Convergence of Gauss-Seidel Method

©2007 Fabian Farelo, Autar Kaw
University of South Florida
United States of America

kaw@eng.usf.edu

NOTE: This worksheet demonstrates the convergence of Gauss-Seidel method, an iterative technique used in solving a system of simultaneous linear equations.

****************************** Introduction ****************************

Gauss-Seidel method is an advantageous approach to solving a system of simultaneous linear equations because it allows the user to control round-off error that is inherent in elimination methods such as Gaussian Elimination. However, this method is not without its pitfalls. Gauss-Seidel method is an iterative technique whose solution may or may not converge.Convergence is ensured only if the coefficient matrix, [A]nxn, is diagonally dominant, otherwise the method may or may not converge.

A diagonally dominant square matrix [A] is defined by the following:

(Sum(|A(i, j)|, j = 1 .. n),[i <> j] <= |(a(i, i))| (1.1)

for all i, and

(Sum(|A(i, j)|, j = 1 .. n),[i <> j] < |(a(i, i))| (1.2)

for at least one i.

Fortunately, many physical systems that result in simultaneous linear equations have diagonally dominant coefficient matrices, or with the exchange of a few equations, the coefficient matrix can become diagonally dominant.
The following simulation illustrates the convergence of the Gauss-Seidel method.

********************************** Input Data **************************

n = number of equations
[A] = nxn coefficient matrix
[RHS] = nx1 right hand side array
[Xold] = nx1 initial guess of the solution vector
maxit = maximum number of iterations

NOTE: These are the default values. Input data can be changed at the beginning of the M-file

```
n= 4

A =

    12     7     3     1
     1     5     1     2
     2     7   -11     1
     9     2     1    13


RHS =

    22
     7
    -2
     3


X =

     1
     2
     1
     1


maxit =

     8
```

********************** Gauss-Seidel Procedure ***********************

Gauss-Seidel method utilizes the equation

```
      x[i] = [rhs[i]-sum(A[i,j]*X*[j])[i<>j]/A[i,j]]          (3.1)
```

to compute an approximate value for a solution vector [X].
The following procedure uses Gauss-Seidel method to calculate the value
of the solution for the above system of equations using maxit iterations.
It will then store each approximate solution, Xi, from each iteration
in a matrix with maxit columns. Thereafter, Matlab will plot the solutions
as a function of the iteration number.

********************************** Results ***************************

The following matrix stores the value of the solution for Xi
in the ith row after each given iteration

```
Xstore =

 0.33333    1.2201    1.0168   0.83943   0.84662   0.87713    0.8805   0.87594
 0.73333    1.0657    1.3274    1.3113    1.2655    1.2613    1.2683      1.27
     0.8     1.066    1.1332    1.0994    1.0832    1.0864    1.0895    1.0895
-0.17436  -0.85986  -0.76456  -0.63668  -0.63336  -0.65408  -0.65774  -0.65484


The following matrix stores the maximum absolute relative approximate error
percentage after each given iteration.

epsmax =

673.53    79.722    19.991    21.131    3.6211    3.4785   0.55525   0.52042

>>
```