Integration Using the Trapezoidal Rule - Method

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NOTE: This worksheet demonstrates the use of Maple to illustrate the Trapezoidal rule method of estimating integrals of continuous functions.

Section I: Introduction

Trapezoidal rule is based on the Newton-Cotes formula that if one approximates the integrand of the integral by an nth order polynomial, then the integral of the function is approximated by the integral of that nth order polynomial. Integration of polynomials is simple and is based on the calculus. Trapezoidal rule is the area under the curve for a first order polynomial (straight line) that approximates the integrand. [click here for textbook notes] [click here for power point presentation].

Section II: Data

The following simulation illustrates the Trapezoidal rule of integration. This section is the only section where the user interacts with the program. The user enters any function *f(x)* and the lower and upper limit of the integration. By entering this data, the program will calculate the exact value of the integral, followed by the results using Trapezoidal rule *n = 1, 2, 3, 4* segments.

```
> restart;
```

Integrand f(x)

```
> f:=x->300*x/(1+exp(x));
```

$$f := x \rightarrow \frac{300\,x}{1+e^{x}} \tag{2.1}$$

The lower limit of the integral *a*

```
> a:=0.0;
```

$$a := 0. \tag{2.2}$$

The upper limit of the integral *b*

```
> b:=10.0;
```

$$b := 10.0 \tag{2.3}$$

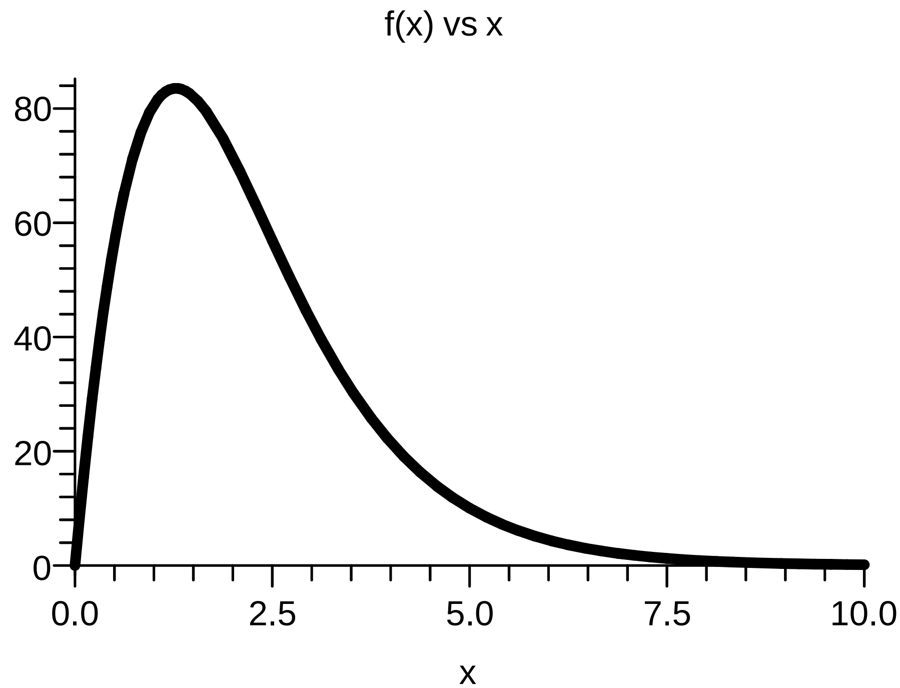
This is the end of the user's section. All information must be entered before proceeding to the next section. Re-execute the program.

Section III: The exact value of the integral

In this section, the program will evaluate the Maple value for the integral of the function *f(x)* evaluated at the limits *a* and *b*.

```
> plot(f(x),x=a..b,title="f(x) vs x",thickness=3, color=black);
  s_exact:=int(f(x),x=a..b);
```

(3.1)



$$s\_exact := 246.5902935$$

Section IV: The value of the integral using the Trapezoidal rule

One segment (*n* = *1*)

```
> n:=1;
```

$$n := 1 \tag{4.1.1}$$

```
> h[1]:=(b-a)/n;
```

$$h_1 := 10.0 \tag{4.1.2}$$

The integral of the function *f(x)* from *a* to *b* using the trapezoidal rule with one segment will be equal to

```
> s[1]:=(b-a)*(f(a)+f(b))/2;
```

$$s_1 := 0.6809680305 \tag{4.1.3}$$

The approximate error is

```
> E_a[1]:=undefined;
```

$$E\_a_1 := undefined \tag{4.1.4}$$

The absolute approximate relative percentage error is

```
> E_arel[1]:=undefined;
```

$$E\_arel_1 := undefined \tag{4.1.5}$$

Two segments ($n = 2$)

```
> n:=2;
```

$$n := 2 \tag{4.2.1}$$

```
> h[2]:=(b-a)/n;
```

$$h_2 := 5.000000000 \tag{4.2.2}$$

The integral of the function *f(x)* from *a* to *b* using the trapezoidal rule with two segment will be equal to

```
> s[2]:=(b-a)*(f(a)+2*f(a+h[2])+f(b))/(2*n);
```

$$s_2 := 50.53686600 \tag{4.2.3}$$

The approximate error is

```
> E_a[2]:=s[2]-s[1];
```

$$E\_a_2 := 49.85589797 \tag{4.2.4}$$

The absolute approximate relative percentage error is

```
> E_arel[2]:=abs(E_a[2]/s[2]*100);
```

$$E\_arel_2 := 98.65253213 \tag{4.2.5}$$

Three segments ($n = 3$)

```
> n:=3;
```

$$n := 3 \tag{4.3.1}$$

```
> h[3]:=(b-a)/n;
```

$$h_3 := 3.333333333 \tag{4.3.2}$$

The integral of the function *f(x)* from *a* to *b* using the trapezoidal rule with three segment will be equal to

```
> s[3]:=(b-a)*(f(a)+2*f(a+h[3])+2*f(a+2*h[3])+f(b))/(2*n);
```

$$s_3 := 123.5177500 \tag{4.3.3}$$

The approximate error is

```
> E_a[3]:=s[3]-s[2];
```

$$E\_a_3 := 72.98088400 \tag{4.3.4}$$

The absolute approximate relative percentage error is

```
> E_arel[3]:=abs(E_a[3]/s[3]*100);
```

$$E\_arel_3 := 59.08534118 \tag{4.3.5}$$

Four segments ($n = 4$)

```
> n:=4;
```

$$n := 4 \tag{4.4.1}$$

```
> h[4]:=(b-a)/n;
```

$$h_4 := 2.500000000 \tag{4.4.2}$$

The integral of the function *f(x)* from *a* to *b* using the trapezoidal rule with four segment will be equal to

```
> s[4]:=(b-a)*(f(a)+2*f(a+h[4])+2*f(a+2*h[4])+2*f(a+3*h[4])+f(b))/(2*n);
```

$$s_4 := 170.6119005 \quad (4.4.3)$$

The approximate error is

```
> E_a[4]:=s[4]-s[3];
```

$$E\_a_4 := 47.0941505 \quad (4.4.4)$$

The absolute approximate relative percentage error is

```
> E_arel[4]:=abs(E_a[4]/s[4]*100);
```

$$E\_arel_4 := 27.60308652 \quad (4.4.5)$$

References

[1] Autar Kaw, Michael Keteltas, Holistic Numerical Methods Institute, See http://numericalmethods.eng.usf.edu/mws/gen/07int/mws_gen_int_txt_trapcontinuous.doc