

# Concepts of True Error: True Error, Absolute True Error, Relative True Error, and Absolute Relative True Error

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## Introduction

The following worksheet demonstrates how to calculate different definitions related to true error, such as true error, absolute true error, relative true error, and absolute relative true error. The concept is demonstrated using an example of a Maclaurin series. The user will choose which function to perform the calculation for in the Inputs section of the program. The choices are given as 1 for  $e^x$ , 2 for  $\sin(x)$ , and 3 for  $\cos(x)$ . The *true value* of these functions will be assumed as given by the Maple commands for these functions.

## Initialization

```
> restart : with(plots) :
```

## Section 1: Input

This is the only section where the user interacts with the program.

Pick the function of your desire by choosing an integer: 1 for  $e^x$ ; 2 for  $\sin(x)$ ; 3 for  $\cos(x)$

```
> funcchoice := 1  
funcchoice := 1 (3.1)
```

Maximum number of terms to use in the Maclaurin series

```
> n := 15  
n := 15 (3.2)
```

Value of x at which the function is calculated

```
> xv := 1.6  
xv := 1.6 (3.3)
```

This is the end of the user section. All information must be entered before proceeding to the next section. **RE-EXECUTE THE PROGRAM.**

## Section 2: Procedure

First, determine which function will be used in the calculations, based on the users input. Once the function is determined, the value is calculated using a Maclaurin series in a repetitive loop.

```
> sumprevious := 0 :
```

```

for  $i$  to  $n$  do
if  $funcchoice = 1$  then

 $sumpresent[i] := sumprevious + \frac{xv^{i-1}}{(i-1)!};$ 

 $f := x \rightarrow e^x$ 
end if;
if  $funcchoice = 2$  then

 $sumpresent[i] := sumprevious + \frac{(-1)^{i-1} xv^{2i-1}}{(2i-1)!};$ 

 $f := x \rightarrow \sin(x)$ 
end if;

if  $funcchoice = 3$  then

 $sumpresent[i] := sumprevious + \frac{(-1)^{i+1} xv^{2i-2}}{(2i-2)!};$ 

 $f := x \rightarrow \cos(x)$ 
end if;

 $sumprevious := sumpresent[i];$ 
 $H[i] := i;$ 
end do;

```

### Section 3: Calculation

Using Maple to calculate true error, absolute true error, relative true error, and absolute relative true error for each term.

Once these error values are calculated, determining if sufficient iterations have been performed by comparing the prespecified tolerance to the absolute relative true error.

```

> for  $i$  to  $n$  do

 $TrueError[i] := f(xv) - sumpresent[i];$ 

 $AbsTrueError[i] := |f(xv) - sumpresent[i]|;$ 

 $RelTrueError[i] := \frac{(f(xv) - sumpresent[i]) \cdot 100}{f(xv)};$ 

 $AbsRelTrueError[i] := \left| \frac{f(xv) - sumpresent[i]}{f(xv)} \right| \cdot 100;$ 

end do;

```

### Section 4: Spreadsheet

This table shows the true value, true error, absolute true error, relative true error, absolute relative true error, and if the prespecified tolerance has been met, all as a function of the number of the number of terms used.

```

>  $with(Spread) :$ 
 $tableoutput := CreateSpreadsheet("Table of Values") :$ 

```

```

SetCellFormula(tableoutput, 1, 1, "Terms Used");
SetCellFormula(tableoutput, 1, 2, "True Value");
SetCellFormula(tableoutput, 1, 3, "True Error");
SetCellFormula(tableoutput, 1, 4, "Abs True Error");
SetCellFormula(tableoutput, 1, 5, "Rel True Error");
SetCellFormula(tableoutput, 1, 6, "Abs Rel True Error") :

```

**for i to n do**

```

SetCellFormula(tableoutput, i + 1, 1, i);
SetCellFormula(tableoutput, i + 1, 2, evalf(f(xv)));
SetCellFormula(tableoutput, i + 1, 3, evalf(TrueError[i]));
SetCellFormula(tableoutput, i + 1, 4, evalf(AbsTrueError[i]));
SetCellFormula(tableoutput, i + 1, 5, evalf(RelTrueError[i]));
SetCellFormula(tableoutput, i + 1, 6, evalf(AbsRelTrueError[i]));
end do;
EvaluateSpreadsheet(tableoutput)

```

Table of Values						
	A	B	C	D	E	F
1	"Terms Used"	"True Value"	"True Error"	"Abs True Error"	"Rel True Error"	"Abs Rel True Error"
2	1	4.953032424	3.953032424	3.953032424	79.81034820	79.81034820
3	2	4.953032424	2.353032424	2.353032424	47.50690532	47.50690532
4	3	4.953032424	1.073032424	1.073032424	21.66415101	21.66415101
5	4	4.953032424	0.390365757	0.390365757	7.881348709	7.881348709
6	5	4.953032424	0.117299090	0.117299090	2.368227784	2.368227784
7	6	4.953032424	0.029917757	0.029917757	0.6040290965	0.6040290965
8	7	4.953032424	0.006616068	0.006616068	0.1335761092	0.1335761092
9	8	4.953032424	0.001289968	0.001289968	0.02604400475	0.02604400475
10	9	4.953032424	0.000224748	0.000224748	0.004537583863	0.004537583863
11	10	4.953032424	0.000035376	0.000035376	0.0007142291221	0.0007142291221
12	11	4.953032424	0.000005076	0.000005076	0.0001024826725	0.0001024826725
13	12	4.953032424	$6.69 \cdot 10^{-7}$	$6.69 \cdot 10^{-7}$	0.00001350687705	0.00001350687705
14	13	4.953032424	$8.1 \cdot 10^{-8}$	$8.1 \cdot 10^{-8}$	0.000001635361796	0.000001635361796
15	14	4.953032424	$9. \cdot 10^{-9}$	$9. \cdot 10^{-9}$	$1.817068662 \cdot 10^{-7}$	$1.817068662 \cdot 10^{-7}$
16	15	4.953032424	$1. \cdot 10^{-9}$	$1. \cdot 10^{-9}$	$2.018965180 \cdot 10^{-8}$	$2.018965180 \cdot 10^{-8}$

(6.1)

## Section 5: Graphs

The following graphs show the true value, calculated value, true error, absolute true error, relative true error, and absolute relative true error, all as a function of the number of terms.

```

> data := [seq([H[i], sumpresent[i]], i = 1 ..n) ] :
plot1 := pointplot(data, connect = true, legend = Calculated Value, color = red) :
plot2 := plot(f(xv), x = 1 ..n, legend = [True Value], legendstyle = [location = right], labels
= ["Number of Terms Used", "Approximate Value"], titlefont = [TIMES, BOLD, 12],
labelfont = [TIMES, ROMAN, 12], color = green) :
display(plot1, plot2, view = [1 ..n, 0 ..f(xv) + .75 f(xv)], thickness = 2, axes = BOXED, title

```

= "Calculated Value of f(x) Using Maclaurin Series vs. Number of Terms");

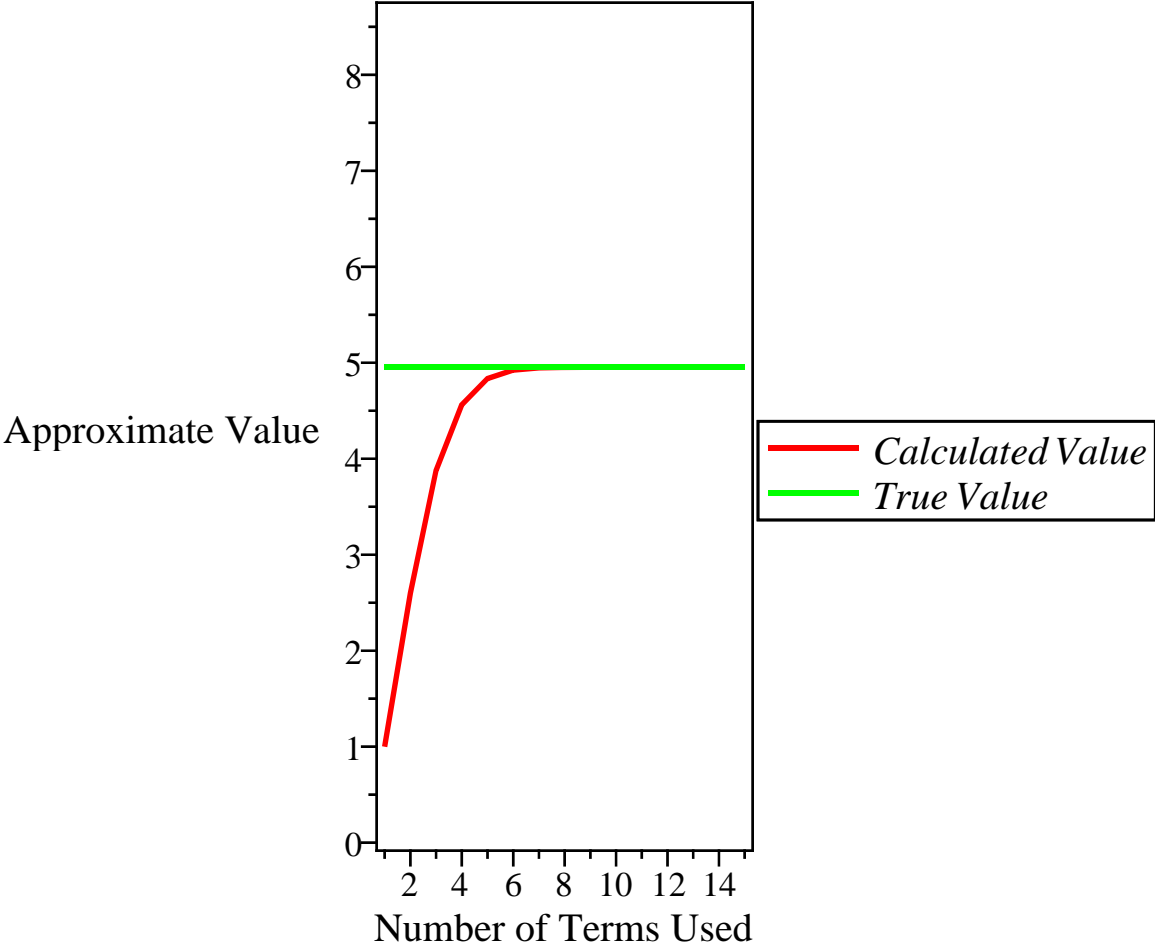
```
data := [seq([H[i], TrueError[i]], i = 1 ..n)] :  
pointplot(data, connect = true, color = blue, title = "True Error vs. Number of Terms", axes  
= BOXED, labels = ["Number of Terms Used", "True Error"], thickness = 2, titlefont  
= [TIMES, BOLD, 12], labelfont = [TIMES, ROMAN, 12]);
```

```
data1 := [seq([H[i], AbsTrueError[i]], i = 1 ..n)] :  
pointplot(data1, connect = true, color = blue, title  
= "Absolute True Error vs. Number of Terms", axes = BOXED, labels  
= ["Number of Terms Used", "Absolute True Error"], thickness = 2, titlefont  
= [TIMES, BOLD, 12], labelfont = [TIMES, ROMAN, 12]);
```

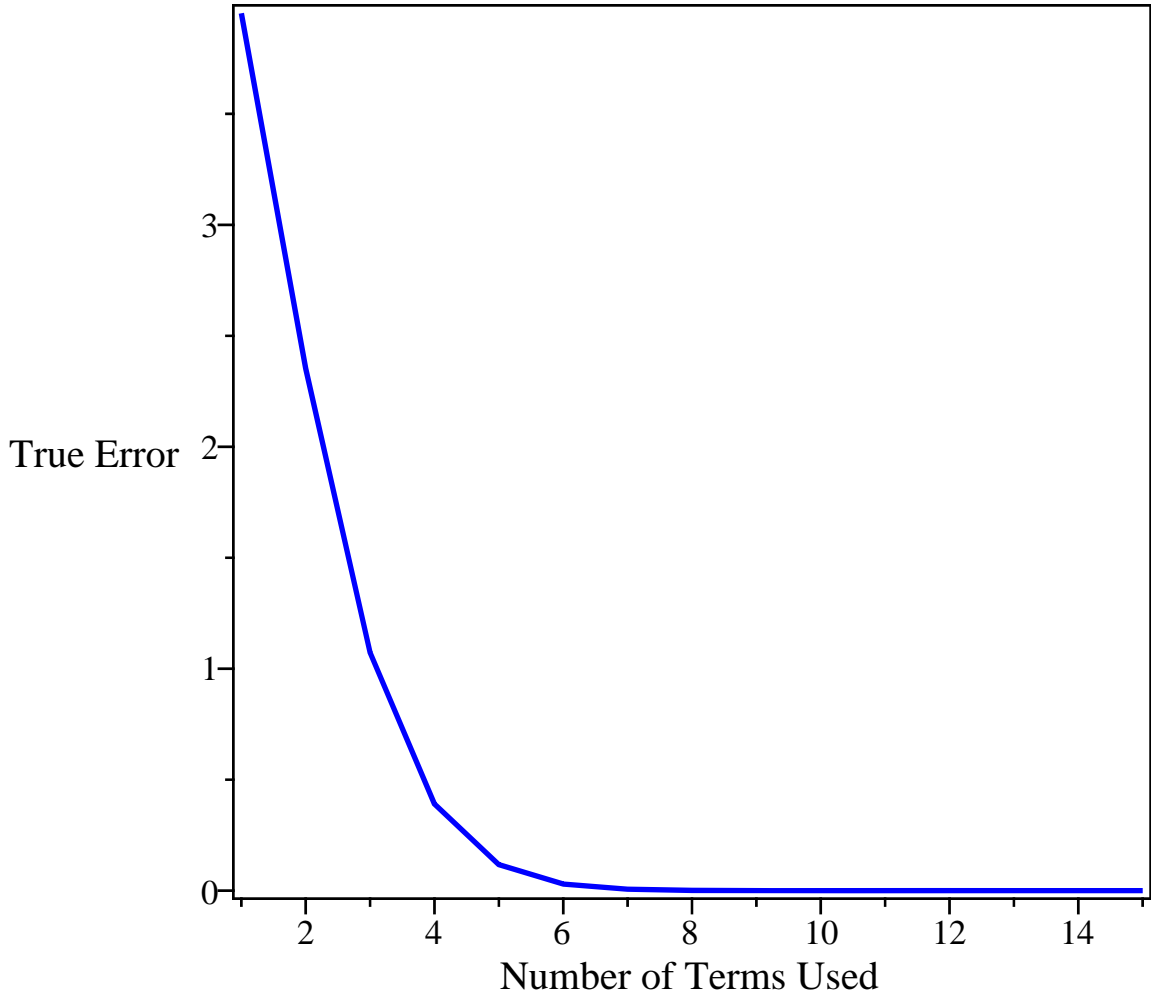
```
data2 := [seq([H[i], RelTrueError[i]], i = 1 ..n)] :  
pointplot(data2, connect = true, color = blue, title  
= "Percentage Relative True Error vs. Number of Terms", axes = BOXED, labels  
= ["Number of Terms Used", "Percentage Relative True Error"], thickness = 2, titlefont  
= [TIMES, BOLD, 12], labelfont = [TIMES, ROMAN, 12]);
```

```
data3 := [seq([H[i], AbsRelTrueError[i]], i = 1 ..n)] :  
pointplot(data3, connect = true, color = blue, title  
= "Percentage Absolute Relative True Error vs. Number of Terms", axes = BOXED,  
labels = ["Number of Terms Used", "Percentage Absolute Relative True Error"],  
thickness = 2, titlefont = [TIMES, BOLD, 12], labelfont = [TIMES, ROMAN, 12]);
```

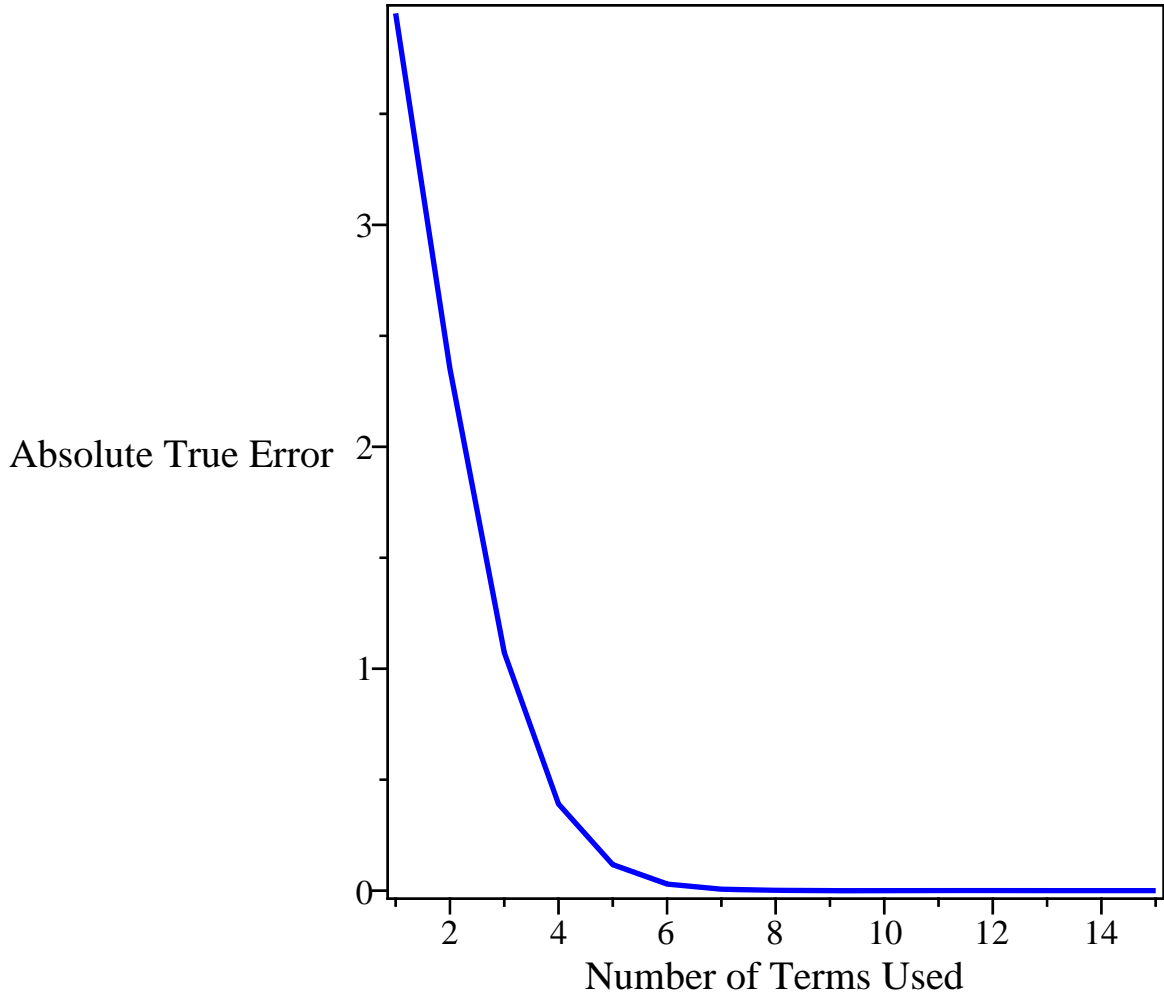
# Calculated Value of $f(x)$ Using Maclaurin Series vs. Number of Terms



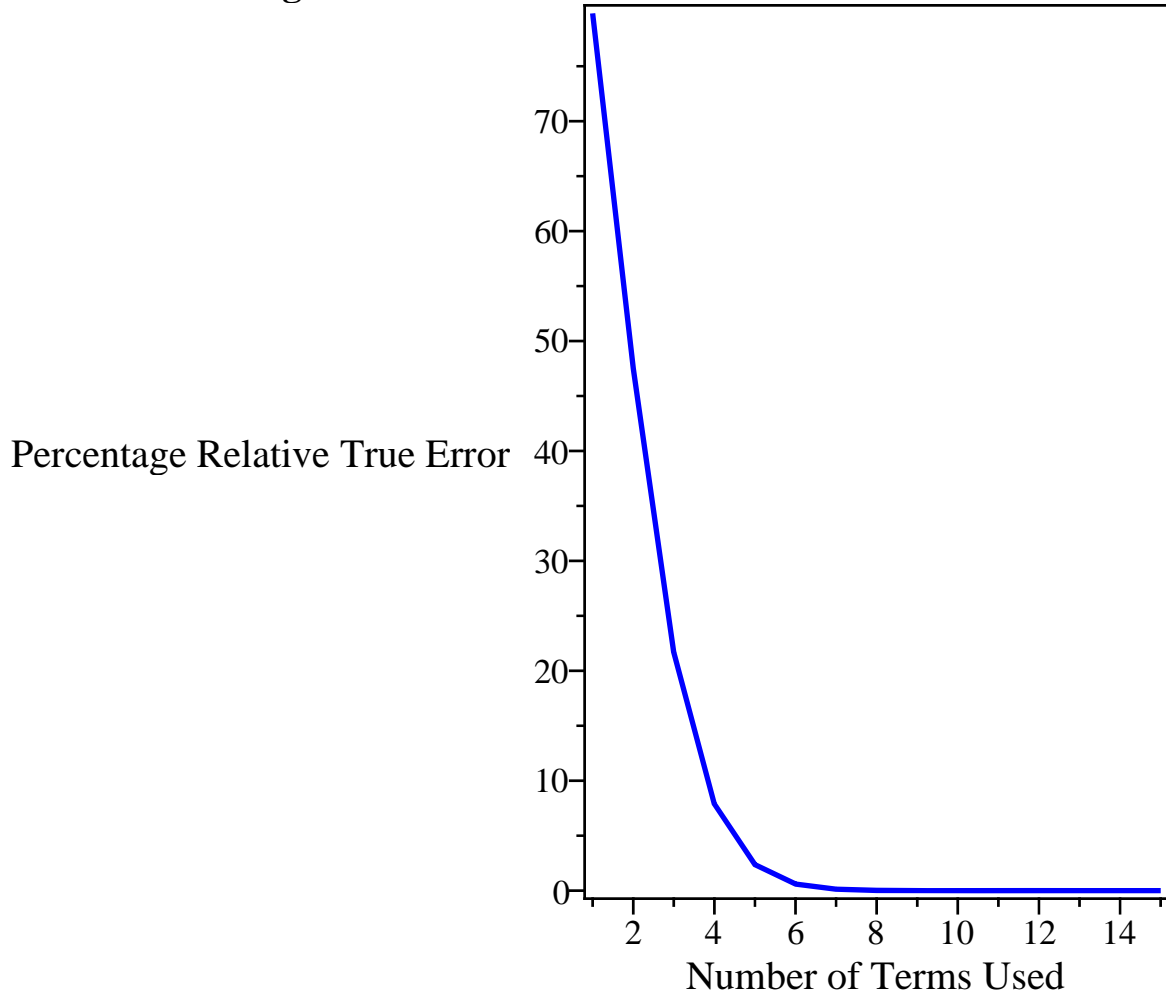
**True Error vs. Number of Terms**



**Absolute True Error vs. Number of Terms**

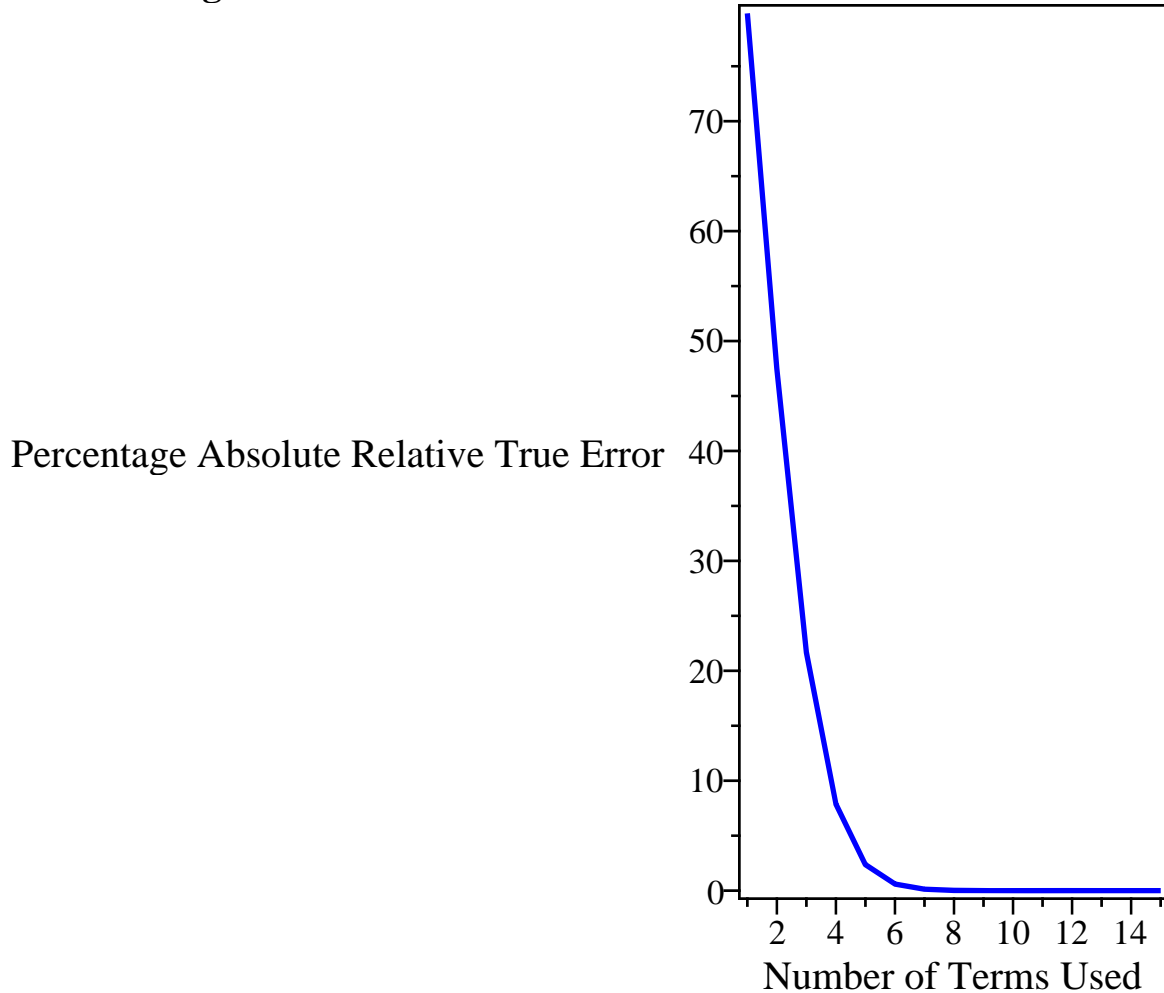


**Percentage Relative True Error vs. Number of Terms**





## Percentage Absolute Relative True Error vs. Number of Terms



### ▼ Conclusion

This worksheet shows how the number of terms taken in a Maclaurin series affects the accuracy of the calculated answer through the analysis of error. Note that though true error shows the magnitude of the error, it does not indicate how bad the error really is. Hence, relative true error is used here to give a more complete picture of the state of error.

### ▼ References

Measuring Errors.

See: [http://numericalmethods.eng.usf.edu/mws/gen/01aae/mws\\_gen\\_aae\\_txt\\_measuringerror.pdf](http://numericalmethods.eng.usf.edu/mws/gen/01aae/mws_gen_aae_txt_measuringerror.pdf)

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