

Zero slope--Newton-Raphson Method.

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NOTE: This worksheet demonstrates the use of Maple to illustrate a pitfall of zero slope in the Newton-Raphson method of finding roots of $f(x)=0$.

- Introduction

Although, Newton-Raphson method [[text notes](#)][[PPT](#)] converges faster than any other method it still has a few drawbacks. One of them is Division of zero or near zero. The formula of Newton-Raphson method is $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$. Consequently if an iteration value, x_i is such that $f'(x_i) \cong 0$, then one can face division by zero or a near-zero number. This will give a large magnitude for the next value, x_{i+1} .

The following simulation illustrates a pitfall of zero slope in the Newton-Raphson method of finding roots of $f(x)=0$.

```
> restart;
```

- Section I : Data.

Function in $f(x)=0$

```
> f(x) := sin(x) :
```

Initial guess

```
> x0 := Pi/2 :
```

Upper bound of range of 'x' that is desired

```
> uxrange := 10.0 :
```

Lower bound of range of 'x' that is desired

```
> lxrange := -10.0 :
```

- Section II: Before you start.

Because the method uses a line tangent to the function at the initial guess, we must calculate the derivative of the function to find the slope of the line at this point. Here we will define the derivative of the function $f(x)$ as $g(x)$.

```
> g(x) := diff(f(x), x) ;
```

```
g(x) := cos(x)
```

We now plot the data. The following function determines the upper and lower ranges on the Y-axis.

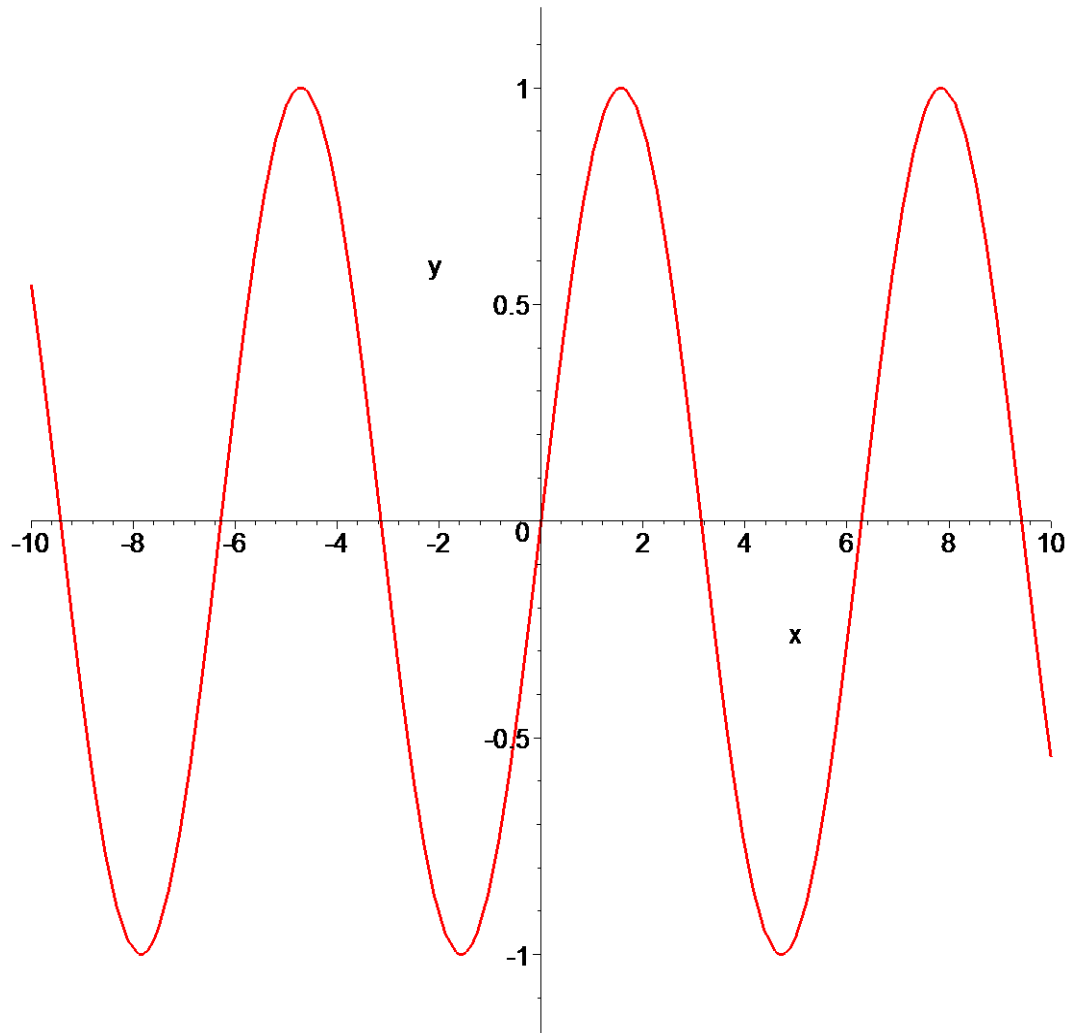
This is done using the upper and lower ranges of the X-axis specified, and the value of the original functional at these values.

```
> yranger:=proc (uxrange ,l xrange)
  local i,maxi,mini,tot;
  maxi:=eval(f(x),x=l xrange);
  mini:=eval(f(x),x=l xrange);
  for i from l xrange by (uxrange-l xrange)/10 to uxrange do
    if eval(f(x),x=i)<mini then mini:=eval(f(x),x=i) end if;
    if eval(f(x),x=i)>maxi then maxi:=eval(f(x),x=i) end if;
  end do;
  tot:=maxi-mini;
  -0.1*tot+mini..0.1*tot+maxi;
end proc:
[ > yrange:=yranger(uxrange,l xrange) :
[ > xrange:=l xrange..uxrange:
```

The following calls are needed to use the plot function

```
> with(plots) :
Warning, the name changecoords has been redefined
[ > with(plottools) :
Warning, the name arrow has been redefined
[ > plot(f(x),x=xrange,y=yrange,title="Entered function on given
interval",legend=["Function"],thickness=3);
```

Entered function on given interval



Function

- Section III: Iteration 1.

The Newton Raphson Method works by taking a tangent line at the value of the function at the initial guess, and seeing where that tangent line crosses the x-axis. This value will be the new estimate for the root..

The first estimate of the root is,

```
> x1:=x0-eval(f(x),x=x0)/eval(g(x),x=x0);  
Error, numeric exception: division by zero
```

How good is that answer? Find the absolute relative approximate error to judge.

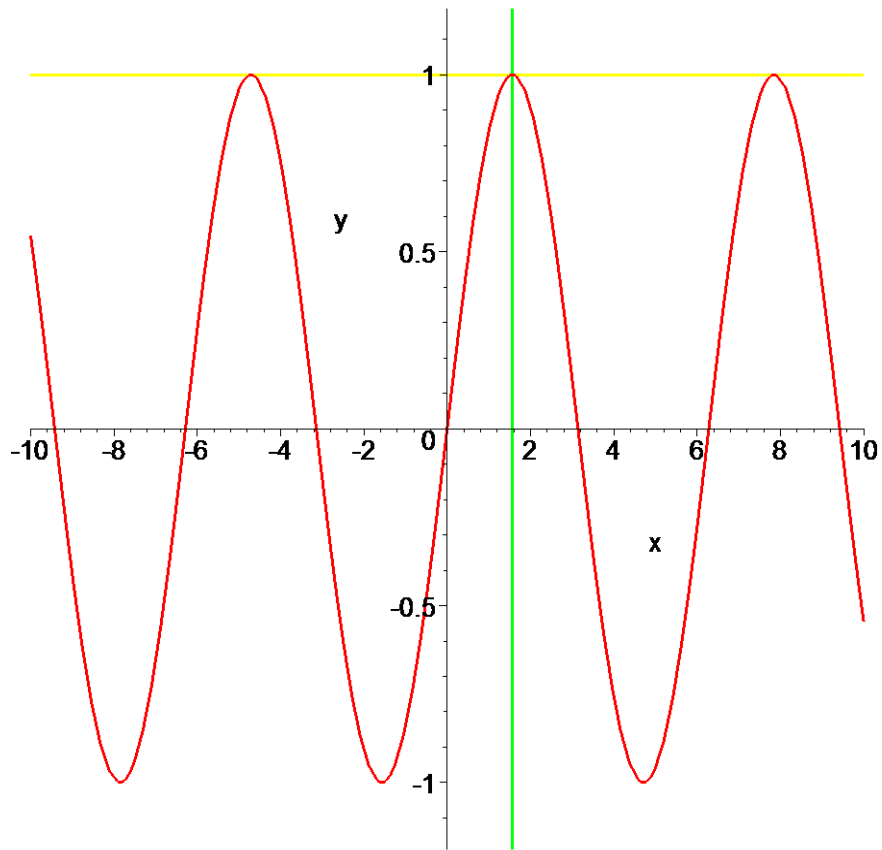
```
> epsilon:=abs((x1-x0)/x1)*100;
```

$$\varepsilon := 100 \left| \frac{x_l - \frac{\pi}{2}}{x_l} \right|$$

While not necessary to the method, for graphing purposes we define the equation of the tangent line touching x_0 .

```
> tanline(x) := eval(g(x), x=x0)*x + eval(f(x), x=x0) - eval(g(x), x=x0)*x0;  
  
> plot([f(x), [x0, t, t=yrange], [x1, t, t=yrange], tanline(x)], x=xrange, y=yrange, title="Entered function on given interval with current and next root\n and tangent line of the curve at the current root", legend=["Function", "x0, Current root", "x1, New root", "Tangent line"], thickness=3);
```

Entered function on given interval with current and next root
and tangent line of the curve at the current root



— Function
— x_0 , Current root
— Tangent line

In the above example, a zero slope is encountered. As the tangent line parallels the x-axis, convergence will not occur. Notice that our first guess has a zero slope, which causes division by zero in the Newton-Raphson formula, and hence an error in our calculation (note the "division by zero" error message, and the undefined x_1).

Section II: Conclusion.

Maple helped us to apply our knowledge of numerical methods of finding roots of a nonlinear equation to understand a pitfall of zero slope in the Newton-Raphson method of finding roots of $f(x)=0$.

References

[1] *Nathan Collier, Autar Kaw, Jai Paul, Michael Keteltas, Holistic Numerical Methods Institute, See http://numericalmethods.eng.usf.edu/mws/gen/03nle/mws_gen_nle_txt_newton.pdf*

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