The Quadratic Formula as a Way to Show the Subtraction of Small Numbers

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Initialization

Clearing the definitions of all symbols in the current context:

```
ClearAll[Evaluate[Context[] <> "*"]]
```

Introduction

The following worksheet illustrates the use of a quadratic equation solution for showing the effect of significant digits on round-off errors The user will enter the a, b and c values as given by the equation for the standard form of a quadratic equation: $ax^2 + bx + c = 0$, as well as the number of significant digits to be displayed in a table that will be created at the end of the program. Two variations of the quadratic equation solution will be used :

$$\text{(A)} \quad x1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$\text{(B)} \quad x1 = \frac{2c}{-b - \sqrt{b^2 - 4ac}}$$

$$x2 = \frac{2c}{-b + \sqrt{b^2 - 4ac}}$$

Section 1: Input Data

This is the only section where the user interacts with the program.

The quadratic formula is derived from the standard form of a quadratic equation: $ax^2 + bx + c = 0$.

- Enter coefficient *a*

```
a = 0.001;
```

- Enter coefficient *b*

```
b = -4.94627;
```

Enter coefficient *c*

```
c = 0.002;
```

- Enter range of significant digits to be used.

```
siglow = 7;
sighigh = 10;
```

This is the end of the user section. All information must be entered before proceeding to the next section. **RE-EVALUATE THE NOTEBOOK**.

Section 2: Significant Digit Arithmetic Functions

The following functions modify standard arithmetic operators allowing computation with the appropriate number of significant digits.

```
sdscale[sd_, x_] := Module[{},
  If[x == 0, m = sd, m = sd - (Floor[Log[10, Abs[x]]] + 1)] ;
  q = N[x * 10 ^ m];
  q = N[Floor[q] * 10 ^ (-m)]]

add[a_, b_] := N[a + b]
sub[a_, b_] := N[a - b]
div[a_, b_] := N[a / b]
mul[a_, b_] := N[a * b]

SdDyadic[op_, sd_, x_, y_] := Module[{},
  z = op[sdscale[sd, x], sdscale[sd, y]];
  sdscale[sd, z]]

sdadd[sd_, x_, y_] := SdDyadic[add, sd, x, y]
sdsub[sd_, x_, y_] := SdDyadic[sub, sd, x, y]
sdmul[sd_, x_, y_] := SdDyadic[mul, sd, x, y]
sddiv[sd_, x_, y_] := SdDyadic[div, sd, x, y]
```

Section 3: Calculation

The following calculations will be performed inside a loop so that the number of significant digits used can be varied as specified by the user.

Variation 1:

$$x1\mathrm{a} = \frac{-b + \sqrt{b^2 - 4\,a\,c}}{2\,a}$$

$$x2\mathrm{a} = \frac{-b - \sqrt{b^2 - 4\,a\,c}}{2\,a}$$

```
root1a = sdsub[dig, sdmul[dig, b, b], sdmul[dig, 4 a, c]];
top1a = sdadd[dig, -b, √root1a];
x1a[dig_] = sddiv[dig, top1a, 2 a];

root2a = sdsub[dig, sdmul[dig, b, b], sdmul[dig, 4 a, c]];
top2a = sdsub[dig, -b, √root2a];
x2a[dig_] = sddiv[dig, top2a, 2 a];
```

Variation 2:

$x1\text{b} = \frac{2c}{-b - \sqrt{b^2 - 4ac}}$

$x2\text{b} = \frac{2c}{-b + \sqrt{b^2 - 4ac}}$

```
root1b = sdsub[dig, sdmul[dig, b, b], sdmul[dig, 4 a, c]];
bott1b = sdsub[dig, -b, √root1b];
final1b = sddiv[dig, 2 c, bott1b];
x1b[dig_] = final1b;

root2b = sdsub[dig, sdmul[dig, b, b], sdmul[dig, 4 a, c]];
bott2b = sdadd[dig, -b, √root2b];
final2b = sddiv[dig, 2 c, bott2b];
x2b[dig_] = final2b;
```

Section 4: Table of Values

This table shows the values of x1a, x2a, x1b, and x2b and the number of significant digits used in their calculation.

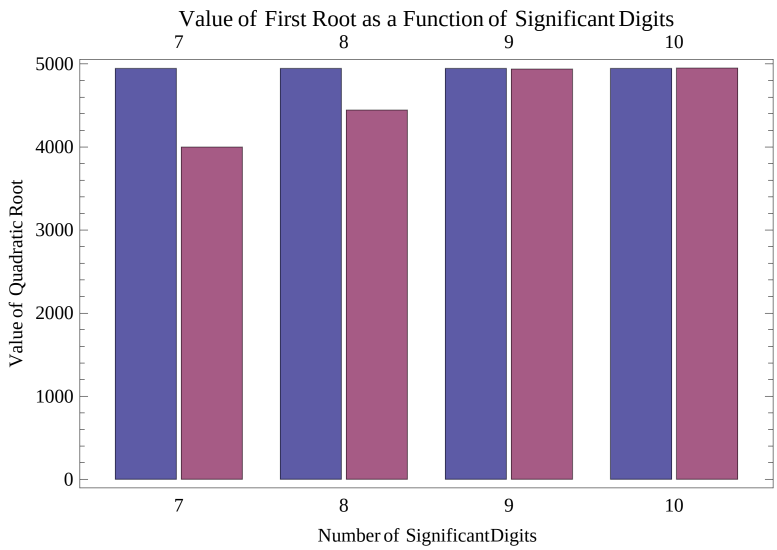
```
TableForm[Table[{i, x1a[i], x1b[i], x2a[i], x2b[i]}, {i, siglow, sighigh}],
 TableHeadings → {None, {"Digits", "x1a", "x1b", "x2a", "x2b"}},
 TableSpacing → {2, 2}]
```

Digits	x1a	x1b	x2a	x2b
7	4946.27	4000.	0.0005	0.000404345
8	4946.27	4444.44	0.00045	0.000404345
9	4946.27	4938.27	0.000405	0.000404345
10	4946.27	4950.5	0.000404	0.000404345

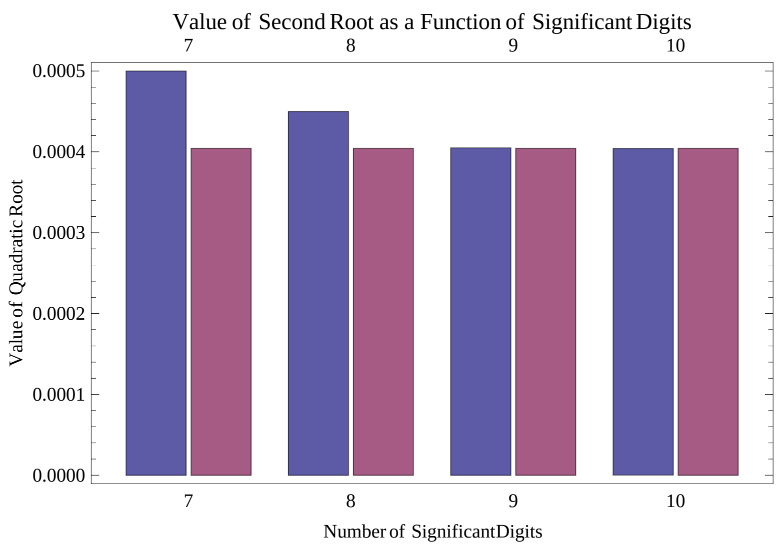
Section 5: Graphs

These bar graphs show the values of x1 and x2 for both variations of the quadratic function.

```
Needs["BarCharts`"];
data1 = Table[x1a[i], {i, siglow, sighigh}];
data2 = Table[x1b[i], {i, siglow, sighigh}];
BarChart[{data1, data2},
 PlotLabel → "Value of First Root as a Function of Significant Digits", Frame → True,
 FrameLabel → {"Number of Significant Digits", "Value of Quadratic Root"},
 BarLabels → Range[siglow, sighigh]]
```




```
data1 = Table[x2a[i], {i, siglow, sighigh}];
data2 = Table[x2b[i], {i, siglow, sighigh}];
BarChart[{data1, data2},
 PlotLabel → "Value of Second Root as a Function of Significant Digits", Frame → True,
 FrameLabel → {"Number of Significant Digits", "Value of Quadratic Root"},
 BarLabels → Range[siglow, sighigh]]
```



Conclusion

Subtraction of numbers that are nearly equal can result in unwanted inaccuracies. The number of significant digits used in calculations plays a large role in the creation of these inaccuracies and the magnitude of the round-off errors. Hence, when the accuracy of calculations is critical, it is necessary to understand possible sources of error and how they are best avoided.

References

Sources of Error. See: http://numericalmethods.eng.usf.edu/nbm/gen/01aae/nbm_gen_aae_txt_sourcesoferror.pdf
Propagation of Errors. See: http://numericalmethods.eng.usf.edu/nbm/gen/01aae/nbm_gen_aae_txt_propagationoferrors.pdf