Topic : Bisection Method - Roots of Equations
Simulation : Graphical Simulation of the Method
Language : Mathmatica 4.1
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Abstract : This simulation shows how the bisection method for finding roots of an equation $f[x] = 0$ works.

■ INPUTS: Enter the Following

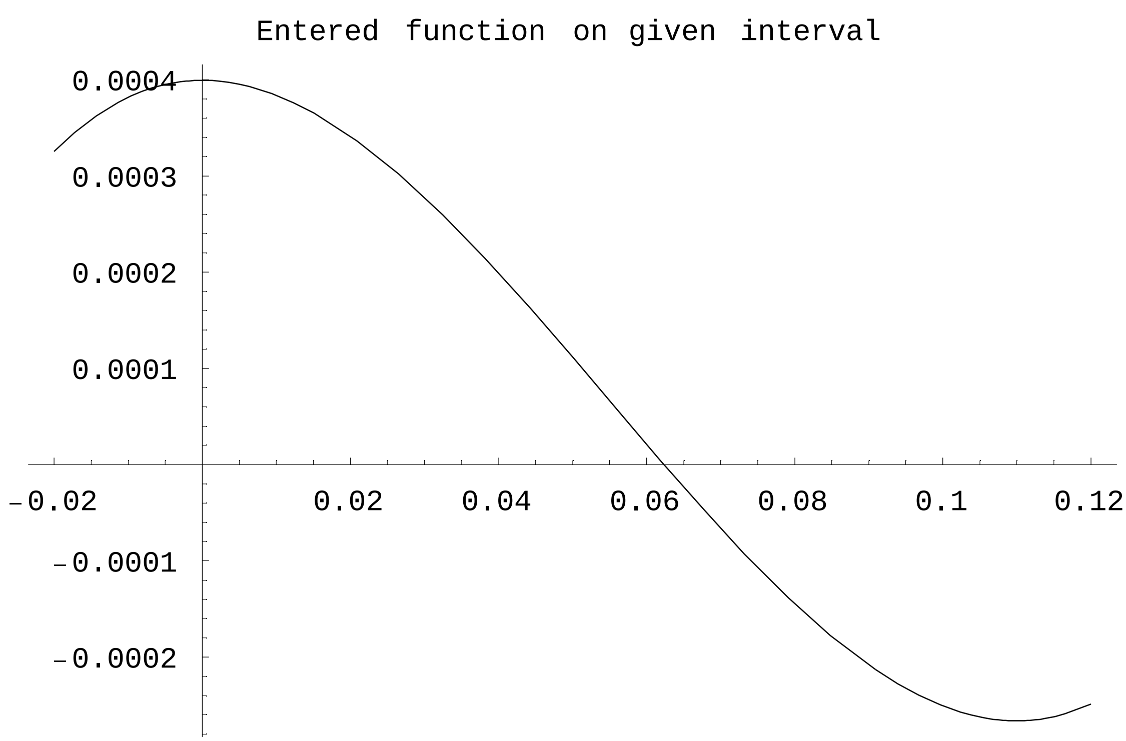
Function in $f[x] = 0$

```
In[91]:= f[x_] := x^3 - 0.165 * x^2 + 3.993 * 10^-4
```

Range of 'x' you want to see the function

```
In[92]:= xb = -0.02;
         xe = 0.12;
```

```
In[94]:= curve = Plot[f[x], {x, xb, xe}, PlotLabel →
             "Entered function on given interval", TextStyle → {FontSize → 11}];
```



Lower initial guess

In[95]:=
```
xl = 0.0;
```

Upper initial guess

In[96]:=
```
xu = 0.11;
```

■ SOLUTION

In[97]:=
```
maxi = f[xb];
mini = f[xb];
step = (xe - xb) / 10;
Do[ If[f[i] > maxi, maxi = f[i]];
  If[f[i] < mini, mini = f[i]], {i, xb, xe, step}];
tot = maxi - mini;
mini = mini - 0.1 * tot;
maxi = maxi + 0.1 * tot;
```

Check first if the lower and upper guesses bracket the root of the equation

In[104]:=
```
f[xl]
```

Out[104]= 0.0003993

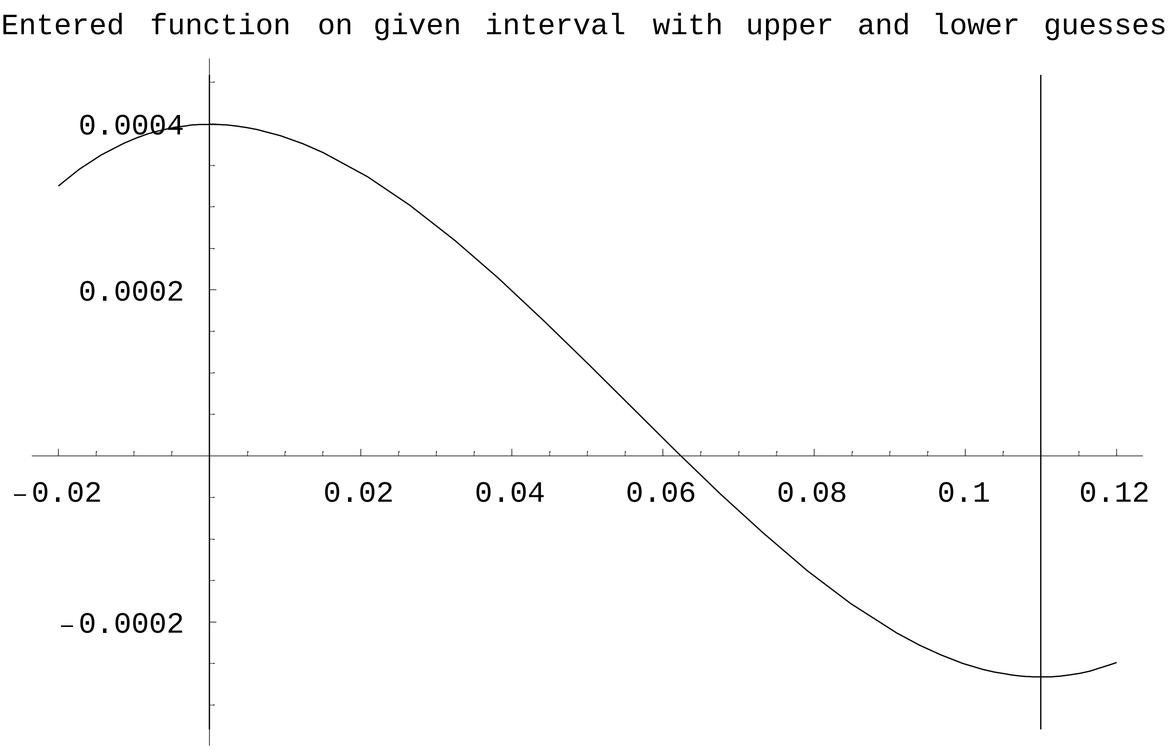
In[105]:=
```
f[xu]
```

Out[105]= -0.0002662

In[106]:=
```
% * %%
```

Out[106]= -1.06294×10^{-7}

```
In[107]:= Show[Graphics[Line[{{xu, maxi}, {xu, mini}}]], curve,
  Graphics[Line[{{xl, maxi}, {xl, mini}}]], Axes → True, PlotLabel →
   "Entered function on given interval with upper and lower guesses",
  TextStyle → {FontSize → 11}];
```



Iteration 1

New estimate of root

In[108]:= $x_r = (x_u + x_l)/2$

Out[108]= 0.055

Finding the value of the function at the lower and upper guesses and the estimated root

In[109]:= $f[x_l]$

Out[109]= 0.0003993

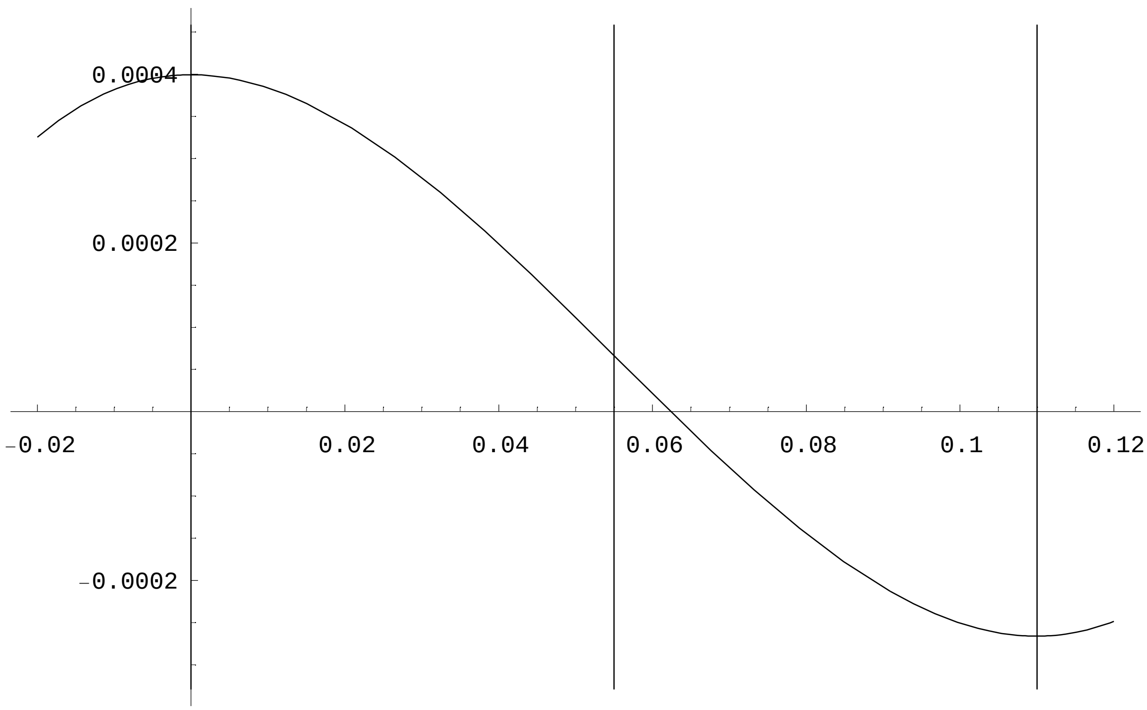
In[110]:= $f[x_u]$

Out[110]= -0.0002662

In[111]:=
```
f[x_r]
```
Out[111]=
```
0.00006655
```

In[112]:=
```
Show[Graphics[Line[{{x_u, maxi}, {x_u, mini}}]],
  curve, Graphics[Line[{{x_l, maxi}, {x_l, mini}}]],
  Graphics[Line[{{x_r, maxi}, {x_r, mini}}]], Axes → True,
  PlotLabel → "Entered function on given interval with upper and
     lower guesses and estimated root", TextStyle → {FontSize → 9}];
```

Entered function on given interval with upper and lower guesses and estimated root



Check the interval between which the root lies. Does it lie in (x_l, x_r) or (x_r, x_u)?

In[113]:=
```
If[f[x_r] * f[x_u] ≤ 0, x_l = x_r, x_u = x_r];
```

In[114]:=
```
x_u
```
Out[114]=
```
0.11
```

In[115]:=
```
x_l
```
Out[115]=
```
0.055
```

Store the value of the previous guess in x_p to calculate absolute relative approximate error.

In[116]:=
```
x_p = x_r;
```

Iteration 2

New estimate of root

In[117]:= $x_r = (x_l + x_u)/2$

Out[117]= 0.0825

Finding the value of the function at the lower and upper guesses and the estimated root

In[118]:= $f[x_l]$

Out[118]= 0.00006655

In[119]:= $f[x_u]$

Out[119]= −0.0002662

In[120]:= $f[x_r]$

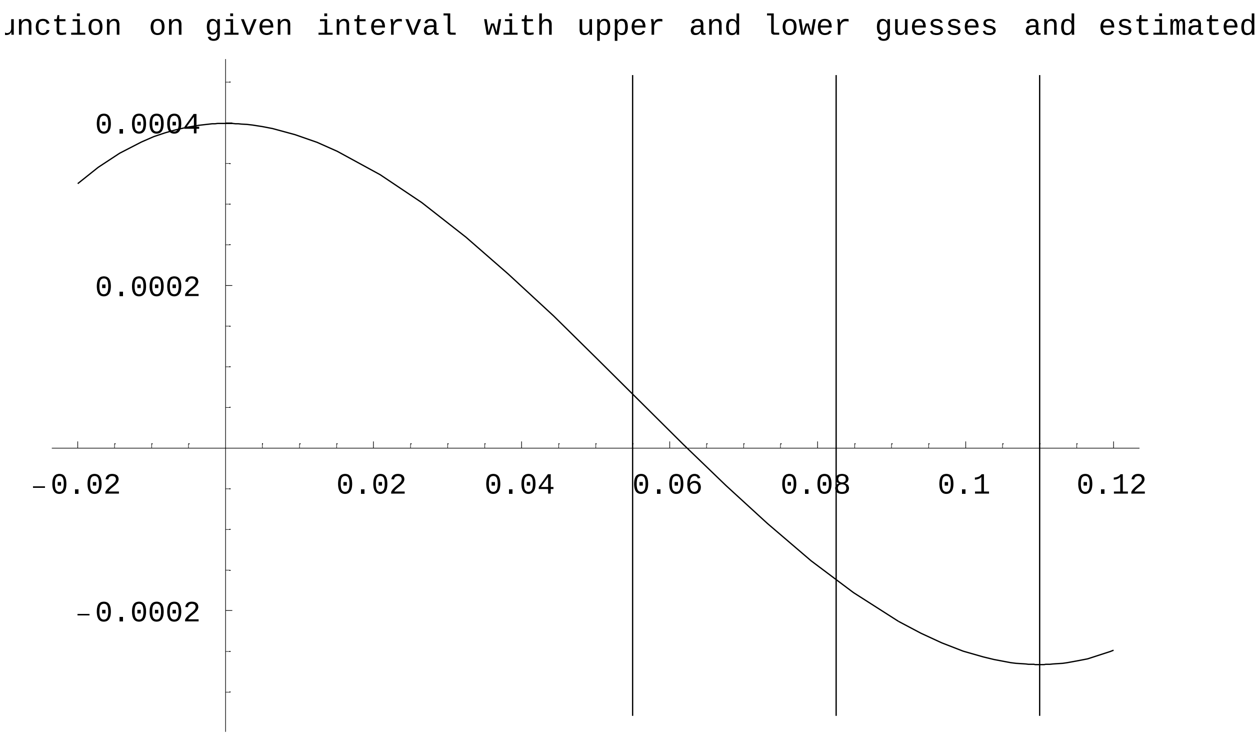
Out[120]= −0.000162216

Absolute relative approximate error, Abs[$_a$].

In[121]:= $\epsilon_a = \text{Abs}[(x_r - x_p)/x_r * 100]$

Out[121]= 33.3333

In[122]:=
```
Show[Graphics[Line[{{xu, maxi}, {xu, mini}}]],
  curve, Graphics[Line[{{xl, maxi}, {xl, mini}}]],
  Graphics[Line[{{xr, maxi}, {xr, mini}}]], Axes → True,
  PlotLabel → "Entered function on given interval with upper and
    lower guesses and estimated root", TextStyle → {FontSize → 11}];
```



Check the interval between which the root lies. Does it lie in (x_l, x_r) or (x_r, x_u)?

In[123]:=
```
If[f[xr] * f[xu] ≤ 0, xl = xr, xu = xr];
```

In[124]:=
```
xu
```

Out[124]= 0.0825

In[125]:=
```
xl
```

Out[125]= 0.055

Store the value of the previous guess in x_p to calculate absolute relative approximate error.

In[126]:=
```
xp = xr;
```

Iteration 3

New estimate of root

In[127]:= $x_r = (x_l + x_u) / 2$

Out[127]= 0.06875

Finding the value of the function at the lower and upper guesses and the estimated root

In[128]:= $f[x_l]$

Out[128]= 0.00006655

In[129]:= $f[x_u]$

Out[129]= −0.000162216

In[130]:= $f[x_r]$

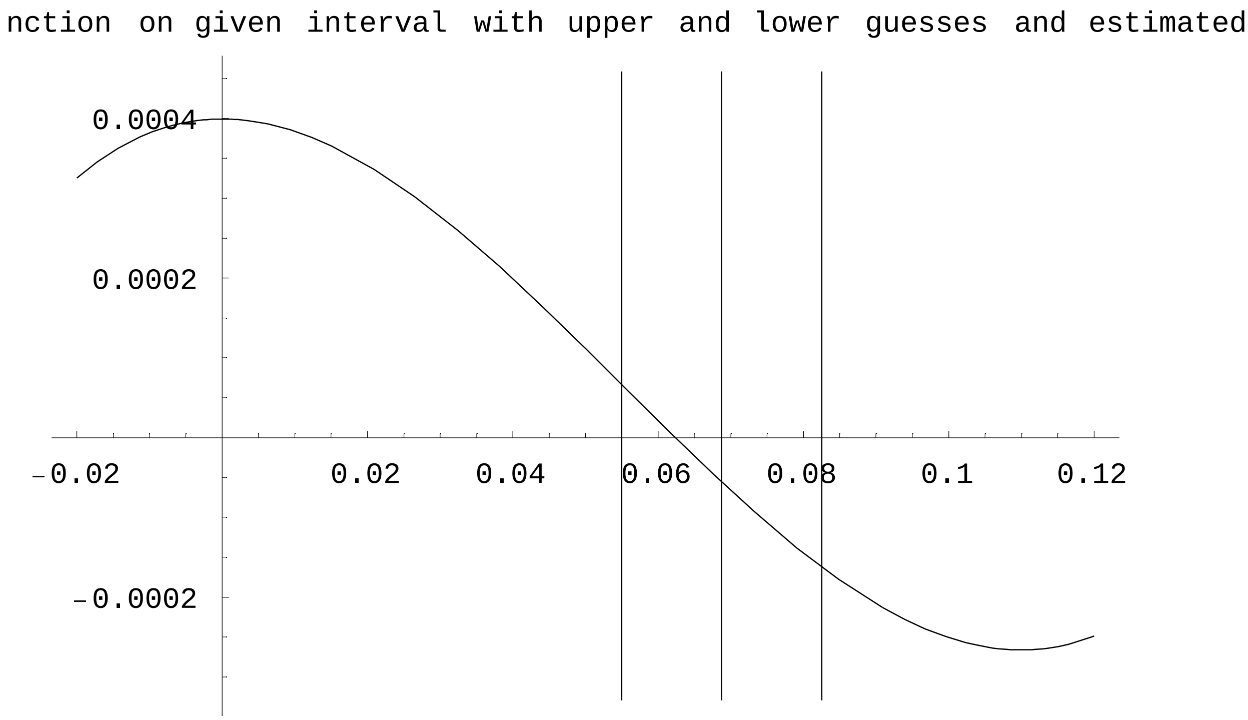
Out[130]= −0.0000556316

Absolute relative approximate error, Abs[$_a$].

In[131]:= $\epsilon_a = \text{Abs}[(x_r - x_p) / x_r * 100]$

Out[131]= 20.

```
In[132]:= Show[Graphics[Line[{{xu, maxi}, {xu, mini}}]],
            curve, Graphics[Line[{{xl, maxi}, {xl, mini}}]],
            Graphics[Line[{{xr, maxi}, {xr, mini}}]], Axes → True,
            PlotLabel → "Entered function on given interval with upper and
               lower guesses and estimated root", TextStyle → {FontSize → 11}];
```



Check the interval between which the root lies. Does it lie in (x_l, x_r) or (x_r, x_u)?

```
In[133]:= If[f[xr] * f[xu] ≤ 0, xl = xr, xu = xr];

In[134]:= xu

Out[134]= 0.06875

In[135]:= xl

Out[135]= 0.055
```

Store the value of the previous guess in x_p to calculate absolute relative approximate error.

```
In[136]:= xp = xr;
```

Iteration 4

New estimate of root

In[137]:= $x_r = (x_l + x_u)/2$

Out[137]= 0.061875

Finding the value of the function at the lower and upper guesses and the estimated root

In[138]:= $f[x_l]$

Out[138]= 0.00006655

In[139]:= $f[x_u]$

Out[139]= −0.0000556316

In[140]:= $f[x_r]$

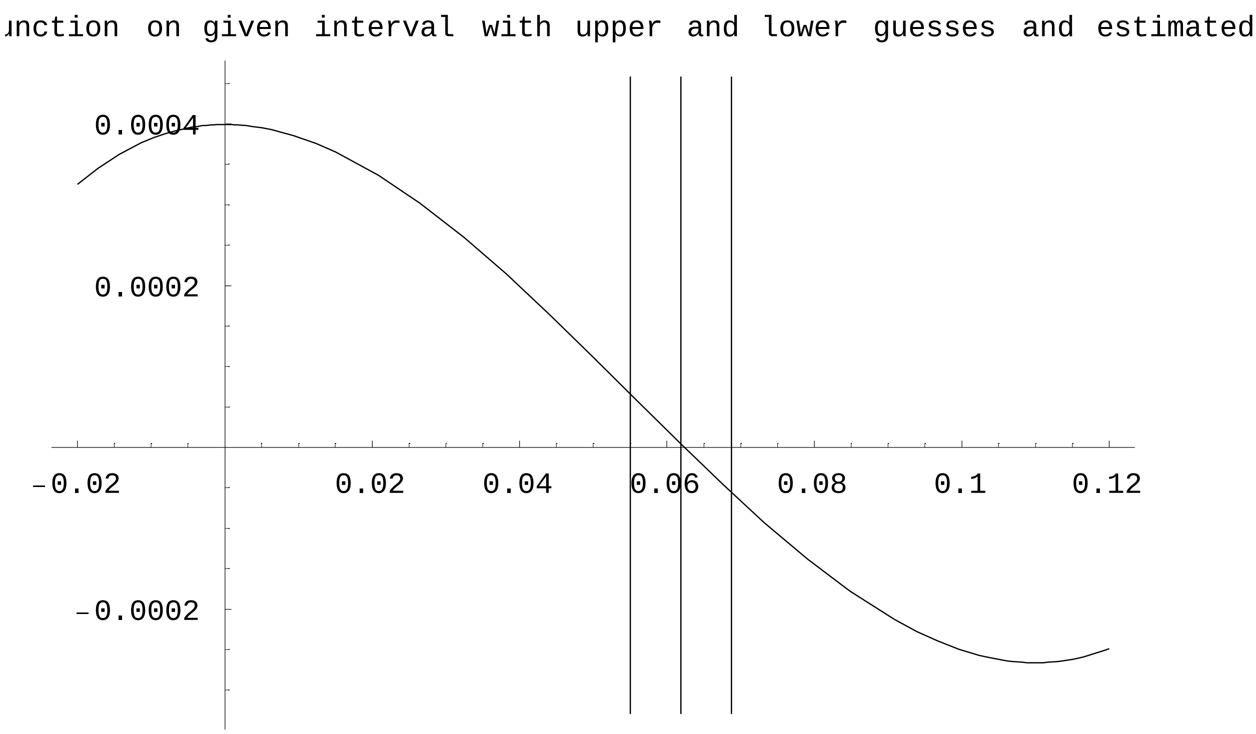
Out[140]= 4.48433×10^{-6}

Absolute relative approximate error, Abs[$_a$].

In[141]:= $\epsilon_a = \text{Abs}[(x_r - x_p)/x_r * 100]$

Out[141]= 11.1111

```
In[142]:= Show[Graphics[Line[{{xu, maxi}, {xu, mini}}]],
            curve, Graphics[Line[{{xl, maxi}, {xl, mini}}]],
            Graphics[Line[{{xr, maxi}, {xr, mini}}]], Axes → True,
            PlotLabel → "Entered function on given interval with upper and
               lower guesses and estimated root", TextStyle → {FontSize → 11}];
```



Check the interval between which the root lies. Does it lie in (x_l,x_r) or (x_r,x_u)?

```
In[143]:= If[f[xr] * f[xu] ≤ 0, xl = xr, xu = xr];

In[144]:= xu

Out[144]= 0.06875

In[145]:= xl

Out[145]= 0.061875
```

Store the value of the previous guess in x_p to calculate absolute relative approximate error.

```
In[146]:= xp = xr;
```

Iteration 5

New estimate of root

In[147]:= $x_r = (x_l + x_u) / 2$

Out[147]= 0.0653125

Finding the value of the function at the lower and upper guesses and the estimated root

In[148]:= $f[x_l]$

Out[148]= 4.48433×10^{-6}

In[149]:= $f[x_u]$

Out[149]= −0.0000556316

In[150]:= $f[x_r]$

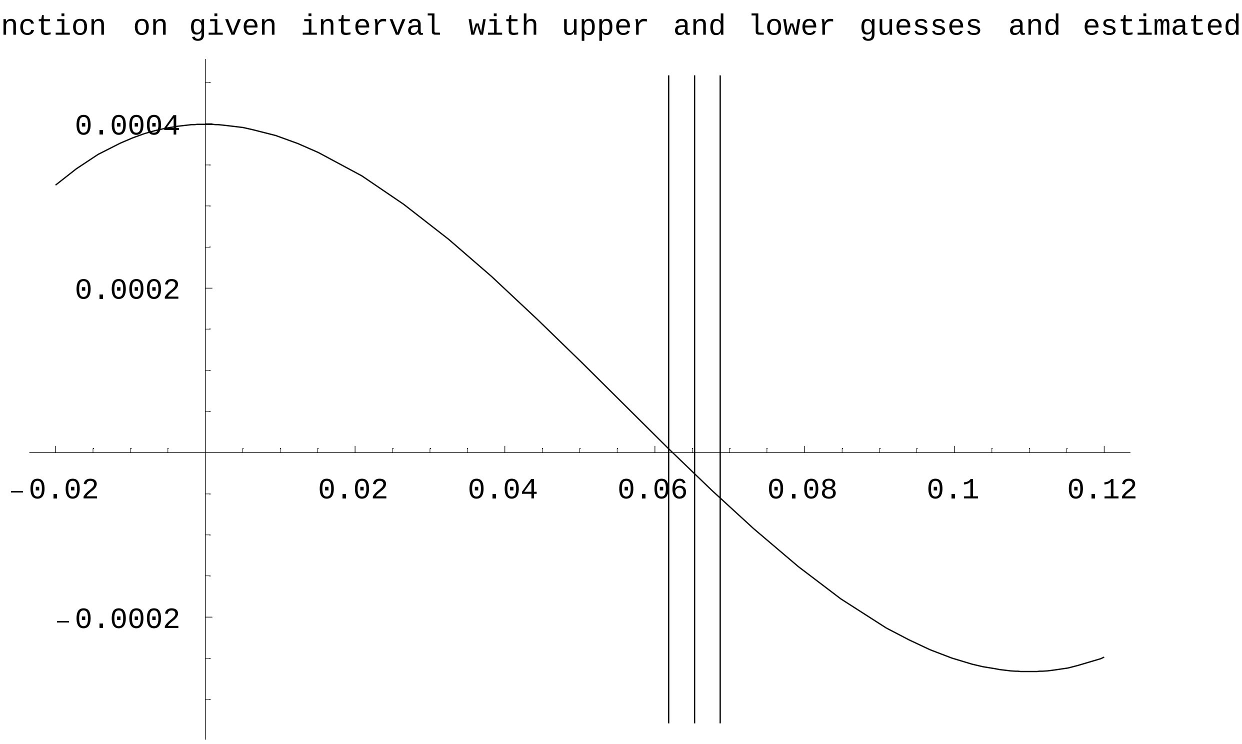
Out[150]= −0.0000259392

Absolute relative approximate error, Abs[$_a$].

In[151]:= $\epsilon_a = \text{Abs}[(x_r - x_p) / x_r * 100]$

Out[151]= 5.26316

```
In[152]:= Show[Graphics[Line[{{xu, maxi}, {xu, mini}}]],
            curve, Graphics[Line[{{xl, maxi}, {xl, mini}}]],
            Graphics[Line[{{xr, maxi}, {xr, mini}}]], Axes → True,
            PlotLabel → "Entered function on given interval with upper and
               lower guesses and estimated root", TextStyle → {FontSize → 11}];
```



Check the interval between which the root lies. Does it lie in (x_l,x_r) or (x_r,x_u)?

```
In[153]:= If[f[xr] * f[xu] ≤ 0, xl = xr, xu = xr];

In[154]:= xu

Out[154]= 0.0653125

In[155]:= xl

Out[155]= 0.061875
```

Store the value of the previous guess in x_p to calculate absolute relative approximate error.

```
In[156]:= xp = xr;
```