

# Concepts of True Error: True Error, Absolute True Error, Relative True Error, and Absolute Relative True Error

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## Initialization

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Clearing the definitions of all symbols in the current context:

```
ClearAll [Evaluate [Context [] <> "*" ]]
```

## Introduction

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The following worksheet demonstrates how to calculate different definitions related to true error, such as true error, absolute true error, relative true error, and absolute relative true error. The concept is demonstrated using an example of a Maclaurin series. The user will choose which function to perform the calculation for in the *Input* section of the program. The choices are given as 1 for  $e^x$ , 2 for  $\sin(x)$ , and 3 for  $\cos(x)$ . The *true value* of these functions will be assumed as given by the *Mathematica* commands for these functions.

## Section 1: Input Data

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This is the only section where the user interacts with the program.

- Pick the function of your desire by choosing an integer: 1 for  $e^x$ ; 2 for  $\sin(x)$ ; 3 for  $\cos(x)$

```
funcchoice = 1;
```

- Maximum number of terms to use in the Maclaurin series

```
n = 15;
```

- Value of  $x$  at which the function is calculated

```
xv = 1.6;
```

This is the end of the user section. All information must be entered before proceeding to the next section. **RE-EVALUATE THE NOTEBOOK.**

## Section 2: Procedure

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First, determine which function will be used in the calculations, based on the user input. Once the function is determined, the value is calculated using a Maclaurin series in a repetitive loop.

```

sumprevious = 0;
If[funcchoice == 1,
  Do[sumpresenti = sumprevious +  $\frac{xv^{i-1}}{(i-1)!}$ ;
    sumprevious = sumpresenti,
    {i, 1, n}];
  f[x_] := N[e^x];
If[funcchoice == 2,
  Do[sumpresenti = sumprevious +  $\frac{(-1)^{i-1} * xv^{2+i-1}}{(2 * i - 1)!}$ ;
    sumprevious = sumpresenti,
    {i, 1, n}];
  f[x_] := N[Sin[x]];
If[funcchoice == 3,
  Do[sumpresenti = sumprevious +  $\frac{(-1)^{i+1} * xv^{2+i-2}}{(2 * i - 2)!}$ ;
    sumprevious = sumpresenti,
    {i, 1, n}];
  f[x_] := N[Cos[x]];

```

Using *Mathematica* to calculate true error, absolute true error, relative true error, and absolute relative true error for each term. Once these error values are calculated, determining if sufficient iterations have been performed by comparing the prespecified tolerance to the absolute relative true error.

```

Do[
  TrueErrori = f[xv] - sumpresenti;
  AbsTrueErrori = Abs[f[xv] - sumpresenti];
  RelTrueErrori =  $\frac{(f[xv] - sumpresenti) * 100}{f[xv]}$ ;
  AbsRelTrueErrori = Abs[ $\frac{(f[xv] - sumpresenti) * 100}{f[xv]}$ ];,
  {i, 1, n}];

```

## Section 3: Table of Values

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This table shows the true value, true error, absolute true error, relative true error, absolute relative true error, and if the prespecified tolerance has been met, all as a function of the number of the number of terms used.

```
TableForm [
  Table[{i, f[xv], TrueErrori, AbsTrueErrori, RelTrueErrori, AbsRelTrueErrori}, {i, 1, n}],
  TableHeadings → {None, {"Terms Used", "True Value", "True Error", "Abs True Error",
    "Rel True Error", "Abs Rel True Error"}},
  TableSpacing →
  {2,
  2}]
```

Terms Used	True Value	True Error	Abs True Error	Rel True Error	Abs Rel True Err
1	4.95303	3.95303	3.95303	79.8103	79.8103
2	4.95303	2.35303	2.35303	47.5069	47.5069
3	4.95303	1.07303	1.07303	21.6642	21.6642
4	4.95303	0.390366	0.390366	7.88135	7.88135
5	4.95303	0.117299	0.117299	2.36823	2.36823
6	4.95303	0.0299178	0.0299178	0.604029	0.604029
7	4.95303	0.00661607	0.00661607	0.133576	0.133576
8	4.95303	0.00128997	0.00128997	0.026044	0.026044
9	4.95303	0.000224748	0.000224748	0.00453759	0.00453759
10	4.95303	0.000035376	0.000035376	0.000714229	0.000714229
11	4.95303	$5.07641 \times 10^{-6}$	$5.07641 \times 10^{-6}$	0.000102491	0.000102491
12	4.95303	$6.69196 \times 10^{-7}$	$6.69196 \times 10^{-7}$	0.0000135108	0.0000135108
13	4.95303	$8.1568 \times 10^{-8}$	$8.1568 \times 10^{-8}$	$1.64683 \times 10^{-6}$	$1.64683 \times 10^{-6}$
14	4.95303	$9.24447 \times 10^{-9}$	$9.24447 \times 10^{-9}$	$1.86643 \times 10^{-7}$	$1.86643 \times 10^{-7}$
15	4.95303	$9.78927 \times 10^{-10}$	$9.78927 \times 10^{-10}$	$1.97642 \times 10^{-8}$	$1.97642 \times 10^{-8}$

## Section 4: Graphs

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The following graphs show the true value, calculated value, true error, absolute true error, relative true error, and absolute relative true error, all as a function of the number of terms.

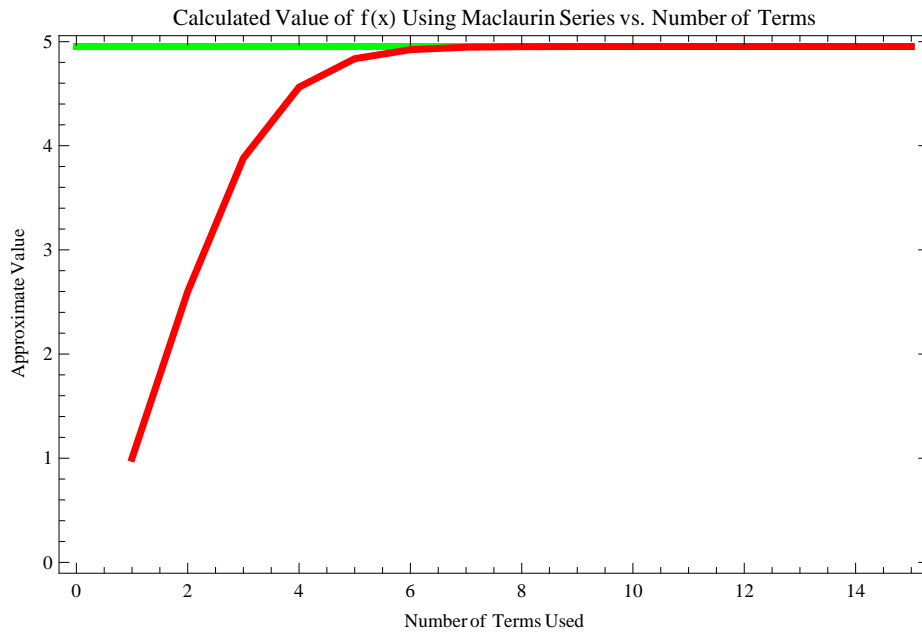
```

Plot1 = Plot[f[xv], {x, 0, n}, PlotStyle -> {Thickness[0.008`], RGBColor[0, 1, 0]};
data = Table[sumpresenti, {i, 1, n}];
Plot2 = ListPlot[data, Joined -> True, PlotStyle -> {Thickness[0.008`], RGBColor[1, 0, 0]};
Needs["PlotLegends`"];
l1 = Graphics[{Green, Line[{{0, 0}, {1, 0}}]}];
l2 = Graphics[{Red, Line[{{0, 0}, {1, 0}}]}];
Legend[{l1, "Exact Value"}, {l2, "Approximate Value"}]
Show[Plot1, Plot2, PlotLabel ->
  "Calculated Value of f(x) Using Maclaurin Series vs. Number of Terms", Frame -> True,
  FrameLabel -> {"Number of Terms Used", "Approximate Value"}, PlotRange -> Automatic]

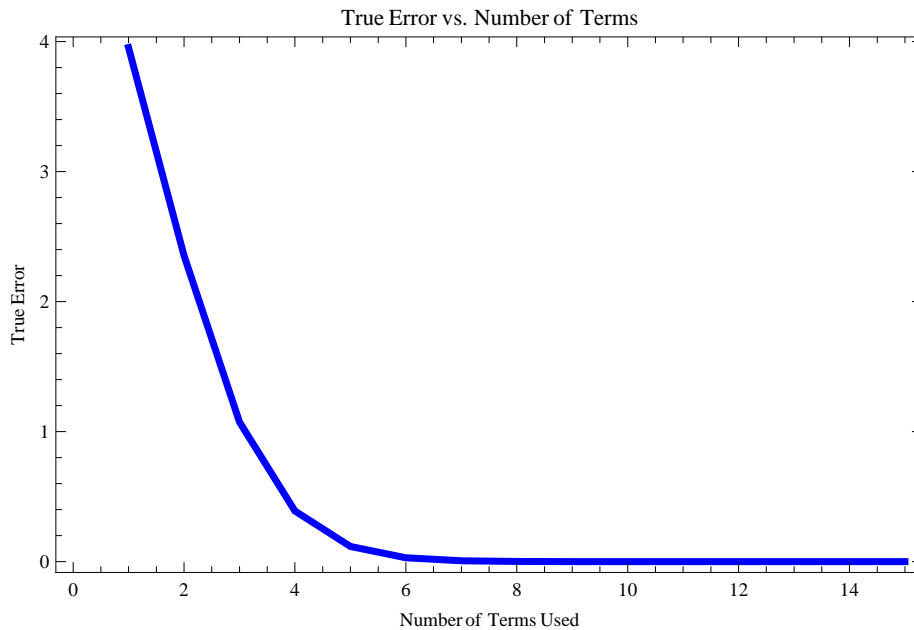
```

Legend[ {  , Exact Value },

{ \_\_\_\_\_ , Approximate Value}]



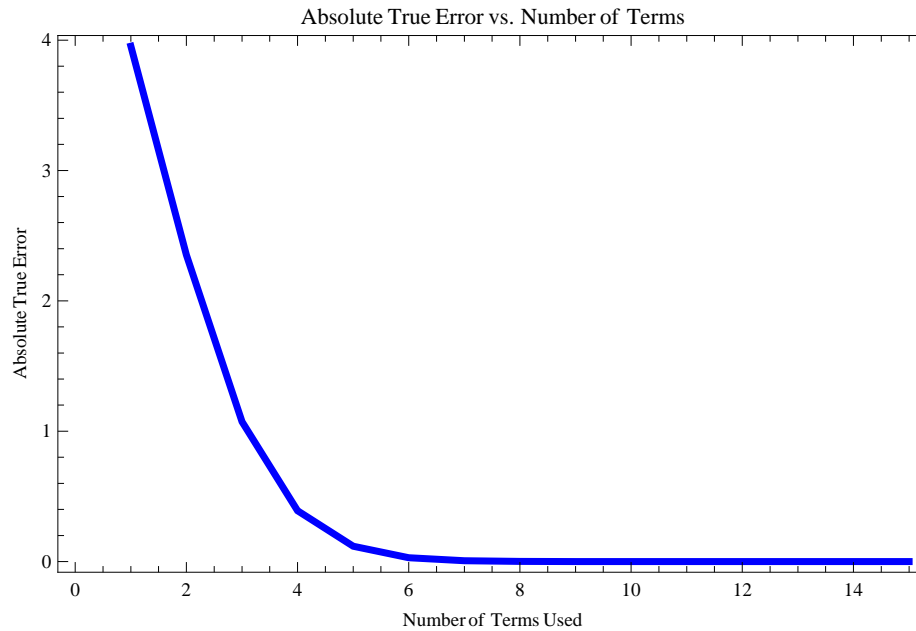
```
data = Table[TrueErrori, {i, 1, n}];
Plot2 = ListPlot[data, Joined → True, PlotStyle → {Thickness[0.008`], RGBColor[0, 0, 1]},
  PlotLabel → "True Error vs. Number of Terms", Frame → True,
  FrameLabel → {"Number of Terms Used", "True Error"}, PlotRange → Full]
```



```

data = Table[AbsTrueErrori, {i, 1, n}];
Plot2 = ListPlot[data, Joined → True, PlotStyle → {Thickness[0.008`], RGBColor[0, 0, 1]},
  PlotLabel → "Absolute True Error vs. Number of Terms", Frame → True,
  FrameLabel → {"Number of Terms Used", "Absolute True Error"}, PlotRange → Full]

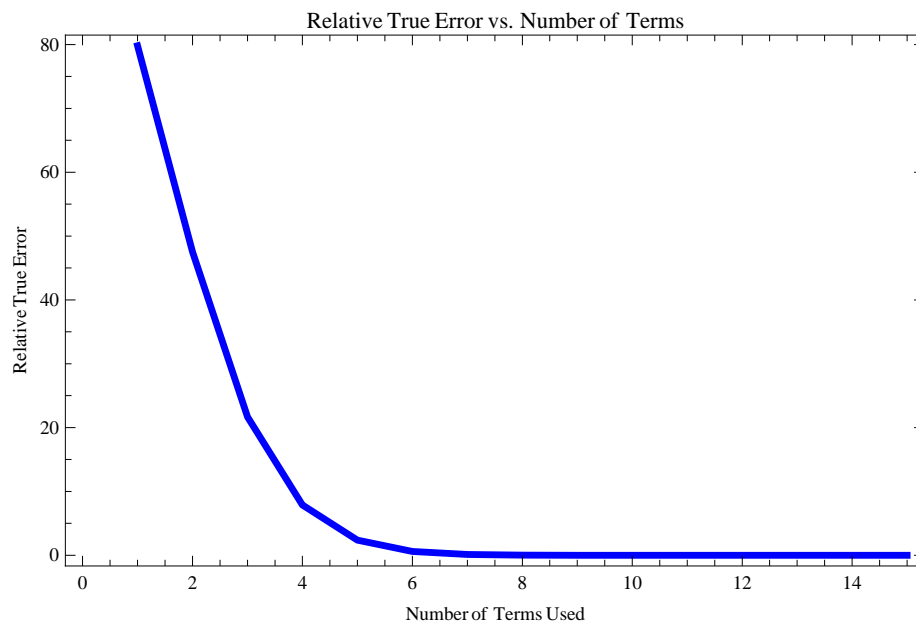
```



```

data = Table[RelTrueErrori, {i, 1, n}];
Plot2 = ListPlot[data, Joined → True, PlotStyle → {Thickness[0.008`], RGBColor[0, 0, 1]},
  PlotLabel → "Relative True Error vs. Number of Terms", Frame → True,
  FrameLabel → {"Number of Terms Used", "Relative True Error"}, PlotRange → Full]

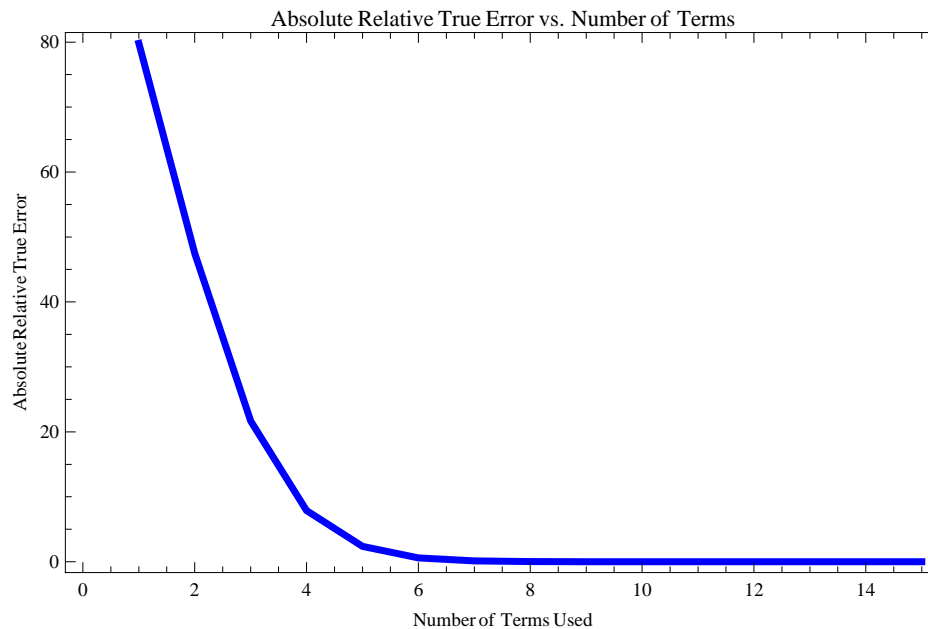
```



```

data = Table[AbsRelTrueErrori, {i, 1, n}];
Plot2 = ListPlot[data, Joined → True, PlotStyle → {Thickness[0.008`], RGBColor[0, 0, 1]},
  PlotLabel → "Absolute Relative True Error vs. Number of Terms", Frame → True,
  FrameLabel → {"Number of Terms Used", "Absolute Relative True Error"}, PlotRange → Full]

```



## Conclusion

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This worksheet shows how the number of terms taken in a Maclaurin series affects the accuracy of the calculated answer through the analysis of error. Note that though true error shows the magnitude of the error, it does not indicate how bad the error really is. Hence, relative true error is used here to give a more complete picture of the state of error.

## References

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Measuring Errors.

See: [http://numericalmethods.eng.usf.edu/nbm/gen/01aae/nbm\\_gen\\_aae\\_txt\\_measuringerror.pdf](http://numericalmethods.eng.usf.edu/nbm/gen/01aae/nbm_gen_aae_txt_measuringerror.pdf)

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