LU Decomposition Method
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NOTE: This worksheet demonstrates the use of Matlab to illustrate LU Decomposition method, a technique used in solving a system of simultaneous linear equations.

**************************************Introduction***************************************

When solving multiple sets of simultaneous linear equations with the same coefficient matrix but different right hand sides,LU Decomposition is advantageous over other numerical methods in that it proves to be numerically more efficient in computational time than other techniques.
In this worksheet, the reader can choose a system of equations and see how each step of LU decomposition method is conducted.

LU Decomposition method is used to solve a set of simultaneous linear equations, [A] [X] = [C], where [A]nxn is a non-singular square coefficient matrix, [X]nx1 is the solution vector,and [C]nx1 is the right hand side array.
When conducting LU decomposition method,one must first decompose the coefficient matrix [A]nxn into a lower triangular matrix [L]nxn, and upper triangular matrix [U]nxn.
These two matrices can then be used to solve for the solution vector [X]nx1 in the following sequence:
Recall that
[A] [X] = [C].
Knowing that
[A] = [L] [U]
then first solving with forward substitution
[L] [Z] = [C]
and then solving with back substitution
[U] [X] = [Z]
gives the solution vector [X].
**************************************Input Data***********************************
Below are the input parameters to begin the simulation.
Input Parameters:
n = number of equations
[A] = nxn coefficient matrix
[RHS] = nx1 right hand side array
n=6

```
A =

 12   0.12346         3       6.7         5         6
  15.053e+009         1         9         7         8
 13        12         4         8         4         6
5.6         3         7     1.003         7         4
  1         2         3         4         5         6
  6         7         5         6         7         5
```

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RHS =

    22
7e-007
29.001
 5.301
     9
    90

************************************************************************************
************************* LU Decomposition Method ********************************
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The following sections divide LU Decomposition method into 3 steps:

1.) Finding the LU decomposition of the coefficient matrix [A]nxn
2.) Forward substitution
3.) Back substitution

-------------------------------Finding the LU Decomposition-------------------------

How does one decompose a non-singular matrix [A], that is how do you find [L] and [U]?
This worksheet decomposes the coefficient matrix [A] into a lower triangular matrix [L] and upper triangular matrix [U], given [A] = [L][U].
 For [U], the elements of the matrix are exactly the same as the coefficient matrix one obtains at the end of forward elimination steps in Naïve Gauss Elimination.
 For [L], the matrix has 1 in its diagonal entries. The non-zero elements are multipliers that made the corresponding elements zero in the upper triangular matrix during forward elimination.

```
L =

       1             0             0             0             0             0
0.083333             1             0             0             0             0
  1.0833    2.3484e-009            1             0             0             0
 0.46667     5.823e-010       7.4667             1             0             0
0.083333    3.9377e-010       3.6667     -0.094267             1             0
     0.5    1.3731e-009       4.6667       0.10587       0.84684             1


U =

12       0.12346             3           6.7             5             6
 0     5.053e+009         0.75        8.4417        6.5833           7.5
 0             0          0.75       0.74167       -1.4167          -0.5
 0             0             0       -7.6614        15.244        4.9333
 0             0             0             0        11.215        7.7984
 0             0             0             0             0        -2.793
```

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------------------------------- Forward Substitution---------------------------------

Now that the [L] and [U] matrices have been formed, the forward substitution step,
[L] [Z] = [C], can be conducted, beginning with the first equation as it has only one
unknown,

z[1] = c[1]/l[1, 1]

Subsequent steps of forward substitution can be represented by the following formula:

z[i] = (c[i]-(Sum(l[i, j]*z[j], j = 1 .. i-1))[i = 2 .. n])/l[i, i]

Z =

           22
      -1.8333
       5.1677
      -43.551
      -15.887
       72.949


----------------------------------Back Substitution-------------------------------------

Now that [Z] has been calculated, it can be used in the back substitution step,
[U] [X] = [Z], to solve for solution vector [X]nx1, where [U]nxn is the upper triangular
matrix calculated in Step 2.1, and [Z]nx1 is the right hand side array.
Back substitution begins with the nth equation as it has only one unknown:

xn = zn/U(n, n)

The remaining unknowns are solved for using the following formula:

xi = (zi-(Sum(U[i, j]*X[j], j = i+1 .. n))[i = n-1 .. 1])/U[i, i]

X =

      -4.2637
 -2.0353e-008
     -0.83104
       22.186
       16.745
      -26.119

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