

Effect of Significant Digits in Solution of Simultaneous Linear Equations
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NOTE: This worksheet demonstrates the use of Matlab to illustrate the effect of significant digits in the solution of simultaneous linear equations

***** Introduction *****

The number of significant digits used in numerical solutions of simultaneous linear equations influences the accuracy of the solution vector, especially if the coefficient matrix is nearly singular. In this worksheet, the reader can choose a system of equations and see the influence of significant digits on each element of the solution vector.

The following simulation uses Naïve Gaussian Elimination method to demonstrate the effect that significant digits have on the accuracy of the solution.

***** Input data *****

The following are the input parameters to begin the simulation. This is the only section that requires user input. In the simulation, Naïve Gaussian Elimination method is used to solve a set of simultaneous linear equations, $[A][X] = [RHS]$, where $[A]_{n \times n}$ is the square coefficient matrix, $[X]_{n \times 1}$ is the solution vector, and $[RHS]_{n \times 1}$ is the right hand side array. To demonstrate the effect that significant digits have on the accuracy of solution, Matlab will return a list of solution vectors that were calculated using the significant digits within the range of your choice. It will then plot each element of the solution vector as a function of the number of significant digits used.

Input parameters:

n = number of equations

[A] = nxn coefficient matrix

[RHS] = nx1 right hand side array

lower_lim = lower limit of range of significant digits

upper_lim = upper limit of range of significant digits

these are the default parameters which can be changed at the top of the M-File

n =

A =

```
12.000000000000000    7.000000000000000    3.000000000000000
 1.000000000000000    5.000000000000000    1.000000000000000
13.000000000000000   12.000000000000000    4.001000000000000
```

RHS =

```
22.000000000000000    7.000000000000000   29.001000000000000
```

lower_lim =

3

upper_lim =

10

***** Naive Gaussian Elimination Method *****

The Naive Gaussian Elimination Method is performed with significant digits from the lower limit to the upper limit. The results obtained for each procedure are stored in a matrix [X_Values] to be plotted further on the simulation. Refer to the M-file to see how the procedure works

***** Results *****

Each [X] solution vector is stored in the Matrix and the number of significant digits used increases with the rows

X_Values =

```
-1.116666666666667   -1.36961451247166   15.000000000000000
 0.475833333333333    0.40919384057971    4.47931034482759
 0.818583333333333    0.79586106960105    2.20203688411781
 0.980775000000000    0.97835694846332    1.12744117647059
 0.998024166666667    0.99777750909849    1.01308823367792
 0.999815916666667    0.99979285282706    1.00121990252979
 0.999981741666667    0.99997945056601    1.00012100891213
 0.999998023333333    0.9999977547170     1.00001309998822
```

The exact solution to the system of equations with default number of significant digits in Matlab is:

X_exact =

1 1 1

The graphs also demonstrate the effect that the number of significant digits has on the solution. Each element of the solution vector is plotted as a function of the number of significant digits used.

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