# The Dangers of Extrapolation. 

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NOTE: This worksheet demonstrates the use of Maple to show how extrapolation can be dangerous and can lead to wrong and erroneous estimations. This is illustrated using NASDAQ reports from 1994-2001.

## - Introduction

Extrapolation can be very dangerous when applied to aspects of real life. The following example illustrates this in estimating the future NASDAQ index. During the technology boom of the late 1990's, the NASDAQ Composite Index that tracks mainly technology stocks had a phenomenal increase in its value. For example, at the end of year 1997, the NASDAQ index was at an all time high of 1570.35 . It increased to 2192.69 by the end of 1998, and to 4069.31 by the end of 1999 . Many people who had never ventured into stocks started investing in the technology stocks and mutual funds. Dreams of doubling their money every year were not considered unrealistic. Given below is the data of the NASDAQ index from 1994 to 1999. If you were extrapolating the data, what would you estimate the NASDAQ index to be at the end of 2000 and 2001? How different did it turn out to be?

```
> restart;
```


## - Section I : Data.

The following is the NASDAQ Composite Data for years actually between 1994 (Year 1) and 1999 (Year 6). End of Year NASDAQ Composite Data taken from www.bigcharts.com.

```
> xy:=[[1,751.96],[2,1052.13],[3,1291.03],[4,1570.35],[5,2192.69]
        ,[6,4069.31]]:
```


## Plotting the data:

$>\operatorname{plot}(x y, x=0 . .6, y=500.4200$, style=POINT, symbol=CIRCLE, symbolsize=30 ,title="Plot of the data points.");

Plot of the data points.


## Section II: Polynomial Extrapolation.

Given the values of the NASDAQ index for six consecutive years from 1994-1999, let us interpolate the data using polynomial interpolation. We will then use the polynomial to estimate the values of the NASDAQ index for the years 2000 and 2001, by extrapolation.

```
> poly_fn:=interp([1,2,3,4,5,6],[751.96,1052.13,1291.03,1570.35,2
    192.69,4069.31],t);
poly_fn:= 3.397916667 t ' - 42.59750001 t + + 81.9899998 +222.0587500 t}\mp@subsup{t}{}{3}+1090.973334
    -603.8625002 t'
```

The NASDAQ index at the end of 2000(year 7) and 2001 (year 8) using polynomial extrapolation would be :
> newvalue2000:=subs(t=7,poly_fn); newvalue2000 :=9127.88000
[ > newvalue2001:=subs( $\left.t=8, p o l y \_f n\right)$;
newvalue2001 := 20720.22996
poly_fn:=t->interp([1,2,3,4,5,6],[751.96,1052.13,1291.03,1570.3 5,2192.69,4069.31],t):
plot([xy,poly_fn,[[7,2470.52],[8,1950.4]],[[7,poly_fn(7)],[8,po ly_fn (8)]]],1..8.1,style=[POINT,LINE, POINT, POINT], symbol=[CIRCL E,DIAMOND, BOX,DIAMOND], symbolsize=[30,30,30], color=[RED,BLUE , GR EEN,BLACK], title="Data from 1994 to 1999 extrapolated to yield results for 2000 and 2001",legend=["Known stock market data", "Extrapolating function","Actual future results","Extrapolated values"],thickness=2);

Data from 1994 to 1999 extrapolated to yield results for 2000 and 2001

$\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc$ Known stock market data
— Extrapolating function
$\square \square \square \square \square \square$
$\diamond \diamond \diamond \diamond \diamond \diamond$
Actual future results
Extrapolated values

## - Section III: Spline Extrapolation.

Let us now use cubic spline interpolation to interpolate the data for the NASDAQ index from 1994-1999 and then, use this to estimate the NASDAQ index for the years 2000 and 2001, by extrapolation. Do we get values closer to the actual values??

$$
\begin{aligned}
& >\text { fcubic:=spline }([1,2,3,4,5,6],[751.96,1052.13,1291.03,1570.35,21 \\
& \quad 92.69,4069.31], t, \text { cubic }) ; \\
& \text { fcubic }:=\left\{433.0358373+318.924162700000010 t-18.7541626800000004(t-1)^{3}, t<2\right. \\
& \quad 526.8066508+262.661674600000026 t-56.2624880382775104(t-2)^{2} \\
& \quad+32.5008133999999984(t-2)^{3}, t<3 \\
& \quad 548.1125836+247.639138800000012 t+41.2399521531100533(t-3)^{2} \\
& \quad-9.55909090699999986(t-3)^{3}, t<4 \\
& \quad 364.582919+301.441770299999973 t+12.5626794258372900(t-4)^{2} \\
& \quad+308.335550200000000(t-4)^{3}, t<5 \\
& \quad-4065.178900+1251.57377999999994 t+937.569330143540696(t-5)^{2} \\
& \quad-312.523109999999974(t-5)^{3}, \text { otherwise } \\
& > \\
& \text { fcubic: }=\mathrm{t}->\operatorname{spline}([1,2,3,4,5,6],[751.96,1052.13,1291.03,1570.35 \\
& \quad, 2192.69,4069.31], t, \text { cubic }):
\end{aligned}
$$

The NASDAQ index at the end of 2000 and 2001 using cubic spline extrapolation would be :

```
new2000:=fcubic(7);
```

$$
\text { new2000 := } 5945.930000
$$

new2001:=fcubic (8);

$$
\text { new2001 := } 5947.411340
$$

> plot([xy,fcubic,[[7,2470.52],[8,1950.4]],[[7,fcubic(7)],[8,fcub ic (8)]]],1..8.1, style=[POINT,LINE,POINT,POINT], symbol=[CIRCLE,D IAMOND, BOX,DIAMOND], symbolsize=[30,30,30], color=[RED,BLUE, GREEN ,BLACK], title="Data from 1994 to 1999 extrapolated to yield results for 2000 and 2001",legend=["Known stock market data","Extrapolating function","Actual future results","Extrapolated values"],thickness=2);


## - Section IV: Conclusion.

Maple helped us to apply our knowledge of numerical methods of extrapolation to see that extrapolation can lead to erroneous estimations. The values obtained for the years 2000 and 2001 using polynomial extrapolation and cubic spline extrapolation were much higher than the actual NASDAQ values in those years. Surely, the people who ventured into stocks, and invested in the technology stocks and mutual funds with dreams of doubling their money every year, after the steep rise in NASDAQ Index from 1994-1999 due to the technology boom, would have had lots to think
about. Can you estimate the end of year NASDAQ Index for the year 2002, and compare it with the actual value of 1335 ? Can you also estimate the end of year NASDAQ Index value for the year 2003?

## References:

[1] Autar Kaw and Michael Keteltas, Holistic Numerical Methods Institute, See http://numericalmethods.eng.usf.edu/mws/ind/05inp/mws_ind_inp_spe_extrapolation.pdf

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