Concepts of Approximate Error: Approximate Error, Absolute Approximate Error, Relative Approximate Error, and Absolute Relative Approximate Error

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Initialization

Clearing the definitions of all symbols in the current context:

```
ClearAll[Evaluate[Context[] <> "*"]]
```

Introduction

The following worksheet demonstrates how to calculate different definitions related to approximate error, such as approximate error, absolute approximate error, relative approximate error, and absolute relative approximate error. The concept is demonstrated using an example of a Maclaurin series. The user will choose which function to perform the calculation for in the *Input* section of the program. The choices are given as 1 for e^x, 2 for *sin(x)* , and 3 for *cos(x)*. The *true value* of these functions will be assumed as given by the *Mathematica* commands for these functions.

Section 1: Input Data

This is the only section where the user interacts with the program.

- Pick the function of your desire by choosing an integer: 1 for e^x; 2 for *sin(x)*; 3 for *cos(x)*

```
funcchoice = 1;
```

- Maximum number of terms to use in the Maclaurin series

```
n = 15;
```

- Value of *x* at which the function is calculated

```
xv = 3.14159;
```

This is the end of the user section. All information must be entered before proceeding to the next section. **RE-EVALUATE THE NOTEBOOK**.

Section 2: Procedure

First, determine which function will be used in the calculations, based on the users input. Once the function is determined, the value is calculated using a Maclaurin series in a repetitive loop.

```
sumprevious = 0;
If[funcchoice == 1,
  Do[sumpresent_i = sumprevious + xv^(i-1)/(i - 1)!;
   sumprevious = sumpresent_i,
   {i, 1, n}];
  f[x_] := N[e^x]];
If[funcchoice == 2,
  Do[sumpresent_i = sumprevious + ((-1)^(i-1) * xv^(2*i-1))/(2 * i - 1)!;
   sumprevious = sumpresent_i,
   {i, 1, n}];
  f[x_] := N[Sin[x]]];
If[funcchoice == 3,
  Do[sumpresent_i = sumprevious + ((-1)^(i+1) * xv^(2*i-2))/(2 * i - 2)!;
   sumprevious = sumpresent_i,
   {i, 1, n}];
  f[x_] := N[Cos[x]]];
```

Using *Mathematica* to calculate approximate error, absolute approximate error, relative approximate error, and absolute relative approximate error for each term. Once these error values are calculated, determining the least number of significant figures guaranteed correct.

```
Do[
 ApproxError_i = sumpresent_i - sumpresent_(i-1);
 AbsApproxError_i = Abs[sumpresent_i - sumpresent_(i-1)];
 RelApproxError_i = ((sumpresent_i - sumpresent_(i-1)) * 100)/sumpresent_i;
 AbsRelApproxError_i = Abs[((sumpresent_i - sumpresent_(i-1)) * 100)/sumpresent_i];
 SigDigits_i = Floor[2 - Log[10, AbsRelApproxError_i/(100 * 0.5)]];
 If[SigDigits_i < 0, SigDigits_i = 0],
 {i, 2, n}]
```

Section 3: Table of Values

This table shows the true value, true error, absolute true error, relative true error, absolute relative true error, and if the prespecified tolerance has been met, all as a function of the number of the number of terms used.

```
TableForm[Table[{i, sumpresent_i, ApproxError_i, AbsApproxError_i, RelApproxError_i,
   AbsRelApproxError_i, SigDigits_i}, {i, 2, n}], TableHeadings → {None,
   {"Terms Used", "Approximate Value", "Approximate Error", "Abs Approximate Error",
    "Rel Approximate Error", "Abs Rel Approximate Error", "Sig Digits"}},
 TableSpacing →
  {2,
   2}]
```

Terms Used	Approximate Value	Approximate Error	Abs Approximate Error	Rel Approximate
2	4.14159	3.14159	3.14159	75.8547
3	9.07638	4.93479	4.93479	54.3696
4	14.2441	5.1677	5.1677	36.2796
5	18.3028	4.0587	4.0587	22.1753
6	20.8529	2.55015	2.55015	12.2292
7	22.1882	1.33526	1.33526	6.01787
8	22.7875	0.599261	0.599261	2.62978
9	23.0228	0.235329	0.235329	1.02216
10	23.1049	0.0821453	0.0821453	0.355531
11	23.1307	0.0258067	0.0258067	0.111569
12	23.1381	0.00737036	0.00737036	0.0318538
13	23.14	0.00192955	0.00192955	0.0083386
14	23.1405	0.000466298	0.000466298	0.00201507
15	23.1406	0.000104637	0.000104637	0.000452179

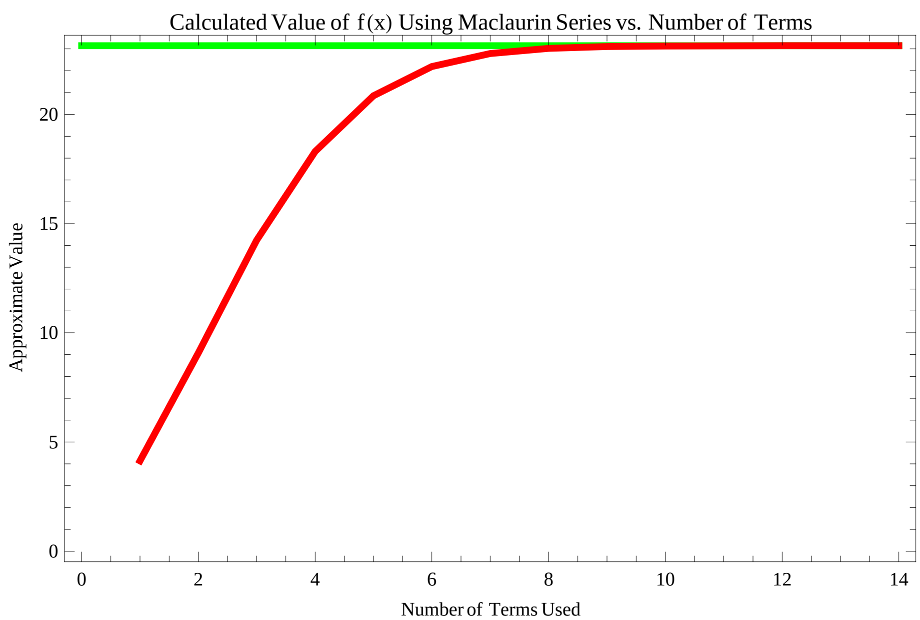
Section 4: Graphs

The following graphs show the calculated value of f(x) using Maclaurin series as a function, approximate error, absolute approximate error, absolute relative approximate error, relative approximate error, and least number of significant digits as a function of step size. Each graph displays the error results of each of the methods of approximation.

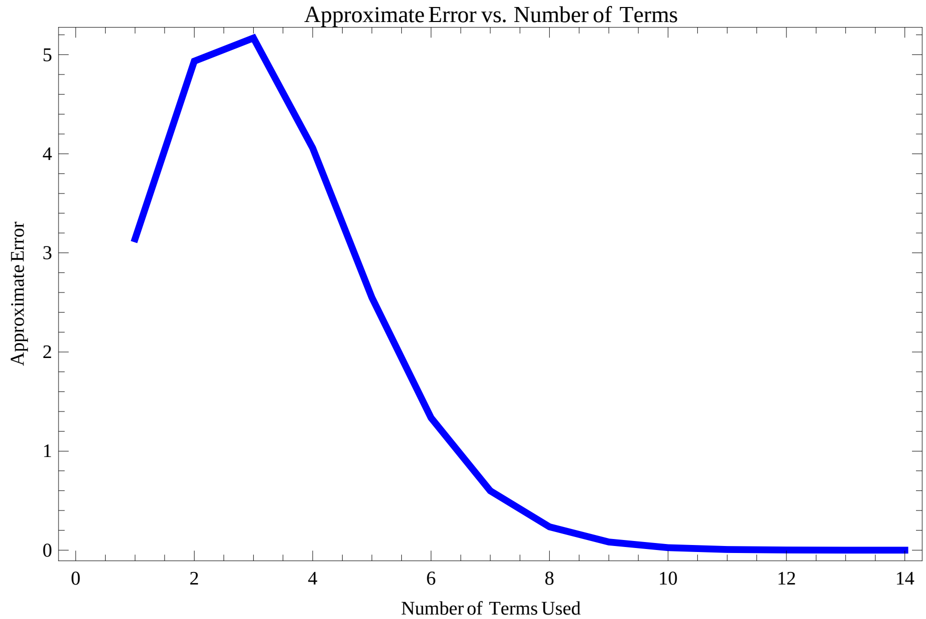
```
Plot1 = Plot[f[xv], {x, 0, n - 1}, PlotStyle → {Thickness[0.008`], RGBColor[0, 1, 0]}];
data = Table[sumpresent_i, {i, 2, n}];
Plot2 = ListPlot[data, Joined → True, PlotStyle → {Thickness[0.008`], RGBColor[1, 0, 0]}];
l1 = Graphics[{Green, Line[{{0, 0}, {1, 0}}]}];
l2 = Graphics[{Red, Line[{{0, 0}, {1, 0}}]}];
Legend[{l1 , "Exact Value"}, {l2, "Approximate Value"}]
Show[Plot1, Plot2, PlotLabel →
  "Calculated Value of f(x) Using Maclaurin Series vs. Number of Terms", Frame → True,
 FrameLabel → {"Number of Terms Used", "Approximate Value"}, PlotRange → Automatic]
```

Legend[{ ——————————————————— , Exact Value},

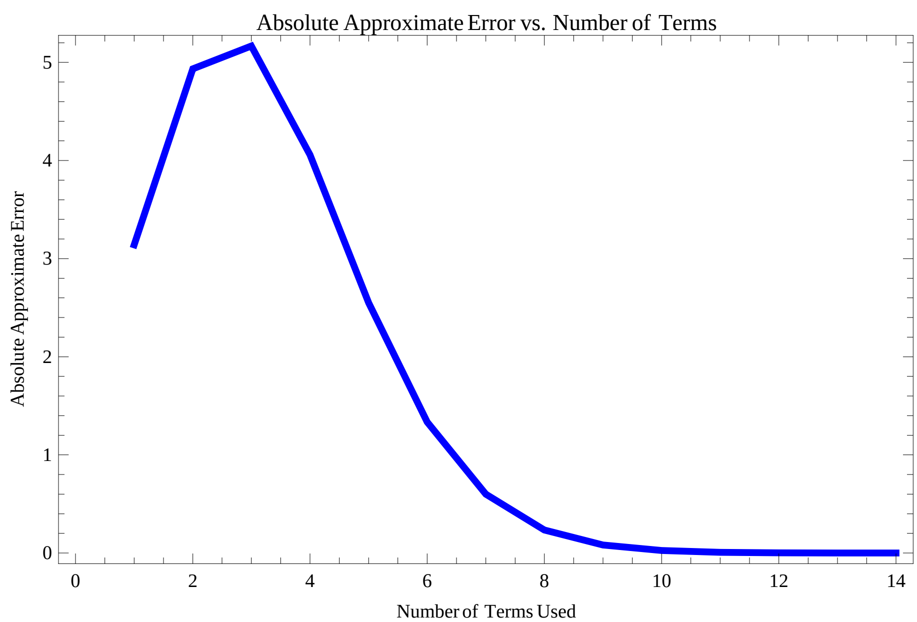
{ ———————————————— , Approximate Value}]



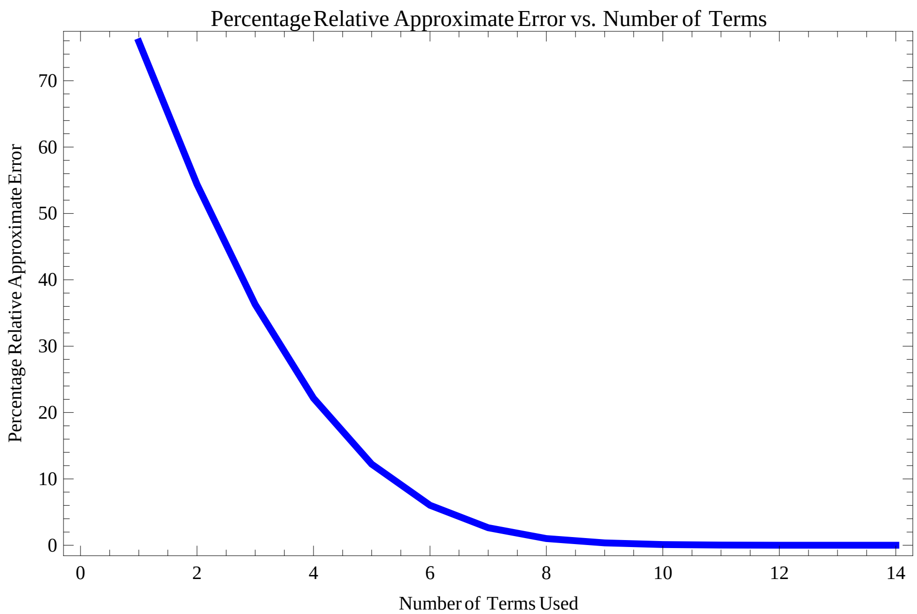
```
data = Table[ApproxError_i, {i, 2, n}];
Plot2 = ListPlot[data, Joined → True, PlotStyle → {Thickness[0.008`], RGBColor[0, 0, 1]},
  PlotLabel → "Approximate Error vs. Number of Terms", Frame → True,
  FrameLabel → {"Number of Terms Used", "Approximate Error"}, PlotRange → Full]
```



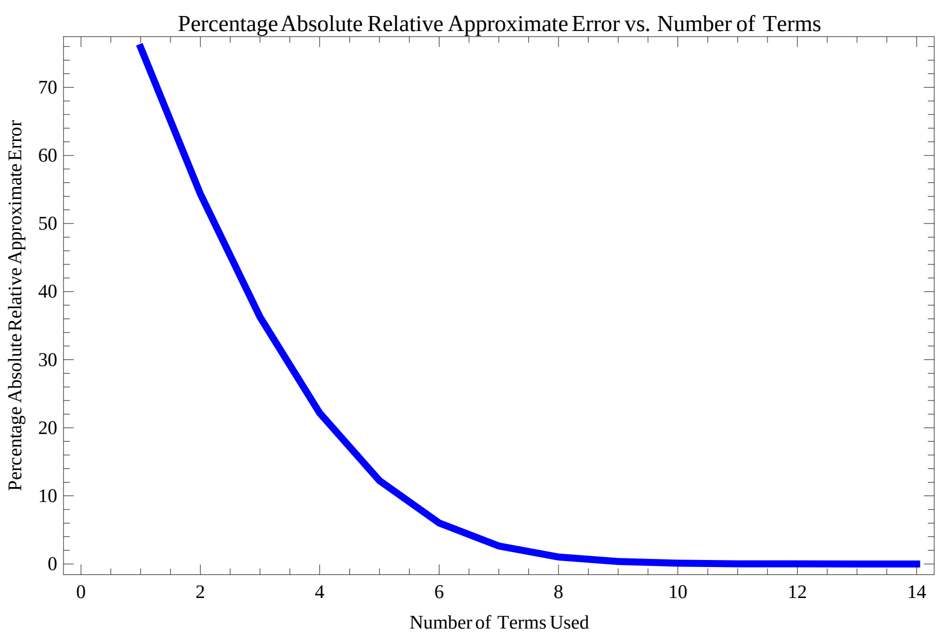

```
data = Table[AbsApproxError_i, {i, 2, n}];
Plot2 = ListPlot[data, Joined → True, PlotStyle → {Thickness[0.008`], RGBColor[0, 0, 1]},
  PlotLabel → "Absolute Approximate Error vs. Number of Terms", Frame → True,
  FrameLabel → {"Number of Terms Used", "Absolute Approximate Error"}, PlotRange → Full]
```



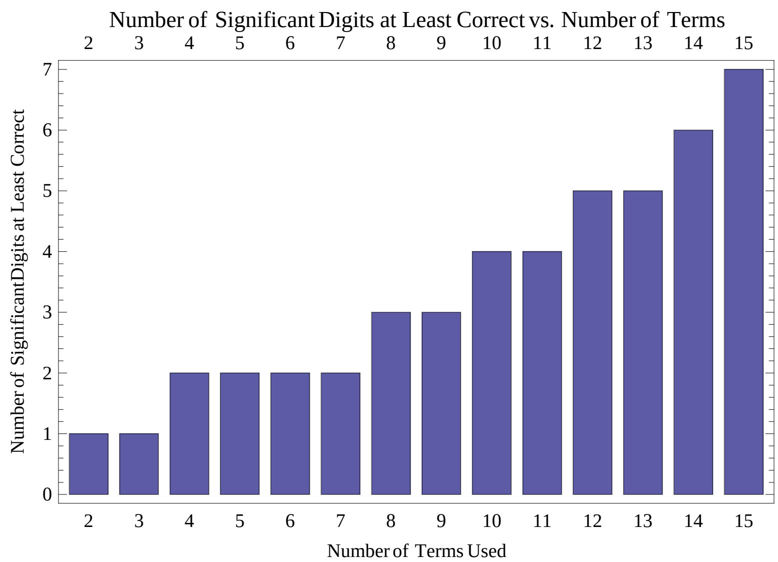
```
data = Table[RelApproxErrori, {i, 2, n}];
Plot2 = ListPlot[data, Joined → True, PlotStyle → {Thickness[0.008`], RGBColor[0, 0, 1]},
  PlotLabel → "Percentage Relative Approximate Error vs. Number of Terms", Frame → True,
  FrameLabel → {"Number of Terms Used", "Percentage Relative Approximate Error"},
  PlotRange → Full]
```




```
data = Table[AbsRelApproxErrori, {i, 2, n}];
Plot2 = ListPlot[data, Joined → True, PlotStyle → {Thickness[0.008`], RGBColor[0, 0, 1]},
  PlotLabel → "Percentage Absolute Relative Approximate Error vs. Number of Terms",
  Frame → True, FrameLabel → {"Number of Terms Used",
    "Percentage Absolute Relative Approximate Error"}, PlotRange → Full]
```



```
Needs["BarCharts`"]
data = Table[SigDigits_i, {i, 2, n}];
BarChart[{data}, PlotLabel →
  "Number of Significant Digits at Least Correct vs. Number of Terms", Frame → True,
 FrameLabel → {"Number of Terms Used", "Number of Significant Digits at Least Correct"},
 BarLabels → Range[2, n]]
```



Conclusion

This worksheet shows how the number of terms taken in a Maclaurin series affects the accuracy of the calculated answer through the analysis of error. Note that though approximate error shows the magnitude of the error, it does not indicate how bad the error really is. Hence, relative approximate error is used here to give a more complete picture of the state of error.

References

Measuring Errors.
See: http://numericalmethods.eng.usf.edu/nbm/gen/01aae/nbm_gen_aae_txt_measuringerror.pdf

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