Simulation of Gauss-Seidel Method

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NOTE: This worksheet demonstrates the use of Matlab to illustrate the Gauss-Seidel Method, an iterative technique used in solving a system of simultaneous linear equations.

*************************** Introduction *****************************

Gauss-Seidel method is used to solve a set of simultaneous linear equations,
[A] [X] = [RHS],where [A]nxn is the square coefficient matrix, [X]nx1 is the solution vector, and [RHS]nx1 is the right hand side array.
The equations can be rewritten as:

```
        x[i] = [rhs[i]-sum(A[i,j]*X*[j])[i<>j]/A[i,j]]                              (1.1)
```

In certain cases, such as when a system of equations is large, iterative methods of solving equations such as Gauss-Seidel method are more advantageous.
Elimination methods, such as Gaussian Elimination, are prone to round-off errors for a large set of equations whereas iterative methods,
such as Gauss-Seidel method, allow the user to control round-off error.
Also if the physics of the problem are well known, initial guesses needed in iterative methods can be made more judiciously for faster convergence.
Steps to apply Gauss-Seidel Method:
1) Make an initial guess for the solution vector [X]. This can be based on the physics of the problem.
(Note: To begin, the initial guess will be considered Xold).

```
               X[old] = [x1, x2, x3 .. xn]                                          (1.2)
```

2) Substitute the initial guess solution vector [X] in Equation (1.1).

```
        xi = [RHS[i]-sum(A[i,j]*X[j])[i<>j]/A[i,j]]                                 (1.3)
```

3) The new xi guess that is obtained will replace the previous guess in the [X] vector.

```
               Xold = [xnew, x2, x3 .. xn]                                          (1.4)
```

[X] will then be used to calculate the next xi value by repeating Step 2.
This will be repeated n times until the new solution vector [X] is complete.

```
               Xnew = [xnew, x2new, x3new .. xnnew]                                 (1.5)
```

4) At this point, the first iteration is completed and the absolute relative approximate error (abs_ea) is calculated by comparing the new guess [X] with the previous guess [Xold].

```
               abs_ea_i = 100*|xi new - xi old] /[xi new]|                          (1.6)
```

The maximum of these errors is the absolute relative approximate error at the end of the iteration.

5) Repeat Steps 1-4, replacing the new solution vector with the old solution vector in Step 1. Repeat until you have conducted either the maximum number of iterations or met the pre-specified tolerance.

$$Xold = Xnew \qquad (1.7)$$

A simulation of Gauss-Seidel method follows.

********************************** Input Data ******************************************

```
n = number of equations
[A] = nxn coefficient matrix
[RHS] = nx1 right hand side array
[Xold] = nx1 initial guess of the solution vector
maxit = maximum number of iterations
```

NOTE: These are the default values. Input data can be changed at the beginning of the M-file

```
n= 4

A =

    10     3     4     5
     2    24     7     4
     2     2    34     3
     2     5     2    12


RHS =

    22    32    41    18


X =

     1    23     4    50


maxit =

     5
```

*********************************** Iterations *******************************************

Below, "maxit" iterations are conducted and values of the previous approximations, present approximations,absolute relative percentage approximate error, and maximum absolute relative percentage approximate error are calculated at the end of each iteration.

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Iteration number1

Previous iteration values of the solution vector

```
Xold =

     1    23     4    50

----------------------New iterative values of the solution vector-----------------------

X =

  Columns 1 through 3

 -31.30000000000000  -5.55833333333333  -1.03774509803922

  Column 4

   9.20559640522876

---------------------Absolute relative percentage approximate error---------------------

abs_e =

  1.0e+002 *

  Columns 1 through 3

   1.03194888178914   5.13793103448276   4.85451110061408

  Column 4

   4.43147861355296

-----------------Maximum absolute relative percentage approximate error-----------------

Max_abs_ea =

    5.137931034482758e+002

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Iteration number2
Previous iteration values of the solution vector

Xold =

  Columns 1 through 3

 -31.30000000000000  -5.55833333333333  -1.03774509803922

  Column 4

   9.20559640522876
```

```
----------------------New iterative values of the solution vector----------------------

X =

  Columns 1 through 3

  -0.32020016339869   0.12842626633987   0.40490466407151

  Column 4

   1.43237163891292

---------------------Absolute relative percentage approximate error---------------------

abs_e =

  1.0e+003 *

  Columns 1 through 3

   9.67513554889327   4.42803467058964   0.35629368839672

  Column 4

   0.54268211930077

-----------------Maximum absolute relative percentage approximate error-----------------

Max_abs_ea =

    9.675135548893273e+003

=========================================================================================
=========================================================================================

Iteration number3
Previous iteration values of the solution vector

Xold =

  Columns 1 through 3

  -0.32020016339869   0.12842626633987   0.40490466407151

  Column 4

   1.43237163891292

----------------------New iterative values of the solution vector----------------------

X =
```

```
  Columns 1 through 3

   1.28332443501298   0.86956383024258   0.95285613390442

  Column 4

   0.76498497591270

---------------------Absolute relative percentage approximate error---------------------

abs_e =

  1.0e+002 *

  Columns 1 through 3

   1.24950835086021   0.85230955811025   0.57506212148483

  Column 4

   0.87241800037180

-----------------Maximum absolute relative percentage approximate error-----------------

Max_abs_ea =

    1.249508350860206e+002

=========================================================================================
=========================================================================================

Iteration number4
Previous iteration values of the solution vector

Xold =

  Columns 1 through 3

   1.28332443501298   0.86956383024258   0.95285613390442

  Column 4

   0.76498497591270

----------------------New iterative values of the solution vector----------------------

X =

  Columns 1 through 3

   1.17549590940911   0.82996147250834   1.02041559730668
```

```
  Column 4

   0.78819746866889

---------------------Absolute relative percentage approximate error---------------------

abs_e =

  Columns 1 through 3

   9.17302431601522   4.77158989254654   6.62077917865818

  Column 4

   2.94500980768190

-----------------Maximum absolute relative percentage approximate error-----------------

Max_abs_ea =

   9.17302431601522

=========================================================================================
=========================================================================================

Iteration number5
Previous iteration values of the solution vector

Xold =

  Columns 1 through 3

   1.17549590940911   0.82996147250834   1.02041559730668

  Column 4

   0.78819746866889

----------------------New iterative values of the solution vector----------------------

X =

  Columns 1 through 3

   1.14874658499038   0.80861699059154   1.02119648361263

  Column 4

   0.80141907581969

---------------------Absolute relative percentage approximate error---------------------
```

```
abs_e =

  Columns 1 through 3

   2.32856617536377   2.63962817565631   0.07646778249690

  Column 4

   1.64977445006191

-----------------Maximum absolute relative percentage approximate error-----------------

Max_abs_ea =

   2.63962817565631

>>
```