Integration Using the Simpson's 1/3rd Rule - Method

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NOTE: This worksheet demonstrates the use of Maple to illustrate the multiple segment Simpson's 1/3rd rule of integration.

Section I: Introduction

Simpson's rule is based on the Newton-Cotes formula that if one approximates the integrand of the integral by an nth order polynomial, then the integral of the function is approximated by the integral of that nth order polynomial. Integration of polynomials is simple and is based on the calculus. Simpson's 1/3rd rule is the area under the curve where the function is approximated by a second order polynomial. [click here for textbook notes] [click here for power point presentation].

Section II: Data

The following simulation illustrates the Simpson's 1/3rd rule of integration. This section is the only section where the user interacts with the program. The user enters any function *f(x)*, the lower and upper limit of the integration. By entering this data, the program will calculate the exact value of the integral, followed by the results using the Simpson's 1/3rd rule with $n = 2, 4, 6, 8$ segments.

```
> restart;
```

Integrand *f(x)*

```
> f:=x->300*x/(1+exp(x));
```

$$f := x \to \frac{300\,x}{1+\mathbf{e}^{x}}$$

The lower limit of the integral *a*

```
> a:=0.0;
```

$$a := 0.$$

The upper limit of the integral *b*

```
> b:=10.0;
```

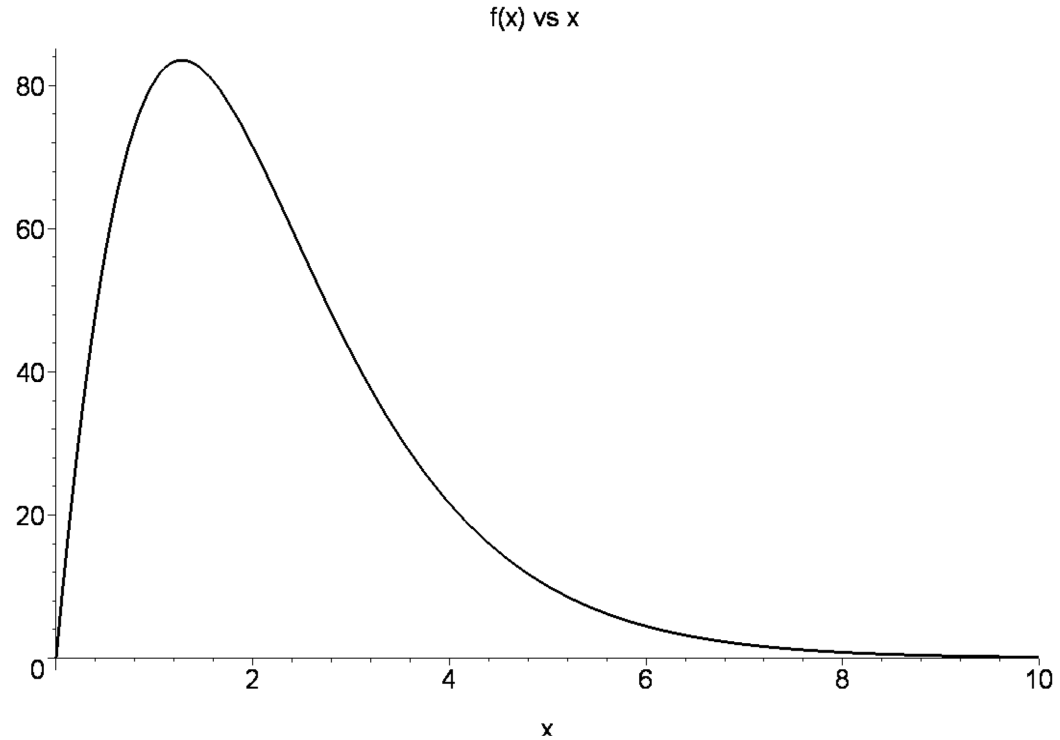
$$b := 10.0$$

This is the end of the user's section. All information must be entered before proceeding to the next section. Re-execute the program.

Section III: The exact value of the integral

In this section, the program will evaluate the exact value for the integral of the function *f(x)* evaluated at the limits *a* and *b*.

```
> plot(f(x),x=a..b,title="f(x) vs x",thickness=3,color=black);
  s_exact:=int(f(x),x=a..b);
```



$s\_exact := 246.5902935$

Section IV: The value of the integral using the simpson's rule

Two segments ($n = 2$)

```
> n:=2;
```

$$n := 2$$

```
> h[2]:=(b-a)/n;
```

$$h_2 := 5.000000000$$

The integral of the function $f(x)$ from a to b using the simpson's rule with two segments will be equal to:

```
> s[2]:=(b-a)*(f(a)+4*f(a+h[2])+f(b))/(3*n);
```

$$s_2 := 67.15549860$$

The approximate error (E_a):

```
> E_a[2]:=undefined;
```

$$E\_a_2 := undefined$$

The absolute approximate percentage relative error (E_arel):

```
> E_arel[2]:=undefined;
```

$$E\_arel_2 := undefined$$

Four segments ($n = 4$)

```
> n:=4;
```

$$n := 4$$

```
> h[4]:=(b-a)/n;
```

$$h_4 := 2.500000000$$

The integral of the function *f(x)* from *a* to *b* using the simpson's rule with four segments will be equal to:

```
> s[4]:=(b-a)*(f(a)+4*(f(a+h[4])+f(a+3*h[4]))+2*f(a+2*h[4])+f(b
  ))/(3*n);
```

$$s_4 := 210.6369118$$

The approximate error (E_a):

```
> E_a[4]:=s[4]-s[2];
```

$$E\_a_4 := 143.4814132$$

The absolute approximate percentage relative error (E_arel):

```
> E_arel[4]:=abs(E_a[4]/s[4]*100);
```

$$E\_arel_4 := 68.11788683$$

Six segments (*n* = *6*)

```
> n:=6;
```

$$n := 6$$

```
> h[6]:=(b-a)/n;
```

$$h_6 := 1.666666667$$

The integral of the function *f(x)* from *a* to *b* using the simpson's rule with six segments will be equal to:

```
> s[6]:=(b-a)*(f(a)+4*(f(a+h[6])+f(a+3*h[6])+f(a+5*h[6]))+2*(f(
  a+2*h[6])+f(a+4*h[6]))+f(b))/(3*n);
```

$$s_6 := 241.3383791$$

The approximate error (E_a):

```
> E_a[6]:=s[6]-s[4];
```

$$E\_a_6 := 30.7014673$$

The absolute approximate percentage relative error (E_arel):

```
> E_arel[6]:=abs(E_a[6]/s[6]*100);
```

$$E\_arel_6 := 12.72133650$$

Eight segments (*n* = *8*)

```
> n:=8;
```

$$n := 8$$

```
> h[8]:=(b-a)/n;
```

$$h_8 := 1.250000000$$

The integral of the function *f(x)* from *a* to *b* using the simpson's rule with eight segments will be equal to:

```
> s[8]:=(b-a)*(f(a)+4*(f(a+h[8])+f(a+3*h[8])+f(a+5*h[8])+f(a+7*
  h[8]))+2*(f(a+2*h[8])+f(a+4*h[8])+f(a+6*h[8]))+f(b))/(3*n);
```

$$s_8 := 245.8549932$$

The approximate error (E_a):

```
> E_a[8]:=s[8]-s[6];
```

$$E\_a_8 := 4.5166141$$

The absolute approximate percentage relative error (E_arel):

```
> E_arel[8]:=abs(E_a[8]/s[8]*100);
```

$$E\_arel_8 := 1.837104889$$

```
>
```

References

[1] Autar Kaw, Michael Keteltas, Holistic Numerical Methods Institute, See http://numericalmethods.eng.usf.edu/mws/gen/07int/mws_gen_int_txt_simpson13.doc