Integration Using the Simpson's 1/3rd Rule - Convergence

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NOTE: This worksheet demonstrates the use of Maple to illustrate the convergence of Simpson's 1/3rd rule of integration.

Introduction

Simpson's rule is based on the Newton-Cotes formula that if one approximates the integrand of the integral by an nth order polynomial, then the integral of the function is approximated by the integral of that nth order polynomial. Integration of polynomials is simple and is based on calculus. Simpson's 1/3rd rule is the area under the curve where the function is approximated by a second order polynomial. [click here for textbook notes] [click here for power point presentation].

Section I: Data

The following simulation illustrates the convergence of Simpson's 1/3rd rule of integration. This section is the only section where the user interacts with the program. The user enters any function *f(x)*, the lower and upper limit of the integration, and the maximum number of segments, *n*. The program will display the true error, the absolute relative percentage true error, the approximate error, the absolute relative percentage approximate error, and the least number of significant digits correct all as a function of number of segments.

```
> restart;
```

Integrand *f(x)*

```
> f:=x->300*x/(1+exp(x));
```

$$f := x \to \frac{300\,x}{1+\mathbf{e}^{x}}$$

The lower limit of the integration, *a*

```
> a:=0.0;
```

$$a := 0.$$

The upper limit of the integration, *b*

```
> b:=10.0;
```

$$b := 10.0$$

Maximum number of segments, *n*. Note *n* needs to be an even number.

```
> n:=20;
```

$$n := 20$$

This is the end of the user's section. All information must be entered before proceeding to the next section. Re-execute the program.

Section II: Procedure for Simpson's 1/3rd rule

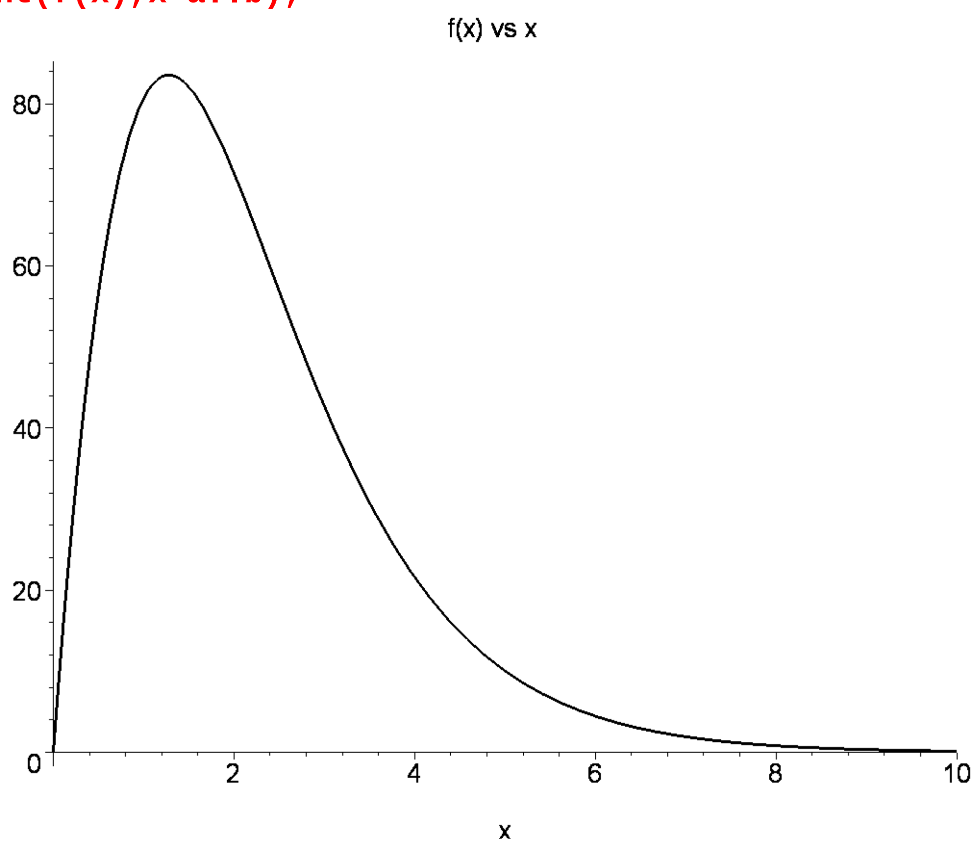
The following procedure determines the value of the integral using Simpson's 1/3rd rule with n segments.

```
> simp:=proc(n,a,b,f)
  local AV,sum_1,sum_2,h,i:
  h:=(b-a)/n:
  sum_1:=0:
  for i from 1 by 2 to n-1 do
  sum_1:=sum_1+f(a+i*h):
  end do:
  sum_2:=0:
  for i from 2 by 2 to n-2 do
  sum_2:=sum_2+f(a+i*h):
  end do:
  AV:=(h/3)*(f(a)+4*sum_1+2*sum_2+f(b)):
  return (AV):
  end proc:
```

Section III: Calculations

The exact value of the integral, *EV* :

```
> plot(f(x),x=a..b,title="f(x) vs x",thickness=3,color=black);
  EV:=int(f(x),x=a..b);
```



$EV := 246.5902935$

```
> for i from 2 by 2 to n do
```
AV is the value of the integral using n segment
```
AV[i]:=simp(i,a,b,f);
```
Et is the true error
```
Et[i]:=EV-AV[i]:
```
abs_et is the absolute relative percentage true rror
```
abs_et[i]:=abs(Et[i]/EV)*100.0:
if (i>2) then
```
Ea is the approximate error
```
Ea[i]:=AV[i]-AV[i-2]:
```
ea is the absolute realtive approximate percentage error
```
ea[i]:=abs(Ea[i]/AV[i])*100.0:
```
sig is the least significant digits correct in your answer
```
sig[i]:=floor((2-log10(ea[i]/0.5))):
if sig[i]<0 then
sig[i]:=0:
end if:
end if:
end do:
```

Section IV: Spreadsheet

This table shows the approximate value, true error, absolute relative true error, approximate error and relative approximate error as a function of the number of segments.

```
> with( Spread ):
> EvaluateSpreadsheet(Simpson_Results);
```

	A	B	C	D
1	*The number of segments*	*Exact Value*	*Approximate Value*	*True Erro*
2	2	11061.33554	11065.71634	-4.38080
3	4	11061.33554	11061.63613	-0.30059
4	6	11061.33554	11061.39611	-0.06057
5	8	11061.33554	11061.35483	-0.01929
6	10	11061.33554	11061.34347	-0.00793
7	12	11061.33554	11061.33937	-0.00383
8	14	11061.33554	11061.33759	-0.00205

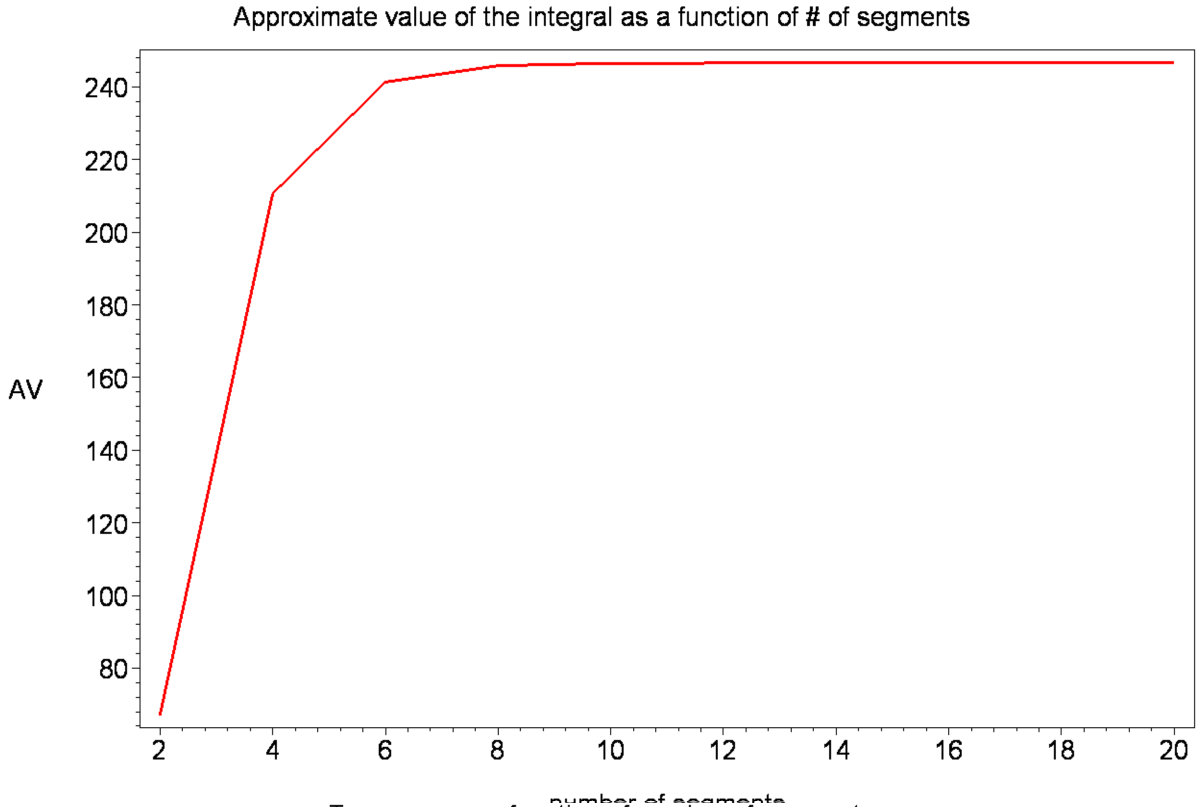
NOTE: To evaluate the spreadsheet, you need to right click on it and select evaluate

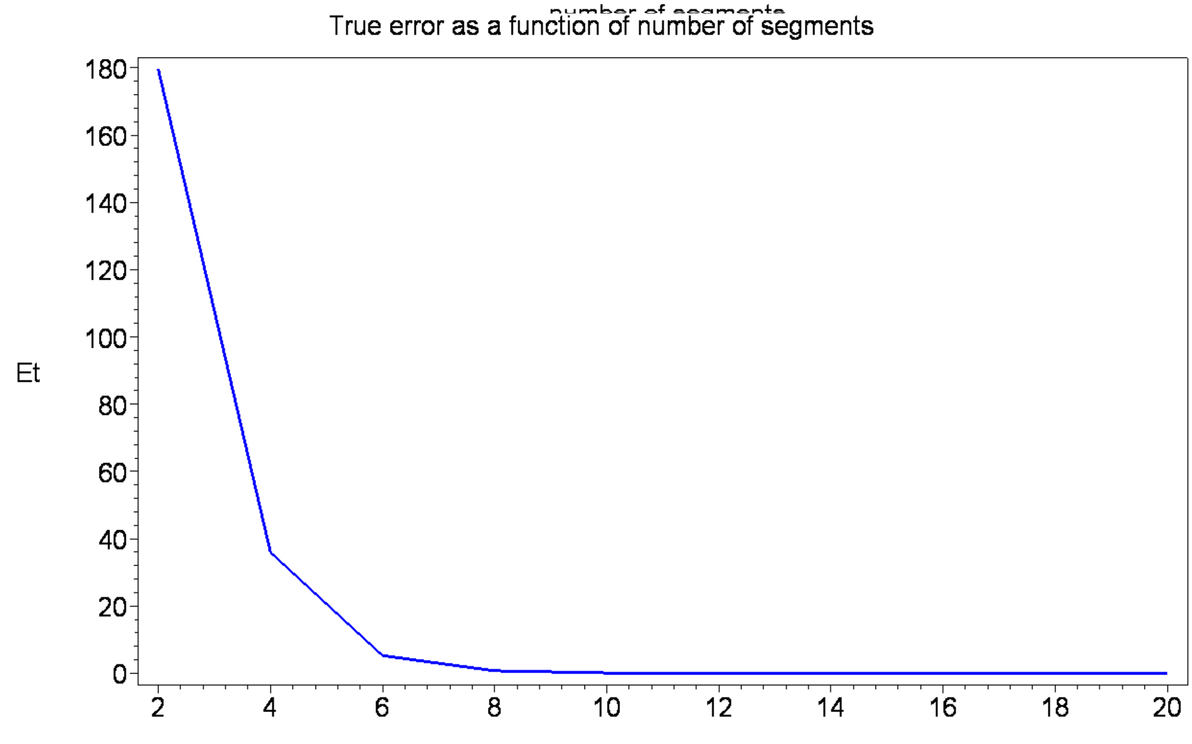
Section V:Graphs

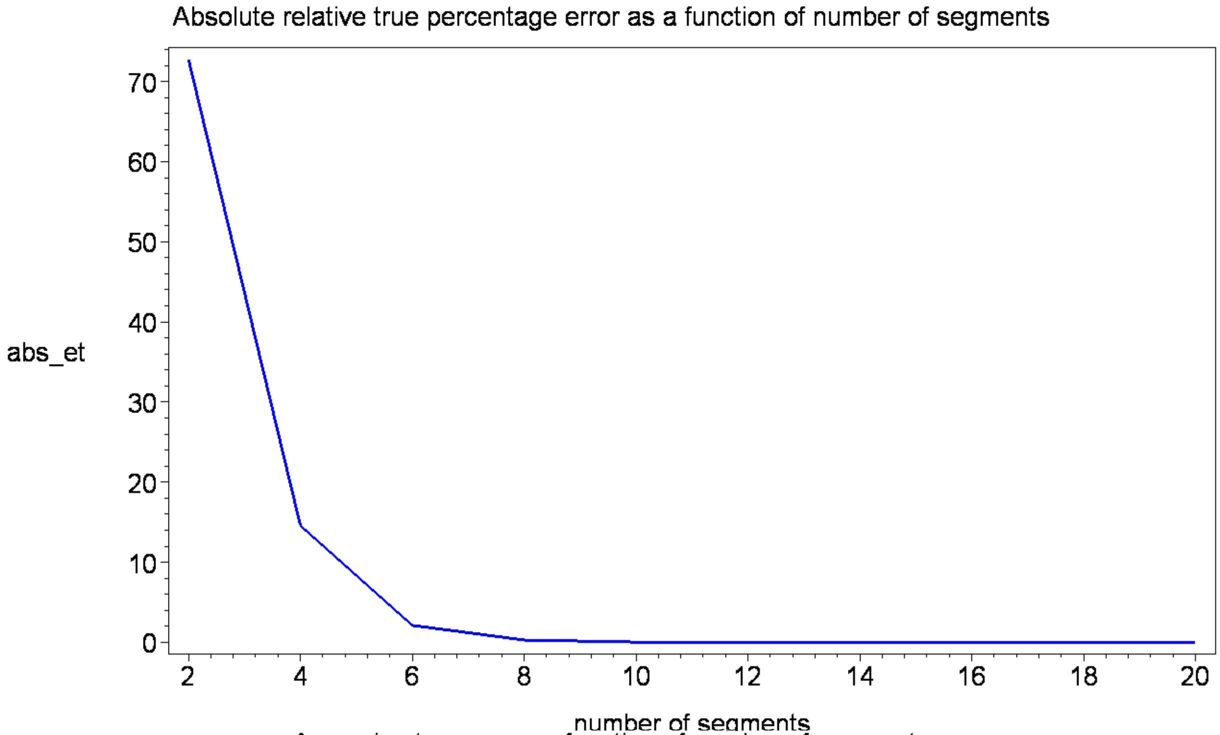
```
> with(plots):
Warning, the name changecoords has been redefined
```

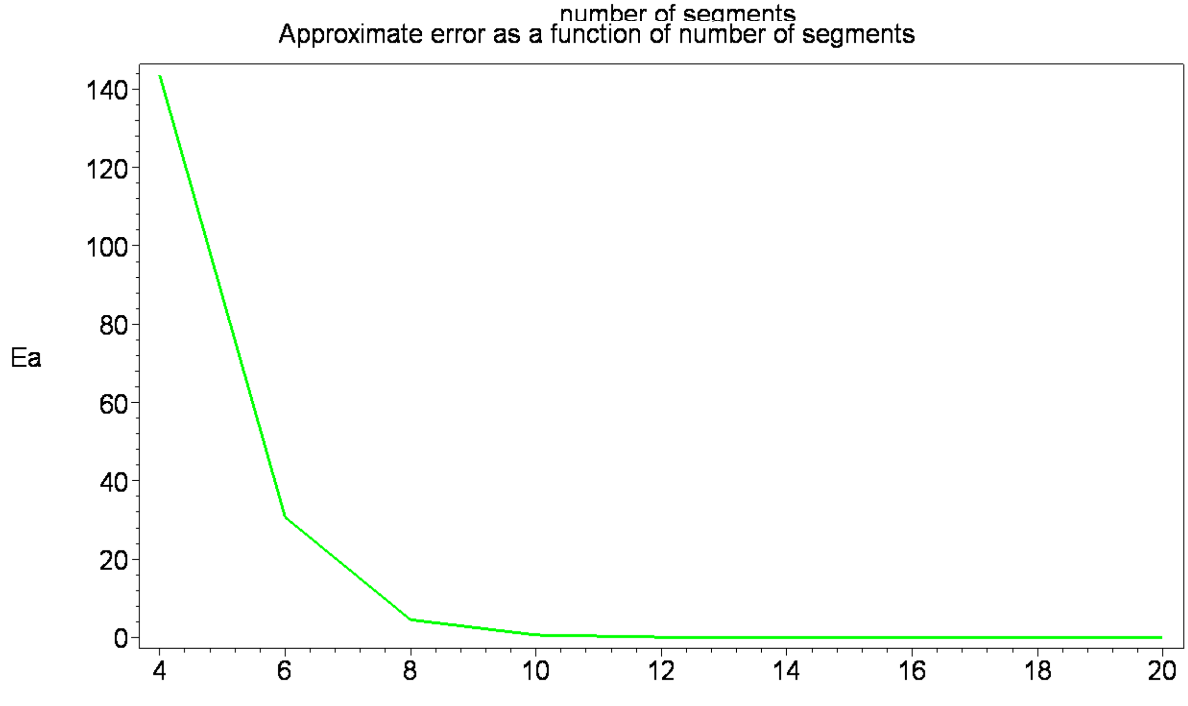
```
> X:=[seq(i*2, i=1..n/2)]:
  data:=[seq([X[i],AV[i*2]],i=1..n/2)]:

> pointplot(data,connect=true,color=red,axes=boxed,title="Approxi
  mate value of the integral as a function of # of
  segments",axes=BOXED,labels=["number of
  segments","AV"],thickness=3);

> data:=[seq([X[i],Et[i*2]],i=1..n/2)]:

> pointplot(data,connect=true,color=blue,axes=boxed,title="True
  error as a function of number of
  segments",axes=BOXED,labels=["number of
  segments","Et"],thickness=3);
```

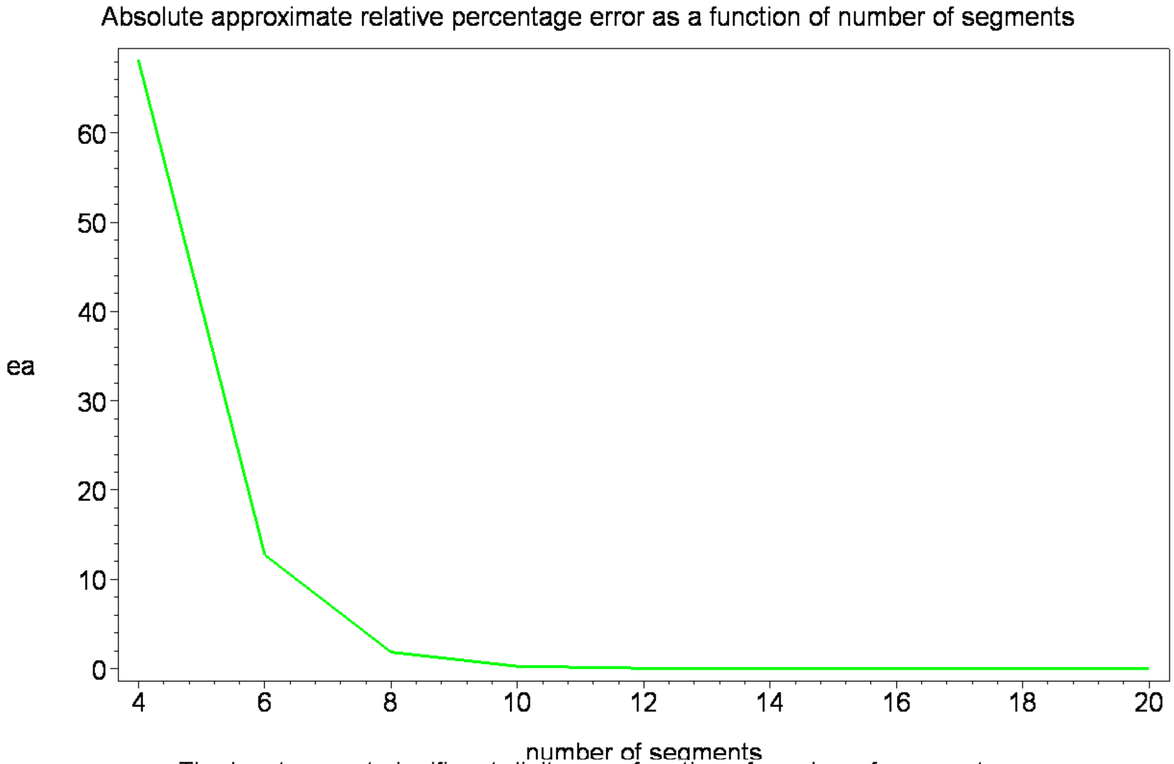
```
> data:=[seq([X[i],abs_et[i*2]],i=1..n/2)]:

> pointplot(data,connect=true,color=blue,axes=boxed,title="Absolu
  te relative true percentage error as a function of number of
  segments",axes=BOXED,labels=["number of
  segments","abs_et"],thickness=3);


> data:=[seq([X[i],Ea[i*2]],i=2..n/2)]:

> pointplot(data,connect=true,color=green,axes=boxed,title="Appro
  ximate error as a function of number of
  segments",axes=BOXED,labels=["number of
  segments","Ea"],thickness=3);


> data:=[seq([X[i],ea[i*2]],i=2..n/2)]:

> pointplot(data,connect=true,color=green,axes=boxed,title="Absol
  ute approximate relative percentage error as a function of
  number of segments",axes=BOXED,labels=["number of
  segments","ea"],thickness=3);


> data:=[seq([X[i],sig[i*2]],i=2..n/2)]:

> pointplot(data,connect=true,color=brown,axes=boxed,title="The
  least correct significant digits as a function of number of
  segments",axes=BOXED,labels=["number of
  segments","sig"],thickness=3);
```

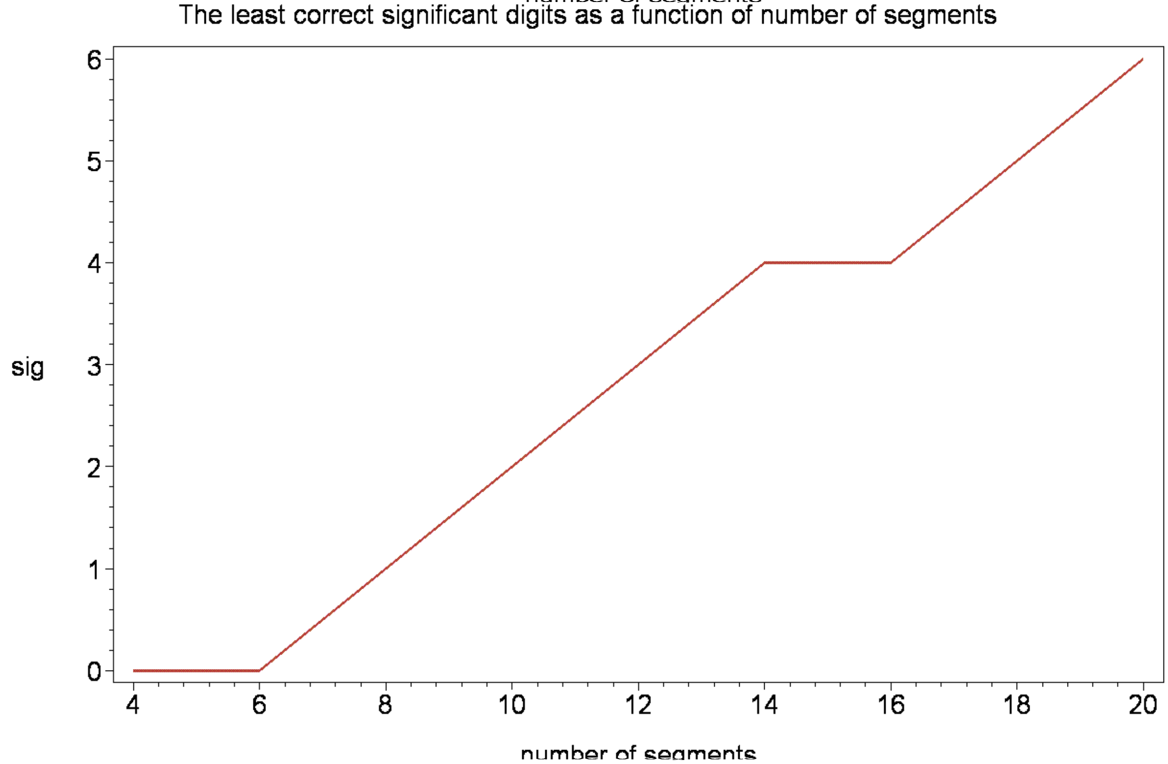

Approximate value of the integral as a function of # of segments
AV
240
220
200
180
160
140
120
100
80
2
4
6
8
10
12
14
16
18
20


True error as a function of number of segments
Et
180
160
140
120
100
80
60
40
20
0
2
4
6
8
10
12
14
16
18
20
number of segments


Absolute relative true percentage error as a function of number of segments
abs_et
70
60
50
40
30
20
10
0
2
4
6
8
10
12
14
16
18
20
number of segments


Approximate error as a function of number of segments
Ea
140
120
100
80
60
40
20
0
4
6
8
10
12
14
16
18
20
number of segments





>

Maple helped us to understand the concept of convergence of multiple segment Simpson's 1/3rd rule.

Question:

1. An integral like $\frac{1}{\sqrt{x}}$ from x=0 to 2 cannot be found using Simpson's 1/3rd rule as the integrand

is infinite at x=0. What if you defined the function as finite at x=0? Would you get close to an accurate solution? Is defining the function at one point x=0 as finite wrong?

References

[1] Autar Kaw, Michael Keteltas, Holistic Numerical Methods Institute, See http://numericalmethods.eng.usf.edu/mws/gen/07int/mws_gen_int_txt_simpson13.doc