The Quadratic Formula as a Way to Show the Subtraction of Small Numbers

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Introduction

The following worksheet illustrates the use of a quadratic equation solution for showing the effect of significant digits on round-off errors The user will enter the *a*, *b* and *c* values as given by the equation for the standard form of a quadratic equation: $ax^2 + bx + c = 0$, as well as the number of significant digits to be displayed in a table that will be created at the end of the program. Two variations of the quadratic equation solution will be used:

$$(A)\quad x1 = \frac{-b + \sqrt{b^2 - 4\,a\cdot c}}{2\,a}$$

$$x2 = \frac{-b - \sqrt{b^2 - 4\,a\cdot c}}{2\,a}$$

$$(B)\quad x1 = \frac{2\,c}{-b - \sqrt{b^2 - 4\,a\cdot c}}$$

$$x2 = \frac{2\,c}{-b + \sqrt{b^2 - 4\,a\cdot c}}$$

Initialization

```
> restart : with(Statistics) :
```

Section 1: Input

This is the only section where the user interacts with the program.

The quadratic formula is derived from the standard form of a quadratic equation: $ax^2 + bx + c = 0$.

Enter coefficient *a*

```
> a := 0.001
```

$$a := 0.001 \quad \textbf{(3.1)}$$

Enter coefficient *b*

```
> b := -4.94627
```

(3.2)

$$b := -4.94627 \tag{3.2}$$

Enter coefficient *c*

```
> c := 0.002
```

$$c := 0.002 \tag{3.3}$$

Enter range of significant digits to be used.

```
> sig_low := 7;
  sig_high := 10
```

$$sig\_low := 7$$

$$sig\_high := 10 \tag{3.4}$$

This is the end of the user section. All information must be entered before proceeding to the next section. **RE-EXECUTE THE PROGRAM.**

Section 2: Simulation

The following calculations will be performed inside a loop so that the number of significant digits used can be varied as specified by the user. The *digits* command will be used to control the number of digits Maple uses when calculating.

Variation 1:

$$x1a = \frac{-b + \sqrt{b^2 - 4\,a \cdot c}}{2\,a}$$

$$x2a = \frac{-b - \sqrt{b^2 - 4\,a \cdot c}}{2\,a}$$

```
> j := sig_low :
   for i from sig_low to sig_high do

   Digits := i;

   x1a[j] := (-b + sqrt(b^2 - 4*a*c))/(2*a);

   x2a[j] := evalf((-b - sqrt(b^2 - 4*a*c))/(2*a));

  j := j + 1;

  end do:
```

Variation 2:

$$x1b = \frac{2\,c}{-b - \sqrt{b^2 - 4\,a \cdot c}}$$

$$x2b = \frac{2\,c}{-b + \sqrt{b^2 - 4\,a \cdot c}}$$

```
> j := sig_low :
   for i from sig_low to sig_high do

   Digits := i;
```

$$x1b[j] := \frac{2\,c}{-b - \sqrt{b^2 - 4\,a \cdot c}};$$

$$x2b[j] := \frac{2\,c}{-b + \sqrt{b^2 - 4\,a \cdot c}};$$

$j := j + 1;$

end do:

Section 3: Spreadsheet

This table shows the values of x1a, x2a, x1b, and x2b and the number of significant digits used in their calculation.

```
> n := 1 :
  with(Spread) :
  tableoutput := CreateSpreadsheet("Table of Values") :
  SetCellFormula(tableoutput, 1, 1, "Sig Digits");
  SetCellFormula(tableoutput, 1, 2, "x1a");
  SetCellFormula(tableoutput, 1, 3, "x1b");
  SetCellFormula(tableoutput, 1, 4, "x2a");
  SetCellFormula(tableoutput, 1, 5, "x2b");
  for j from sig_low to sig_high do
  SetCellFormula(tableoutput, n + 1, 1, j);
  SetCellFormula(tableoutput, n + 1, 2, x1a[j]);
  SetCellFormula(tableoutput, n + 1, 3, x1b[j]);
  SetCellFormula(tableoutput, n + 1, 4, x2a[j]);
  SetCellFormula(tableoutput, n + 1, 5, x2b[j]);
  n := n + 1;
  end do:
  EvaluateSpreadsheet(tableoutput)
```

Table of Values

	A	B	C	D	E	
1	"Sig Digits"	"x1a"	"x1b"	"x2a"	"x2b"	
2	7	4946.270	4000.000	0.0005000000	0.0004043452	
3	8	4946.2696	5000.0000	0.00040000000	0.00040434512	
4	9	4946.26960	4938.27160	0.000405000000	0.000404345126	
5	10	4946.269596	4944.375772	0.0004045000000	0.0004043451254	
6						
7						
8						
9						
10						

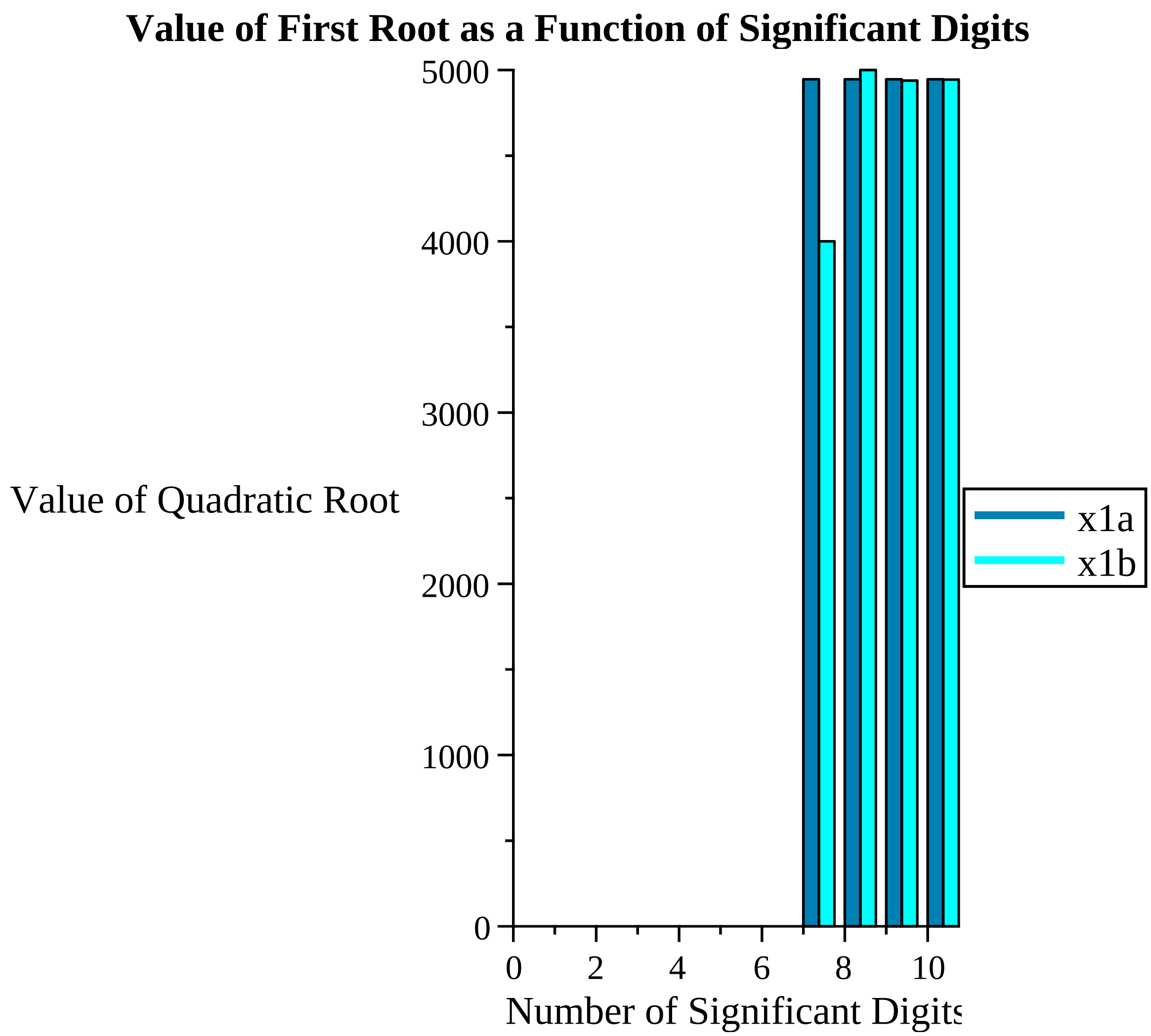
(5.1)

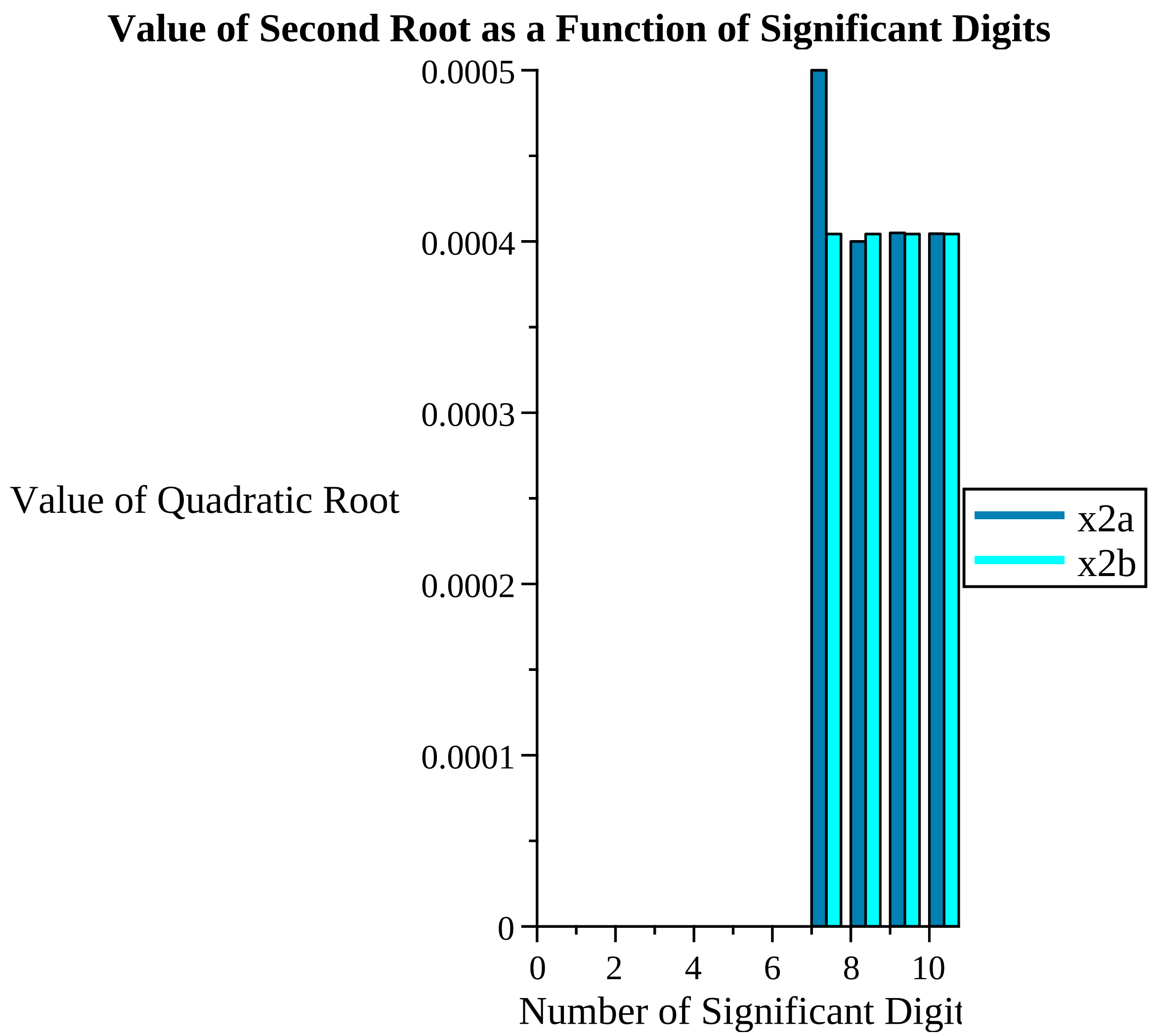
Section 4: Graphs

These bar graphs show the values of x1 and x2 for both variations of the quadratic function.

```
> Data1 := Array([seq(x1a[i], i = sig_low ..sig_high)]) :
  Data2 := Array([seq(x1b[i], i = sig_low ..sig_high)]) :
  ColumnGraph([Data1, Data2], offset = sig_low, title
      = "Value of First Root as a Function of Significant Digits", legend = ["x1a", "x1b"],
      titlefont = [TIMES, BOLD, 12], labelfont = [TIMES, ROMAN, 12], labels
      = ["Number of Significant Digits", "Value of Quadratic Root"], legendstyle = [location
      = right]);

  Data3 := Array([seq(x2a[i], i = sig_low ..sig_high)]) :
  Data4 := Array([seq(x2b[i], i = sig_low ..sig_high)]) :
  ColumnGraph([Data3, Data4], offset = sig_low, title
      = "Value of Second Root as a Function of Significant Digits", legend = ["x2a", "x2b"],
      titlefont = [TIMES, BOLD, 12], labelfont = [TIMES, ROMAN, 12], labels
      = ["Number of Significant Digits", "Value of Quadratic Root"], legendstyle = [location
      = right]);
```





Conclusion

Subtraction of numbers that are nearly equal can result in unwanted inaccuracies. The number of significant digits used in calculations plays a large role in the creation of these inaccuracies and the magnitude of the round-off errors. Hence, when the accuracy of calculations is critical, it is necessary to understand possible sources of error and how they are best avoided.

References

Sources of Error. See: http://numericalmethods.eng.usf.edu/mws/gen/01aae/mws_gen_aae_txt_sourcesoferror.pdf
Propagation of Errors. See: http://numericalmethods.eng.usf.edu/mws/gen/01aae/mws_gen_aae_txt_propagationoferrors.pdf