# The Quadratic Formula as a Way to Show the Subtraction of Small Numbers 

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## Initialization

Clearing the definitions of all symbols in the current context:
ClearAll [Evaluate [Context [] <> "*"]]

## Introduction

The following worksheet illustrates the use of a quadratic equation solution for showing the effect of significant digits on round-off errors The user will enter the $\mathrm{a}, \mathrm{b}$ and c values as given by the equation for the standard form of a quadratic equation: $a x^{2}+b x+c=$ 0 , as well as the number of significant digits to be displayed in a table that will be created at the end of the program. Two variations of the quadratic equation solution will be used :

$$
\begin{aligned}
\text { (A) } \quad x 1 & =\frac{-b+\sqrt{b^{2}-4 a c}}{2 a} \\
x 2 & =\frac{-b-\sqrt{b^{2}-4 a c}}{2 a} \\
\text { (B) } \quad x 1 & =\frac{2 c}{-b-\sqrt{b^{2}-4 a c}} \\
x 2 & =\frac{2 c}{-b+\sqrt{b^{2}-4 a c}}
\end{aligned}
$$

## Section 1: Input Data

This is the only section where the user interacts with the program.
The quadratic formula is derived from the standard form of a quadratic equation: $a x^{2}+b x+c=0$.

- Enter coefficient $a$
$a=0.001 ;$
- Enter coefficientb
b $=-4.94627$;

Enter coefficient $c$

$$
c=0.002
$$

- Enter range of significant digits to be used.

```
siglow = 7;
sighigh = 10;
```

This is the end of the user section. All information must be entered before proceeding to the next section. RE-EVALUATE THE NOTEBOOK.

## Section 2: Significant Digit Arithmetic Functions

The following functions modify standard arithmetic operators allowing computation with the appropriate number of significant digits.

```
sdscale[sd_, x_] := Module [{},
    If[x== 0,m=sd,m=sd - (Floor[LOg[10, Abs[x]]] + 1)];
    q = N[x*10^m];
    q=N[Floor[q] * 10^(-m)]]
add[a_, b_] := N[a+b]
sub[a_, b_] := N[a-b]
div[a_, b_] :=N[a/b]
mul[a_, b_] := N[a*b]
SdDyadic[op_, sd_, x_, y_] := Module[{},
    z = op[sdscale[sd, x], sdscale[sd, y]];
    sdscale[sd, z]]
sdadd[sd_, x_, y_] := SdDyadic[add, sd, x, y]
sdsub[sd_, x_, y_] := SdDyadic[sub, sd, x, y]
sdmul[sd_, x_, y_] := SdDyadic[mul, sd, x, y]
sddiv[sd_, x_, y_] := SdDyadic[div, sd, x, y]
```


## Section 3: Calculation

The following calculations will be performed inside a loop so that the number of significant digits used can be varied as specified by the user.
Variation 1:
$x 1 \mathrm{a}=\frac{-b+\sqrt{b^{2}-4 a c}}{2 a}$
$x 2 \mathrm{a}=\frac{-b-\sqrt{b^{2}-4 a c}}{2 a}$

```
rootla = sdsub[dig, sdmul[dig, b, b], sdmul[dig, 4a, c]];
top1a = sdadd [dig, -b, \sqrt{}{\mathrm{ root1a}}];
x1a[dig_] = sddiv[dig, top1a, 2 a];
root2a = sdsub[dig, sdmul[dig, b, b], sdmul[dig, 4a, c]];
top2a = sdsub[dig, -b, \sqrt{}{\mathrm{ root2a}}];
x2a[dig_] = sddiv[dig, top2a, 2a];
```

Variation 2:

```
\(x 1 \mathrm{~b}=\frac{2 c}{-b-\sqrt{b^{2}-4 a c}}\)
\(x 2 \mathrm{~b}=\frac{2 c}{-b+\sqrt{b^{2}-4 a c}}\)
    root1b = sdsub[dig, sdmul[dig, b, b], sdmul[dig, 4 a, c] ];
    bott1b \(=\) sdsub \([\operatorname{dig},-b, \sqrt{\text { root1b }}]\);
    final1b = sddiv[dig, \(2 \mathrm{c}, \mathrm{bott1b}\) ];
    x1b [dig_] = finallb;
    root \(2 \mathrm{~b}=\) sdsub [dig, sdmul [dig, b, b], sdmul [dig, \(4 \mathrm{a}, \mathrm{c}\) ] ];
    bott \(2 \mathrm{~b}=\) sdadd \([\operatorname{dig},-b, \sqrt{\operatorname{root} 2 \mathrm{~b}}]\);
    final2b = sddiv[dig, \(2 c\), bott2b];
    x2b[dig_] = final2b;
```


## Section 4: Table of Values

This table shows the values of $\mathrm{x} 1 \mathrm{a}, \mathrm{x} 2 \mathrm{a}, \mathrm{x} 1 \mathrm{~b}$, and x 2 b and the number of significant digits used in their calculation.

```
TableForm[Table[{i, x1a[i], x1b[i], x2a[i], x2b[i]}, {i, siglow, sighigh}],
    TableHeadings -> {None, {"Digits", "x1a", "x1b", "x2a", "x2b"}},
    TableSpacing }->{2,2}
\begin{tabular}{lllll} 
Digits & x1a & x1b & x2a & x2b \\
\hline 7 & 4946.27 & 4000. & 0.0005 & 0.000404345 \\
8 & 4946.27 & 4444.44 & 0.00045 & 0.000404345 \\
9 & 4946.27 & 4938.27 & 0.000405 & 0.000404345 \\
10 & 4946.27 & 4950.5 & 0.000404 & 0.000404345
\end{tabular}
```


## Section 5: Graphs

These bar graphs show the values of x 1 and x 2 for both variations of the quadratic function.

```
Needs["BarCharts`"];
data1 = Table[x1a[i], {i, siglow, sighigh}];
data2 = Table[x1b[i], {i, siglow, sighigh}];
BarChart [{data1, data2},
    PlotLabel }->\mathrm{ "Value of First Root as a Function of Significant Digits", Frame t True,
    FrameLabel }->\mathrm{ {"Number of Significant Digits", "Value of Quadratic Root"},
    BarLabels }->\mathrm{ Range [siglow, sighigh]]
```

Value of First Root as a Function of Significant Digits


```
data1 = Table[x2a[i], {i, siglow, sighigh}];
data2 = Table[x2b[i], {i, siglow, sighigh}];
BarChart[{data1, data2},
    PlotLabel }->\mathrm{ "Value of Second Root as a Function of Significant Digits", Frame }->\mathrm{ True,
    FrameLabel }->\mathrm{ {"Number of Significant Digits", "Value of Quadratic Root"},
    BarLabels }->\mathrm{ Range[siglow, sighigh]]
```

Value of Second Root as a Function of Significant Digits


## Conclusion

Subtraction of numbers that are nearly equal can result in unwanted inaccuracies. The number of significant digits used in calculations plays a large role in the creation of these inaccuracies and the magnitude of the round-off errors. Hence, when the accuracy of calculations is critical, it is necessary to understand possible sources of error and how they are best avoided.

## References

Sources of Error. See: http://numericalmethods.eng.usf.edu/nbm/gen/01aae/nbm_gen_aae_txt_sourcesoferror.pdf
Propagation of Errors. See: http://numericalmethods.eng.usf.edu/nbm/gen/01aae/nbm_gen_aae_txt_propagationoferrors.pdf

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