Concepts of Approximate Error: Approximate Error, Absolute Approximate Error, Relative Approximate Error, and Absolute Relative Approximate Error

Sean Rodby, Luke Snyder, Autar Kaw

University of South Florida

United States of America

kaw@eng.usf.edu

Website: http://numericalmethods.eng.usf.edu

Version: Maple 12; Last Revised: August 28, 2008

Introduction

The following worksheet demonstrates how to calculate different types of error, such as approximate error, absolute approximate error, relative approximate error, absolute relative approximate error, and the least number of significant digits associated with these calculations. The concept is demonstrated using an example of a Maclaurin series. The user will choose which function to perform the calculations with in the *Inputs* section of the program. The choices are given as 1 for e^x, 2 for $sin(x)$, and 3 for $cos(x)$. The *true value* of these functions will be assumed as given by the Maple commands for these functions.

Initialization

```
> restart; with(plots) : with(Statistics) :
```

Section 1: Input

This is the only section where the user interacts with the program.

Pick the function of your desire by choosing an integer: 1 for e^x, 2 for $sin(x)$, and 3 for $cos(x)$

```
> funcchoice := 1
```

$$funcchoice := 1 \quad \textbf{(3.1)}$$

Maximum number of terms to use in the Maclaurin series

```
> n := 15
```

$$n := 15 \quad \textbf{(3.2)}$$

Value at which the function is calculated

```
> xv := 3.14159
```

$$xv := 3.14159 \quad \textbf{(3.3)}$$

This is the end of the user section. All information must be entered before proceeding to the next section. **RE-EXECUTE THE PROGRAM.**

Section 2: Procedure

First, determine which function will be used in the calculations, based on the users input.
Once the function is determined, the value is calculated using a Maclaurin series in a repetitive loop.

```
> sumprevious := 0 :
  for i to n do
   if funcchoice = 1 then
  sumpresent[i] := sumprevious + xv^(i-1)/(i-1)!;
  f := x -> e^x;
  end if;

   if funcchoice = 2 then
  sumpresent[i] := sumprevious + (-1)^(i-1) xv^(2i-1)/(2i-1)!;
  f := x -> sin(x)
  end if;
   if funcchoice = 3 then
  sumpresent[i] := sumprevious + (-1)^(i+1) xv^(2i-2)/(2i-2)!;
  f := x -> cos(x)
  end if;
  sumprevious := sumpresent[i];
  H[i] := i end do:
```

Section 3: Calculation

Using Maple to calculate approximate error, absolute approximate error, relative approximate error, and absolute relative approximate error for each term. Once these error values are calculated, determining the least number of significant figures guaranteed correct.

```
> for i from 2 to n do
  ApproxError[i] := sumpresent[i] - sumpresent[i-1];
  AbsApproxError[i] := |sumpresent[i]-sumpresent[i-1]|;
  RelApproxError[i] := (sumpresent[i] - sumpresent[i-1])·100/sumpresent[i];
  AbsRelApproxError[i] := 100 |(sumpresent[i]-sumpresent[i-1])/sumpresent[i]|;
  SigDigits_i := floor(2 - log10(AbsRelApproxError[i]/(100·0.5)));
  if SigDigits[i] < 0 then
  SigDigits[i] := 0;
  end if;
  end do:
```

Section 4: Spreadsheet

This table shows the approximate value, approximate error, absolute approximate error, relative approximate error, absolute relative approximate error, and the number of significant figures, all as a function of the number of the number of terms used.

```
> with(Spread) :
  tableoutput := CreateSpreadsheet("Table of Values") :
```

```
SetCellFormula(tableoutput, 1, 1, "Terms Used");
SetCellFormula(tableoutput, 1, 2, "Approximate Value");
SetCellFormula(tableoutput, 1, 3, "Approximate Error");
SetCellFormula(tableoutput, 1, 4, "Abs Approximate Error");
SetCellFormula(tableoutput, 1, 5, "Rel Approximate Error");
SetCellFormula(tableoutput, 1, 6, "Abs Rel Approximate Error") :
SetCellFormula(tableoutput, 1, 7, "Sig Digits")

for i from 2 to n do
SetCellFormula(tableoutput, i + 1, 1, i);
SetCellFormula(tableoutput, i + 1, 2, evalf(sumpresent[i]));
SetCellFormula(tableoutput, i + 1, 3, evalf(ApproxError[i]));
SetCellFormula(tableoutput, i + 1, 4, evalf(AbsApproxError[i]));
SetCellFormula(tableoutput, i + 1, 5, evalf(RelApproxError[i]));
SetCellFormula(tableoutput, i + 1, 6, evalf(AbsRelApproxError[i]));
SetCellFormula(tableoutput, i + 1, 7, evalf(SigDigits[i]));
end do;
EvaluateSpreadsheet(tableoutput)
```

Table of Values

	A	B	C	D	E	F	G
1	"Terms Used"	"Approximate Value"	"Approximate Error"	"Abs Approximate Error"	"Rel Approximate Error"	"Abs Rel Approximate Error"	"Sig Digits"
2							
3	2	4.14159	3.14159	3.14159	75.85468383	75.85468383	1.
4	3	9.076383864	4.934793864	4.934793864	54.36960289	54.36960289	1.
5	4	14.24408355	5.167699686	5.167699686	36.27962212	36.27962212	2.
6	5	18.30278196	4.05869841	4.05869841	22.17530875	22.17530875	2.
7	6	20.85293523	2.55015327	2.55015327	12.22922932	12.22922932	2.
8	7	22.18819123	1.33525600	1.33525600	6.017867730	6.017867730	2.
9	8	22.78745222	0.59926099	0.59926099	2.629784955	2.629784955	3.
10	9	23.02278126	0.23532904	0.23532904	1.022157303	1.022157303	3.
11	10	23.10492652	0.08214526	0.08214526	0.3555313622	0.3555313622	4.
12	11	23.13073319	0.02580667	0.02580667	0.1115687505	0.1115687505	4.
13	12	23.13810355	0.00737036	0.00737036	0.03185377740	0.03185377740	5.
14	13	23.14003310	0.00192955	0.00192955	0.008338579256	0.008338579256	5.
15	14	23.14049940	0.00046630	0.00046630	0.002015081835	0.002015081835	6.
16	15	23.14060404	0.00010464	0.00010464	0.0004521921719	0.0004521921719	7.

(6.1)

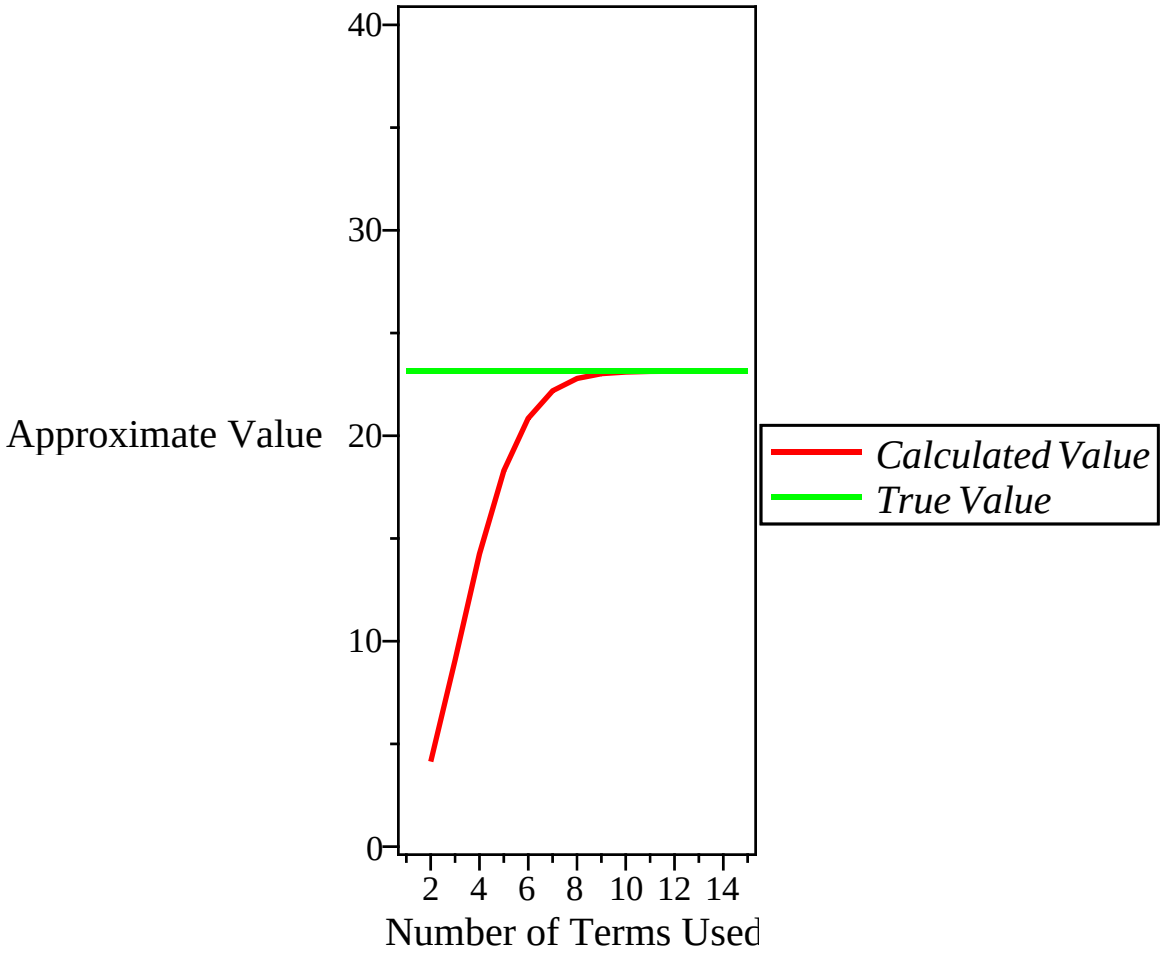
Section 5: Graphs

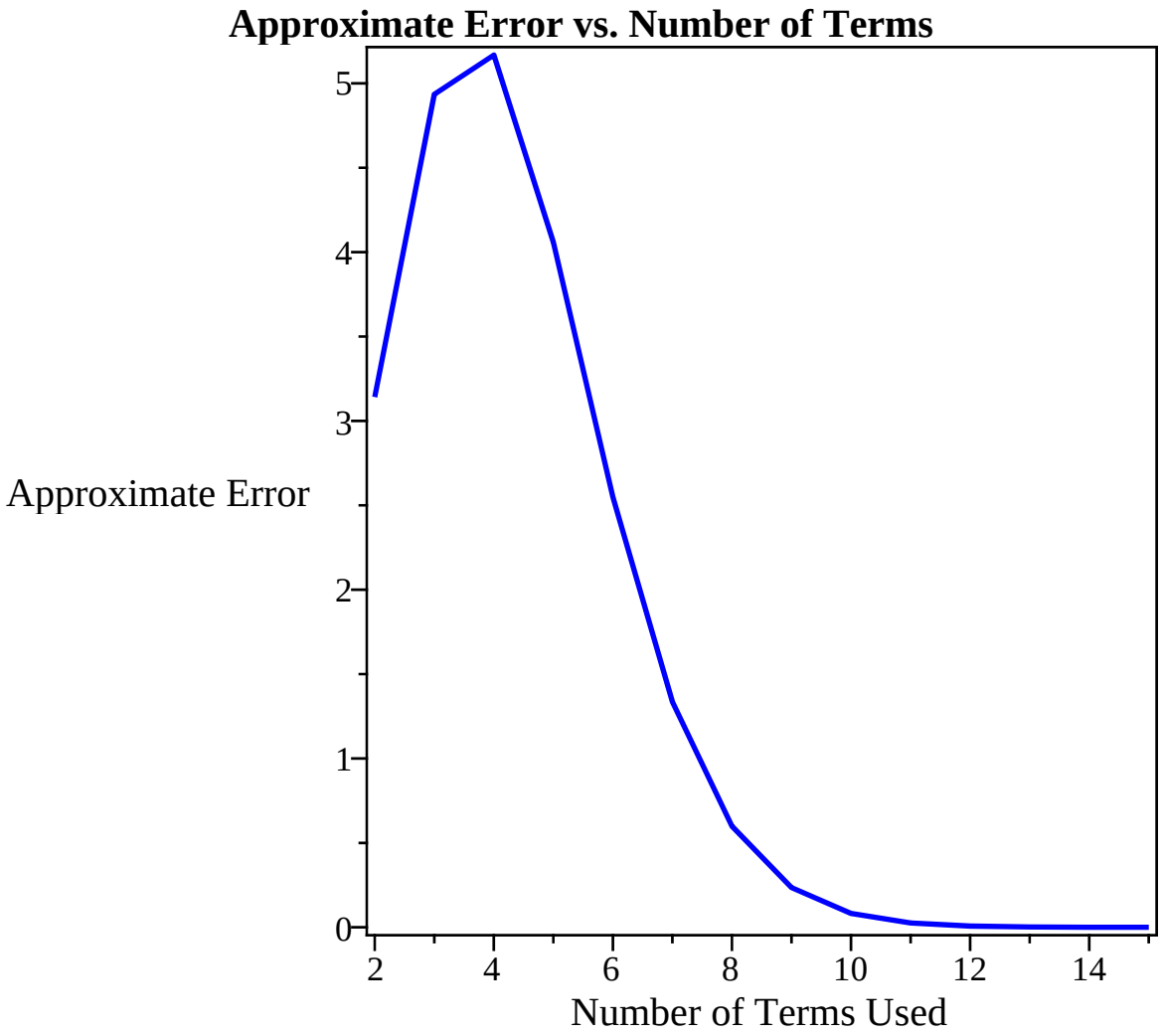
The following graphs show the calculated value of f(x) using Maclaurin series as a function, approximate error, absolute approximate error, absolute relative approximate error, relative approximate error, and least number of significant digits as a function of step size. Each graph displays the error results of each of the methods of approximation.

```
> data := [seq([H[i], sumpresent[i]], i = 2 .. n)] :
plot1 := pointplot(data, connect = true, legend = Calculated Value, color = red) :
plot2 := plot(f(xv), x = 1 .. n, legend = [True Value], legendstyle = [location = right], labels
      = ["Number of Terms Used", "Approximate Value"], titlefont = [TIMES, BOLD, 12],
      labelfont = [TIMES, ROMAN, 12], color = green) :
display(plot1, plot2, view = [1 .. n, 0 .. f(xv) + 0.75 f(xv)], thickness = 2, axes = BOXED,
```

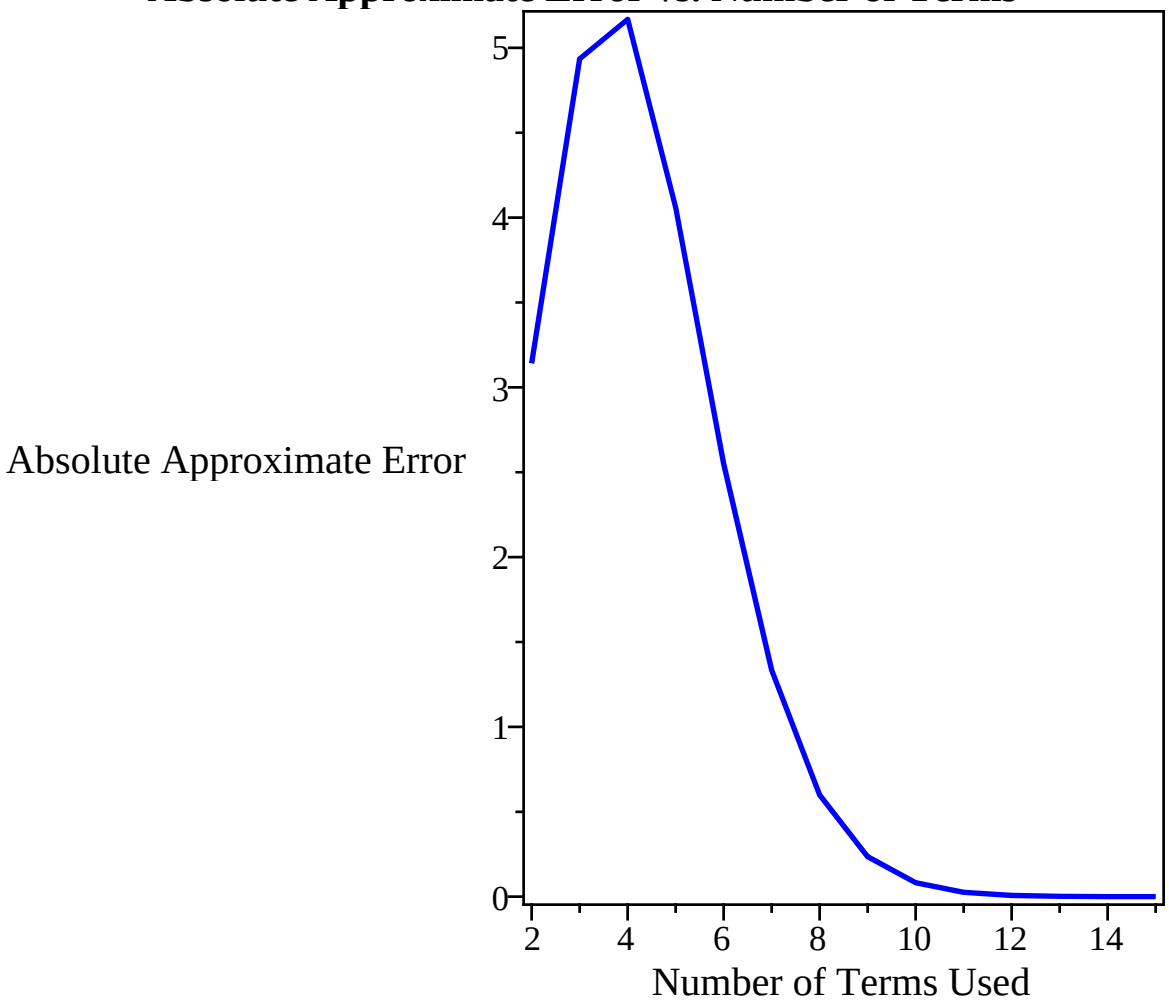
```
    title = "Calculated Value of f(x) Using Maclaurin Series vs. Number of Terms");


data := [seq([H_i, ApproxError_i], i = 2 ..n)]:
pointplot(data, connect = true, color = blue, title
    = "Approximate Error vs. Number of Terms", axes = BOXED, labels
    = ["Number of Terms Used", "Approximate Error"], thickness = 2, titlefont = [TIMES,
    BOLD, 12], labelfont = [TIMES, ROMAN, 12]);

data1 := [seq([H_i, AbsApproxError_i], i = 2 ..n)]:
pointplot(data1, connect = true, color = blue, title
    = "Absolute Approximate Error vs. Number of Terms", axes = BOXED, labels
    = ["Number of Terms Used", "Absolute Approximate Error"], thickness = 2, titlefont
    = [TIMES, BOLD, 12], labelfont = [TIMES, ROMAN, 12]);

data2 := [seq([H_i, RelApproxError_i], i = 2 ..n)]:
pointplot(data2, connect = true, color = blue, title
    = "Percentage Relative Approximate Error vs. Number of Terms", axes = BOXED,
    labels = ["Number of Terms Used", "Percentage Relative Approximate Error"],
    thickness = 2, titlefont = [TIMES, BOLD, 12], labelfont = [TIMES, ROMAN, 12]);

data3 := [seq([H_i, AbsRelApproxError_i], i = 2 ..n)]:
pointplot(data3, connect = true, color = blue, title
    = "Percentage Absolute Relative Approximate Error vs. Number of Terms", axes
    = BOXED, labels = ["Number of Terms Used",
    "Percentage Absolute Relative Approximate Error"], thickness = 2, titlefont = [TIMES,
    BOLD, 12], labelfont = [TIMES, ROMAN, 12]);

data4 := Array([seq(SigDigits_i, i = 2 ..n)]):
ColumnGraph([data4], title
    = "Number of Significant Digits at Least Correct vs. Number of Terms Used", legend
    = ["Sig Digits"], titlefont = [TIMES, BOLD, 12], labelfont = [TIMES, ROMAN, 12],
    labels = ["Number of Terms Used", "Number of Significant Digits at Least Correct"],
    legendstyle = [location = right], offset = 2)
```

Calculated Value of f(x) Using Maclaurin Series vs. Number of Terms

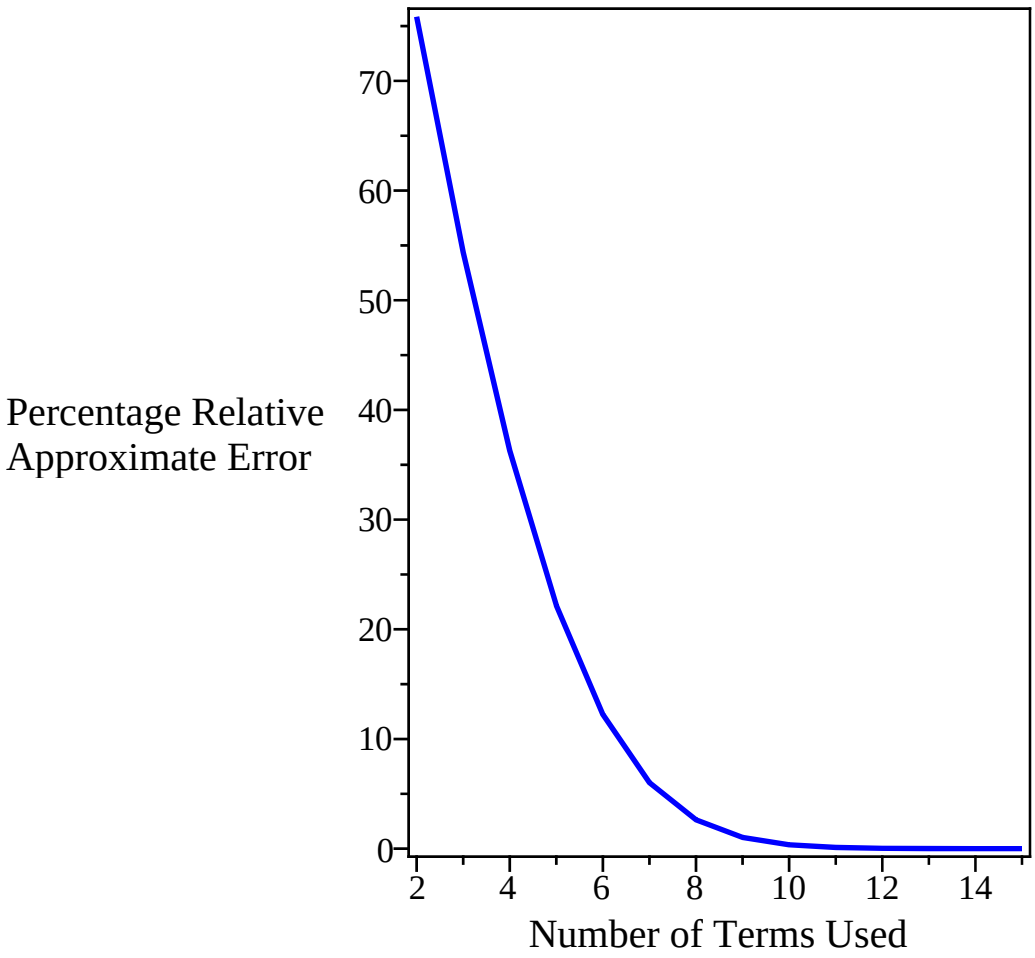



Approximate Error vs. Number of Terms
Approximate Error
0
1
2
3
4
5
2
4
6
8
10
12
14
Number of Terms Used

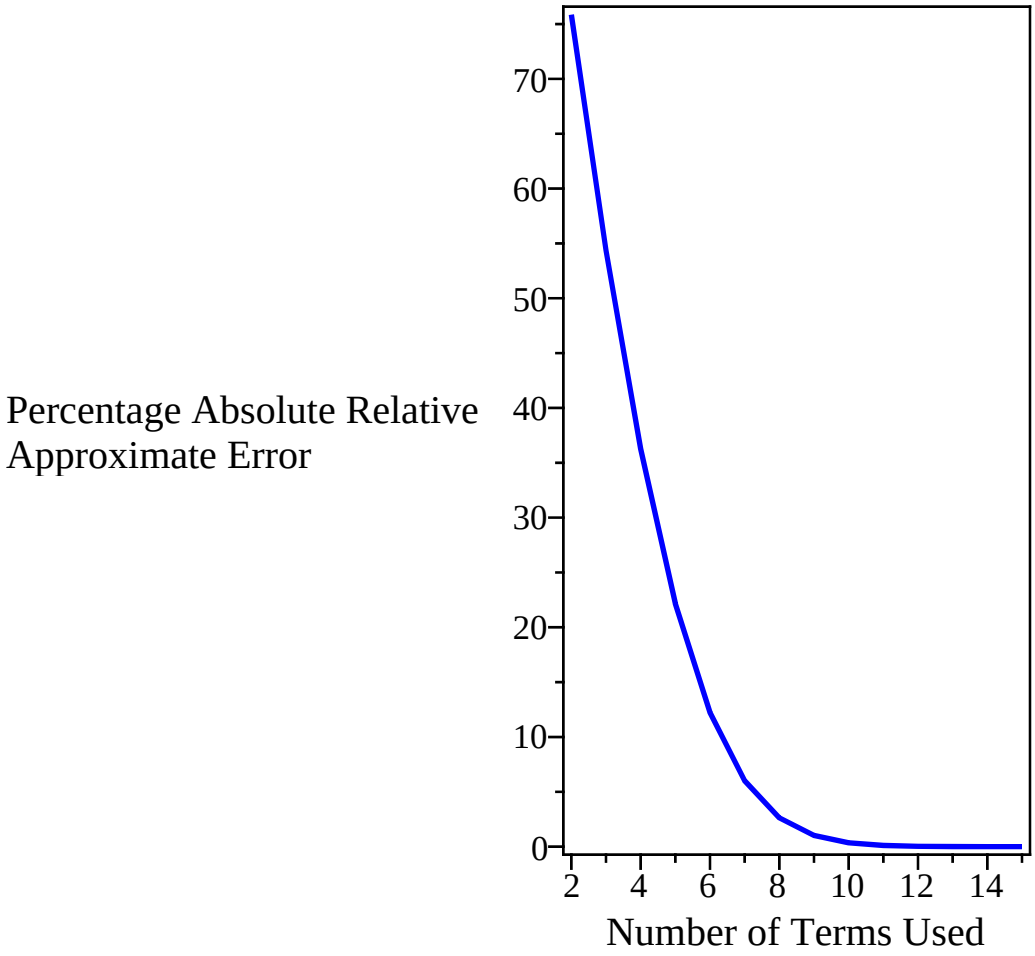
Absolute Approximate Error vs. Number of Terms

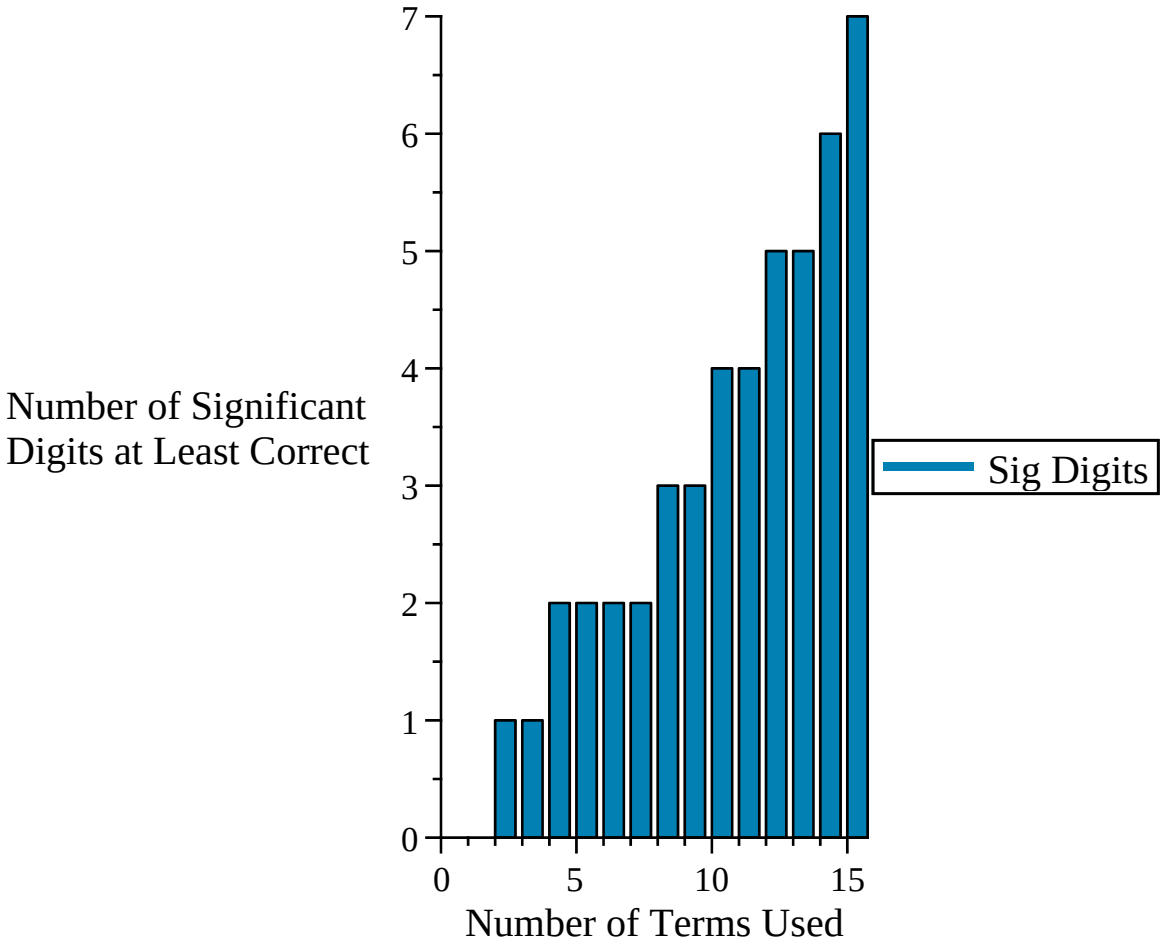


Percentage Relative Approximate Error vs. Number of Terms



Percentage Absolute Relative Approximate Error vs. Number of Terms





Conclusion

This worksheet shows how the number of terms taken in a Maclaurin series affects the accuracy of the calculated answer through the analysis of error. Note that though approximate error shows the magnitude of the error, it does not indicate how bad the error really is. Hence, relative approximate error is used here to give a more complete picture of the state of error.

References

Measuring Errors.
See: http://numericalmethods.eng.usf.edu/mws/gen/01aae/mws_gen_aae_txt_measuringerror.pdf