule with two segments will be equal to

$$s_2 := h_2 \cdot \frac{f(a) + 2 \cdot f\left(a + h_2\right) + f(b)}{2}$$

$$s_2 = 50.53687$$

Figure 3: Integral approximated by 2 segment trapezoidal rule



The approximate error is

$$E_{a2} := s_2 - s_1$$

$$E_{a2} = 49.856$$

The absolute relative approximate error is

$$\varepsilon_{a2} := \left|\frac{E_{a2}}{s_2}\right| \cdot 100$$

$$\varepsilon_{a2} = 98.653$$

3 Segment Trapezoidal Rule

$n := 3$

$$h_3 := \frac{b - a}{n}$$

$h_3 = 3.333$

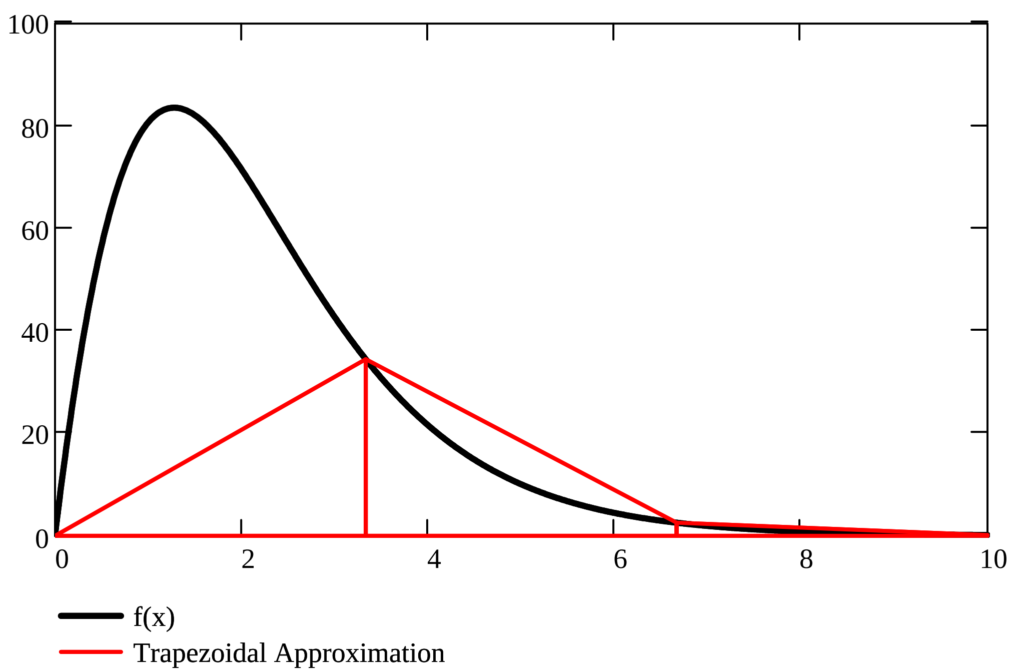
The integral of the function *f(x)* from *a* to *b* using the trapezoidal rule with three segments will be equal to

$$sum := f\left(a + h_3\right) + f\left(a + 2 \cdot h_3\right)$$

$$s_3 := h_3 \cdot \frac{f(a) + 2 \cdot sum + f(b)}{2}$$

$s_3 = 123.51775$

Figure 4: Integral approximated by 3 segment trapezoidal rule



The approximate error is

$E_{a3} := s_3 - s_2$

$E_{a3} = 72.981$

The absolute relative approximate error is

$$\varepsilon_{a3} := \left|\frac{E_{a3}}{s_3}\right| \cdot 100$$

$\varepsilon_{a3} = 59.085$

4 Segment Trapezoidal Rule

$n := 4$

$h_4 := \frac{b - a}{n}$

$h_4 = 2.5$

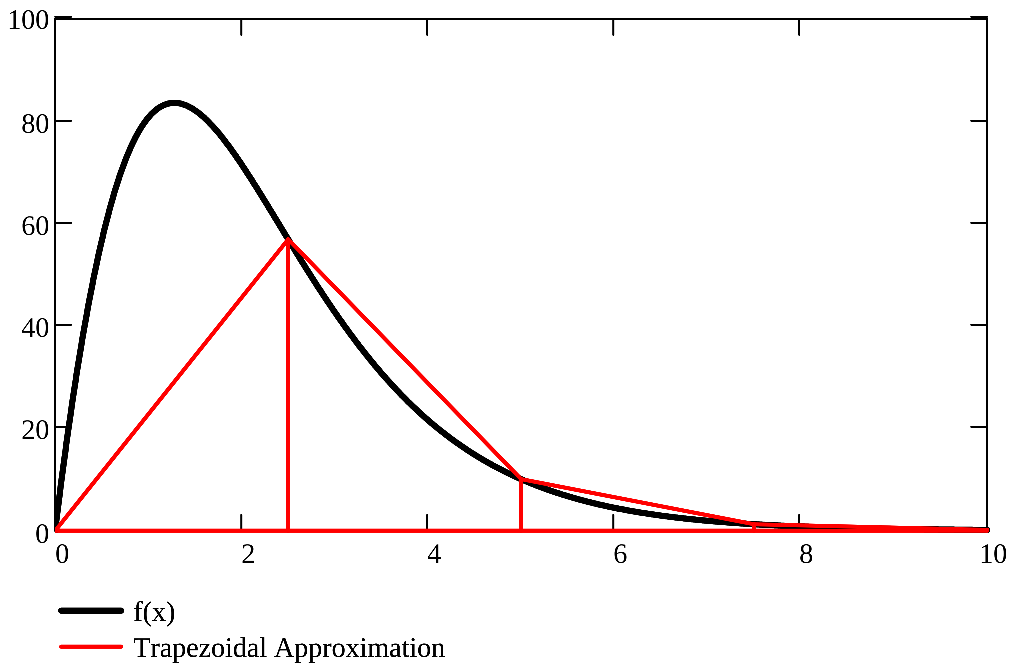
The integral of the function *f(x)* from *a* to *b* using the trapezoidal rule with four segments will be equal to

$sum := f(a + h_4) + f(a + 2 \cdot h_4) + f(a + 3 \cdot h_4)$

$s_4 := h_4 \cdot \frac{f(a) + 2 \cdot sum + f(b)}{2}$

$s_4 = 170.6119$

Figure 5: Integral approximated by 4 segment trapezoidal rule



The approximate error is

$E_{a4} := s_4 - s_3$

$E_{a4} = 47.094$

The absolute relative approximate error is

$$\varepsilon_{a4} := \left|\frac{E_{a4}}{s_4}\right| \cdot 100$$

$\varepsilon_{a4} = 27.603$